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A chance-constrained programming approach to optimal management of car-rental fleets of electric vehicles

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ABSTRACT

In the current context of growing electrification of the transport sector, offering rental and sharing programs for electric vehicles is considered one of the strategies to achieve decarbonization targets. Such programs should be supported by suitable optimization tools to manage the vehicle fleet, and make rental provision profitable for its operator. In this paper, we consider a rental system having a single station for electric vehicle pickup and delivery. For this system, we address the operational problem of simultaneously assigning rental requests to vehicles and determining the charging policies during inactivity intervals. The objective is to maximize the profit for the operator by minimizing the costs for electricity. The considered problem is complicated by uncertainty regarding the battery energy level when a vehicle returns to the station. This leads to a chance-constrained programming formulation, where the request-to-vehicle assignment and charging policies are determined by minimizing electricity costs while ensuring that the energy demand of the served requests is met with a prescribed high probability. Since the formulated mixed-integer problem with probabilistic constraints is hard to solve, a suboptimal approach is proposed, consisting of two sequential steps. In the first step, request-to-vehicle assignment is accomplished via a suitably designed heuristic procedure. Then, for a given assignment, the charging policy of each vehicle is determined by solving a relaxed chance-constrained problem. Numerical results are presented to assess the performance of both the assignment procedure and the optimization problem which determines the electric vehicle charging policies.

1. Introduction

Transport electrification is one of the main solutions to tackle climate change by lowering carbon emissions [1]. It has been estimated that 16.2% of global greenhouse gas emissions in 2016 came from the transportation sector, with road transport (mostly passenger travel) accounting for three quarters of the total transportation emissions [2]. Vehicle electrification is thus regarded as a key pathway towards global sustainable energy transition. It involves replacing fossil-fueled personal cars, commercial fleets of cars and trucks, and public transport like buses and trains, with ones powered by electricity. This process has accelerated over the past decade [3], with a growing number of countries worldwide making commitments to electrify their fleets of vehicles, and adapting their legislation to foster increased use of electric vehicles (EVs). Combined with advancement in battery technology, these supportive government efforts have made EVs more and more popular. Recent projections estimate that EVs will account for 29% to 54% of global new vehicle sales by 2050, reaching cumulative sales between 465 million and 832 million battery EVs, as well as between 218 million and 241 million plug-in hybrid EVs over the period 2022–2050 [4].

In this context of growing EV penetration, offering EV rental and sharing programs is considered among the strategies to promote EV adoption [5]. This calls for suitable optimization tools to manage the EV fleet, and to make the EV rental provision profitable for its operator. Just to mention a specific issue addressed in this paper, EVs require significant charging times. This implies forced inactivity before a vehicle becomes again available. Optimizing inactivity intervals becomes crucial to maximize vehicles utilization, and to ensure economic sustainability of the rental business, also in view of the high purchase cost of EVs. On the other hand, EVs could be advantageous to car rental companies because they require lower maintenance costs, and they could have a marketing effect associated with improved sustainability of urban mobility [6].

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Car-rental and sharing systems have been extensively studied in the operations research literature, since they offer a wide range of decision problems and challenges to cope with. The interested reader is referred to [7,8] for comprehensive reviews on optimization of car-sharing systems, with particular focus on how the related decision problems are mathematically formulated and solved. Since the considered problems often involve discrete decisions, a large number of contributions propose mixed-integer programming (MIP) formulations, that are then tackled via suitable solution techniques, such as Benders or Dantzig–Wolfe decomposition and branch-and-cut algorithms.

While most of the literature on car-sharing systems considers fleets of conventional vehicles, recent years have witnessed an increasing number of contributions where EVs come into play. This is needed to understand the impact that the special characteristics of EVs, in particular the usually limited driving range and, consequently, the need of being recharged, might have on car-sharing planning and operation. Most of the contributions addressing EV fleets focus on fleet size and station location optimization [9–11], vehicle relocation optimization [12–17], or several of the aforementioned decision problems simultaneously [18–20]. Some papers consider mixed fleets of electric and conventional vehicles [6] or heterogeneous EVs [21].

Lots of contributions in the literature on car-sharing systems assume a deterministic setting, where all the problem parameters are known. However, several uncertainties affect the EV sharing decision problems in practice, for instance stochastic demand, car imbalance, parking shortage, and battery state-of-charge (SOC). This motivates the development of stochastic approaches, where uncertainty is accounted for and managed in the formulated problems. In [20], the authors present a method to determine the deployment of one-way electric car-sharing services within a designated region with the objective to maximize the total profit for the operator. Uncertainty on the SOC of EVs parked at one station at a given time is modeled by a continuous probability distribution whose mean changes over time. The deployment of a one-way EV sharing system that serves an urban area is considered also in [22]. Long-term infrastructure planning (charging station location and fleet distribution) and real-time fleet operations (relocation and charging decisions) are jointly optimized under time-varying uncertain demand. To this aim, a multistage stochastic model is proposed. The optimal allocation of a fleet of plug-in EVs to the stations of an EV-sharing system is addressed in [10]. The objective is to maximize the profit of the system operator. A multi-layer time-space network flow technique is adopted to describe the movement of EVs in the system. The study applies robust optimization and chance-constrained techniques to deal with the fleet deployment problem under uncertain, stochastic demand. Optimal placement and type selection of EV charging stations is addressed in [11], considering routing selection and charging management of an EV fleet in a car-sharing business. A stochastic programming framework is exploited to incorporate a variety of operating scenarios, which reflect uncertainties on drivers' itineraries, traffic flow information, and electricity prices. A two-stage stochastic programming formulation is adopted in [15] to tackle the vehicle deployment and relocation problem for hybrid one-way station-based and free-floating EV sharing systems, considering demand and parking space stochasticity. All these examples witness the wide variety of decision problems affected by different types of uncertainties in the context of EV rental and sharing systems, and highlight the potential of stochastic programming techniques [23] to cope successfully with those uncertainties.

In this paper, we consider a rental system hosting a fleet of EVs. The rental system has a single station, and it offers a *round-trip* service, i.e. the station of pickup and that of delivery coincide. The focus of the proposed study is to address the optimal operation problem for the EV rental system. To achieve this goal, two key objectives are identified:

1. Minimize the electricity cost by assigning rental requests to vehicles and determining the charging policy for each EV during inactivity intervals;
2. Handle the uncertainty on rental demand within a stochastic framework.

Although the considered setting might appear oversimplified (single station, round-trip service), it allows one to get valuable insights into the impact of demand uncertainty combined with the peculiar characteristics of EVs. These refer to the need of being recharged and the temporal and operational constraints of the charging process. In this work, we consider uncertainty on the battery energy level when an EV returns to the station. More in detail, we assume that the operator receives rental requests for the next day. Each request is characterized by the pickup time and the delivery time, which are considered to be fixed, and by a number of parameters (e.g., expected mileage, road type, etc.) that, from the modeling point of view, are translated into a probability distribution for the energy required by the request. Given a set of requests, the operator has to determine the request-to-vehicle assignment and the charging policy for each EV on a day-ahead basis. To take into account energy demand uncertainty, this problem is formulated in a chance-constrained programming framework. The request-to-vehicle assignment and the EV charging policies are computed by minimizing the costs for electricity, while guaranteeing that the energy demand of the served requests is met with high probability. This probability is computed with respect to the energy demand probability distributions. The formulated problem turns out to be hard to solve, being a MIP problem (due to integer variables needed for the request-to-vehicle assignment) with probabilistic constraints. For this reason, to cope with the computational burden, a two-step suboptimal approach is proposed, inspired by the inner two-stage structure of the problem. The main paper contributions can therefore be summarized as follows.

1. An efficient heuristic procedure exploiting the problem structure is designed to accomplish the request-to-vehicle assignment. This is the first step of the proposed two-step approach. Necessary and sufficient conditions to quickly check that a given assignment is feasible, are also presented. An assignment is said to be feasible if it ensures feasibility of the problem to be solved at the second step.
2. For a given feasible request-to-vehicle assignment, a chance-constrained problem is formulated to determine the charging policy of each EV. This is the second step of the proposed two-step approach. The problem complexity is managed by means of the Bonferroni inequality, making it possible to approximate a joint chance constraint with a set of individual chance constraints. Then, it is shown that the obtained stochastic formulation can be converted into a linear program by exploiting the Value at Risk (VaR) indicator.
3. Extensive numerical results are reported to assess the good performance of both the assignment procedure and the optimization problem that determines the EV charging policies, thus making the proposed approach suitable for real-world applications.

It is worth highlighting that a similar two-step approach is taken in [20] to tackle a planning problem related to car-sharing services. There, a large-scale MIP model is built, which is then split into two subproblems to handle the computational burden: a strategic planning level that decides the fleet size and the station capacity, and an operational level that decides the required relocation operations.

The paper is structured as follows. Section 2 introduces the notation and the problem formulation. Section 3 describes the optimization problem to determine the EV charging policies given a feasible request-to-vehicle assignment, both in a deterministic and stochastic setting. A heuristic assignment procedure is then presented in Section 4, together with necessary and sufficient conditions to check the feasibility of a given assignment. Numerical results are illustrated in Section 5, and conclusions are drawn in Section 6.

2. Notation and problem formulation

We consider an EV rental system (EVRS) having a single charging station, whose charging units are used to charge a set $\mathcal{V}$ of EVs available for rental. The EVRS offers a round-trip service. Every day the EVRS receives a set $\mathcal{H}$ of vehicle requests for the next day. For each request $h \in \mathcal{H}$, the pickup time t_h^p and the delivery time t_h^d are specified. Let E_h denote the (uncertain) amount of energy needed by the request. As described in Section 1, the operational problem for the EV rental system is split into two subproblems.

First, a request-to-vehicle assignment is determined (if possible), enabling the fulfillment of the whole set of requests. If request h is assigned to vehicle v, it is considered fulfilled if the amount of energy stored in the battery of vehicle v at time t_h^p is greater than or equal to E_h.

Second, for a given request-to-vehicle assignment, the EV charging policies are computed to minimize the electricity cost, while guaranteeing that the energy demand of the whole set of requests is met with high probability.

In the following, $\mathcal{P}(A)$ denotes the probability of event A. We consider a discrete-time setting, where each day is divided into T time intervals. Let $\mathcal{T} = \{0,\dots,T-1\}$ be the set of time intervals for one day. The set of requests assigned to vehicle $v \in \mathcal{V}$ is denoted by $\mathcal{H}_v$. Moreover, we refer to $h_i^v \in \mathcal{H}_v$ as the ith request assigned to vehicle v sorted by pickup time, i.e., $t^p_{h_1^v} < t^p_{h_2^v} < \cdots < t^p_{h_{m_v}^v}$, where m_v is the cardinality of $\mathcal{H}_v$. To streamline the presentation, we will drop the dependency of h_i^v on v when it is clear from the context. The following table contains the list of main symbols used throughout the paper.

Symbol	Description
Δ_T	Sampling time
$\mathcal{T}$	Set of time instants of the day
ε	Probability level of constraint violation
$\mathcal{V}$	Set of electric vehicles
$\mathcal{H}$	Set of requests
$\mathcal{H}_v$	Set of requests assigned to vehicle v
h_i^v	ith request assigned to vehicle v
$t^p_{h_i^v}$	Pickup time of request h_i^v
$t^d_{h_i^v}$	Delivery time of request h_i^v
$E_{h_i^v}$	Energy required by request h_i^v
$\hat{E}_{h_i^v}$	Estimated energy required by request h_i^v
$S_v(t)$	Battery energy level of vehicle v at time t
$P_v(t)$	Average charging power of vehicle v over $[t, t+1)$
η_v	Battery charging efficiency of vehicle v
$\overline{S}_v$	Battery capacity of vehicle v
$\overline{P}_v$	Maximum charging power of vehicle v
$\lambda_{h_i^v}$	Energy charged into vehicle v from $t^d_{h_{i-1}^v}$ to $t^p_{h_i^v}$
$\gamma_{h_i^v}$	Maximum between $\hat{E}_{h_i^v}$ and $\lambda_{h_i^v}$
$\overline{E}_{h_i^v}(\alpha)$	Value at Risk for $\sum_{j=1}^{i-1} E_{h_j^v}$ of level $1-\alpha$
$\underline{E}_{h_i^v}(\alpha)$	Value at Risk for $-\sum_{j=1}^{i-1} E_{h_j^v}$ of level $1-\alpha$
$\pi(t)$	Electricity price at time t
$\overline{\pi}$	Peak power price

3. EV charging policies

In this section, we describe the optimization problem to compute the EV charging policies for a given request-to-vehicle assignment, both in a deterministic setting where the energy demand of each request is assumed to be known, and in a stochastic setting where it is described by a random variable with known probability distribution.

3.1. Deterministic case

To model the EV battery dynamics, for a given vehicle $v \in \mathcal{V}$, let $S_v(t)$ denote the battery energy level at time $t \in \mathcal{T} \cup \{T\}$, and $P_v(t)$ the average charging power between time t and $t+1$ for $t \in \mathcal{T}$. Moreover, for each request $h \in \mathcal{H}$ let us define the following binary coefficient:

$$\sigma_h(t) = \begin{cases} 1 & \text{if } t = t_h^d \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

being equal to 1 only at the delivery time of request h. Thus, for each $v \in \mathcal{V}$ and $t \in \mathcal{T}$, the battery energy level satisfies

$$S_v(t+1) = S_v(t) + \Delta_T \eta_v P_v(t) - \sum_{h_i \in \mathcal{H}_v} \sigma_{h_i}(t+1) E_{h_i}, \tag{2}$$

where Δ_T is the time step and η_v denotes the battery charging efficiency. In (2), if request h_i is assigned to vehicle v, the energy level of the battery of vehicle v is reduced by E_{h_i} at time $t^d_{h_i}$, when the vehicle returns to the charging station.

Concerning technical specifications for the battery of vehicle v, the battery energy level and charging power must not exceed the battery capacity $\overline{S}_v$ and the maximum power $\overline{P}_v$, respectively. Thus, the following constraints are enforced for all $t \in \mathcal{T}$:

$$0 \le S_v(t) \le \overline{S}_v \tag{3}$$

$$0 \le P_v(t) \le \overline{P}_v. \tag{4}$$

In addition, at the end of the day it is further imposed that the battery energy level for each vehicle v must be at least a specified quantity S_v^f. Hence,

$$S_v^f \le S_v(T) \le \overline{S}_v. \tag{5}$$

Finally, to satisfy a request h_i, two conditions must be met:

- the battery energy level for vehicle v must be at least E_{h_i} at time $t^p_{h_i}$;
- the battery of vehicle v cannot be charged when the vehicle is out of the station, i.e., during the time interval from $t^p_{h_i}$ to $t^d_{h_i} - 1$.

Such conditions are ensured by considering the following constraints for all $h_i \in \mathcal{H}_v$:

$$S_v(t^p_{h_i}) \ge E_{h_i} \tag{6}$$

$$P_v(t) = 0, \qquad \forall t \in \{t^p_{h_i}, \dots, t^d_{h_i} - 1\}. \tag{7}$$

3.2. Stochastic case

Assume now that the energy demand E_{h_i} of each request is uncertain, and described by a random variable with known probability distribution. Random variables corresponding to different requests are assumed to be independent.

Due to the stochasticity of E_{h_i}, which translates into the stochasticity of $S_v(t)$ through (2), constraints (3) and (6) can be satisfied only in probability. For this reason, the following chance constraints are imposed for all $h_i \in \mathcal{H}_v$, $v \in \mathcal{V}$:

$$\mathcal{P}\begin{pmatrix} S_v(t^p_{h_i}) \ge \hat{E}_{h_i} & \\ S_v(t) \ge 0 & \forall t = t^d_{h_{i-1}}, \dots, t^p_{h_i} \\ S_v(t) \le \overline{S}_v & \forall t = t^d_{h_{i-1}}, \dots, t^p_{h_i} \end{pmatrix} \ge 1 - \varepsilon, \tag{8}$$

where $\varepsilon > 0$ is a given probability level, $t^p_{h_0}$, $t^d_{h_0}$ and $\hat{E}_{h_0}$ are set to 0, $S_v(t)$ evolves according to (2), and $\hat{E}_{h_i}$ is the percentile of rank $1-\beta$ of the probability distribution of E_{h_i}, i.e., the minimum value $\hat{E}_{h_i}$ such that

$$\mathcal{P}\left(E_{h_i} \le \hat{E}_{h_i}\right) \ge 1 - \beta, \tag{9}$$

with $\beta > 0$ a given probability level.

Concerning (5), for each vehicle v a chance-constraint on the final battery energy level is further imposed:

$$\mathcal{P}\begin{pmatrix} S_v(T) \geq S_v^f & \\ S_v(t) \geq 0 & t = t^d_{h_{m_v}}, \dots, T \\ S_v(t) \leq \overline{S}_v & t = t^d_{h_{m_v}}, \dots, T \end{pmatrix} \geq 1 - \varepsilon. \tag{10}$$

Note that by adding to each vehicle a final fictitious request h_{m_v+1} where $t^d_{h_{m_v+1}} = t^p_{h_{m_v+1}} = T$ and $\widehat{E}_{h_{m_v+1}} = S_v^f$, (10) can be reformulated as (8). Hence, from now on, for each vehicle v we will consider an extended set of requests composed of $\mathcal{H}_v \cup \{h_{m_v+1}\}$, where the last request represents the final condition.

Constraints defined in (8) are joint chance-constraints, that are typically hard to handle. In order to derive a tractable formulation, note that a vehicle can be only charged during inactivity intervals, i.e., when it is plugged-in at the charging station. This implies that the battery energy level cannot decrease over an inactivity interval, and it is sufficient to check the left-hand inequality of (3) only at the beginning of the interval, and the right-hand inequality only at the end. Hence, (8) is equivalent to:

$$\mathcal{P}\begin{pmatrix} S_v(t^p_{h_i}) \geq \widehat{E}_{h_i} \\ S_v(t^d_{h_{i-1}}) \geq 0 \\ S_v(t^p_{h_i}) \leq \overline{S}_v \end{pmatrix} \geq 1 - \varepsilon \quad \forall h_i \in \mathcal{H}_v \cup \{h_{m_v+1}\}, \forall v \in \mathcal{V}. \tag{11}$$

Let λ_{h_i} be the energy charged into vehicle v between two consecutive requests h_{i-1} and h_i, i.e.

$$\lambda_{h_i} = \sum_{t=t^d_{h_{i-1}}}^{t^p_{h_i}-1} \Delta_T \eta_v P_v(t) \quad \forall h_i \in \mathcal{H}_v \cup \{h_{m_v+1}\}, \forall v \in \mathcal{V}. \tag{12}$$

Then, when a vehicle leaves the charging station, the battery energy level can be written as

$$S_v(t^p_{h_i}) = \lambda_{h_i} + S_v(t^d_{h_{i-1}}) \quad \forall h_i \in \mathcal{H}_v \cup \{h_{m_v+1}\}, \forall v \in \mathcal{V}, \tag{13}$$

and hence the second inequality in (11) can be written as

$$S_v(t^p_{h_i}) \geq \lambda_{h_i}. \tag{14}$$

Let us now define γ_{h_i} as

$$\gamma_{h_i} = \max\left\{\widehat{E}_{h_i}, \lambda_{h_i}\right\} \quad \forall h_i \in \mathcal{H}_v \cup \{h_{m_v+1}\}, \forall v \in \mathcal{V}. \tag{15}$$

Then, by exploiting (14)–(15), (11) can be rearranged as follows

$$\mathcal{P}\begin{pmatrix} S_v(t^p_{h_i}) \geq \gamma_{h_i} \\ S_v(t^p_{h_i}) \leq \overline{S}_v \end{pmatrix} \geq 1 - \varepsilon \quad \forall h_i \in \mathcal{H}_v \cup \{h_{m_v+1}\}, \forall v \in \mathcal{V}. \tag{16}$$

Since the probability in (16) involves two joint events, the Bonferroni inequality [24] can be exploited to derive the final formulation. In particular, (16), and hence (8), is satisfied if the following inequalities hold:

$$\mathcal{P}\left(S_v(t^p_{h_i}) \geq \gamma_{h_i}\right) \geq 1 - \varepsilon_1 \quad \forall h_i \in \mathcal{H}_v \cup \{h_{m_v+1}\}, \forall v \in \mathcal{V}, \tag{17}$$

$$\mathcal{P}\left(S_v(t^p_{h_i}) \leq \overline{S}_v\right) \geq 1 - \varepsilon_2 \quad \forall h_i \in \mathcal{H}_v \cup \{h_{m_v+1}\}, \forall v \in \mathcal{V}, \tag{18}$$

where ε_1, ε_2 are chosen positive parameters such that $\varepsilon_1 + \varepsilon_2 = \varepsilon$.

3.3. Chance-constraint feasible set

In order to characterize the charging power profiles $P_v(t)$ satisfying (17)–(18), let us introduce the following two quantities for each request h_i assigned to vehicle v:

$$\begin{aligned} \overline{E}_{h_i}(\alpha) = &\min E \\ &\text{s.t. } \mathcal{P}\left(\sum_{j=1}^{i-1} E_{h_j} \leq E\right) \geq 1 - \alpha \end{aligned} \tag{19}$$

$$\begin{aligned} \underline{E}_{h_i}(\alpha) = &\max E \\ &\text{s.t. } \mathcal{P}\left(\sum_{j=1}^{i-1} E_{h_j} \geq E\right) \geq 1 - \alpha, \end{aligned} \tag{20}$$

where α is a given probability level. The quantities in (19)–(20) are commonly known as VaR of level $1-\alpha$ [25]. It is stressed that the probability distribution of $\sum_{j=1}^{i-1} E_{h_j}$ can be computed by performing the convolution of the distributions of the single E_{h_j}, since these are assumed to be independent.

To derive an equivalent formulation of (17)–(18), it is worthwhile to show that (13) can be written as

$$\begin{aligned} S_v(t^p_{h_i}) &= \lambda_{h_i} + S_v(t^d_{h_{i-1}}) \\ &= \lambda_{h_i} - E_{h_{i-1}} + S_v(t^p_{h_{i-1}}) \\ &\dots \\ &= \sum_{j=1}^{i} \lambda_{h_j} - \sum_{j=1}^{i-1} E_{h_j} + S_v(0) \quad \forall h_i \in \mathcal{H}_v \cup \{h_{m_v+1}\}, \forall v \in \mathcal{V}. \end{aligned}$$

Using the above expression for $S_v(t^p_{h_i})$ and after some straightforward manipulations, (17) and (18) are equivalent to the following set of constraints:

$$\mathcal{P}\left(\sum_{j=1}^{i} \lambda_{h_j} + S_v(0) - \gamma_{h_i} \geq \sum_{j=1}^{i-1} E_{h_j}\right) \geq 1 - \varepsilon_1 \quad \forall h_i \in \mathcal{H}_v \cup \{h_{m_v+1}\}, \forall v \in \mathcal{V},$$

$$\mathcal{P}\left(\sum_{j=1}^{i} \lambda_{h_j} + S_v(0) - \overline{S}_v \leq \sum_{j=1}^{i-1} E_{h_j}\right) \geq 1 - \varepsilon_2 \quad \forall h_i \in \mathcal{H}_v \cup \{h_{m_v+1}\}, \forall v \in \mathcal{V},$$

where λ_{h_j} and γ_{h_i} are values computed from $P_v(t)$ according to (12) and (15). Now notice that, if $\mathcal{P}\left(\sum_{j=1}^{i} \lambda_{h_j} + S_v(0) - \gamma_{h_i} \geq \sum_{j=1}^{i-1} E_{h_j}\right) \geq 1-\varepsilon_1$, then $\sum_{j=1}^{i} \lambda_{h_j} + S_v(0) - \gamma_{h_i} \geq \overline{E}_{h_i}(\varepsilon_1)$ by definition of $\overline{E}_{h_i}(\varepsilon_1)$ in (19). Vice versa, if $\sum_{j=1}^{i} \lambda_{h_j} + S_v(0) - \gamma_{h_i} \geq \overline{E}_{h_i}(\varepsilon_1)$, then exploiting the fact that a probability distribution is nondecreasing and (19), one gets:

$$\mathcal{P}\left(\sum_{j=1}^{i} \lambda_{h_j} + S_v(0) - \gamma_{h_i} \geq \sum_{j=1}^{i-1} E_{h_j}\right) \geq \mathcal{P}\left(\overline{E}_{h_i}(\varepsilon_1) \geq \sum_{j=1}^{i-1} E_{h_j}\right) \geq 1 - \varepsilon_1.$$

Using similar arguments and (20), one also obtains that $\sum_{j=1}^{i} \lambda_{h_j} + S_v(0) - \overline{S}_v \leq \underline{E}_{h_i}(\varepsilon_2)$ and $\mathcal{P}\left(\sum_{j=1}^{i} \lambda_{h_j} + S_v(0) - \overline{S}_v \leq \sum_{j=1}^{i-1} E_{h_j}\right) \geq 1 - \varepsilon_2$ are equivalent conditions. Hence, constraints (17)–(18) can be equivalently reformulated as follows:

$$\sum_{j=1}^{i} \lambda_{h_j} + S_v(0) - \gamma_{h_i} \geq \overline{E}_{h_i}(\varepsilon_1) \qquad \forall h_i \in \mathcal{H}_v \cup \{h_{m_v+1}\}, \forall v \in \mathcal{V}, \tag{21}$$

$$\sum_{j=1}^{i} \lambda_{h_j} + S_v(0) - \overline{S}_v \leq \underline{E}_{h_i}(\varepsilon_2) \qquad \forall h_i \in \mathcal{H}_v \cup \{h_{m_v+1}\}, \forall v \in \mathcal{V}. \tag{22}$$

3.4. Optimization problem

For a given request-to-vehicle assignment, the optimal EV charging profiles $P_v(t)$ need to be computed. We assume that the cost incurred by the charging station is composed of two terms, i.e., the electricity cost and the peak power cost. Then, the objective function of the formulated problem can be defined as follows:

$$\sum_{t\in\mathcal{T}} \sum_{v\in\mathcal{V}} \Delta_T P_v(t)\pi(t) + \max_{t\in\mathcal{T}} \left\{\sum_{v\in\mathcal{V}} P_v(t)\right\} \overline{\pi}, \tag{23}$$

where $\pi(t)$ and $\overline{\pi}$ are the electricity and the peak power prices, respectively. To get rid of the max operator in (23), we replace $\max_{t\in\mathcal{T}} \left\{\sum_{v\in\mathcal{V}} P_v(t)\right\}$ with a new variable Ψ, defined by the constraints:

$$\Psi \geq \sum_{v\in\mathcal{V}} P_v(t), \quad \forall t \in \mathcal{T}. \tag{24}$$

Then, the optimization problem providing the optimal EV charging profiles is the following linear program:

Problem 1.

$$\min_{P,\Psi} \quad \sum_{t\in\mathcal{T}}\sum_{v\in\mathcal{V}} \Delta_T P_v(t)\pi(t) + \Psi\overline{\pi}$$
$$\text{s.t.} \quad (4),(12),(15),(21),(22),$$
$$\Psi \geq \sum_{v\in\mathcal{V}} P_v(t), \quad \forall t\in\mathcal{T}.$$

In this formulation, $\boldsymbol{P}$ is a matrix collecting the charging powers $P_v(t)$, where $P_v(t)$ is the entry in position (v,t) of $\boldsymbol{P}$.

4. Assignment procedure

All the models presented in the previous section have been obtained by considering a given request-to-vehicle assignment. However, in real-world scenarios the EVRS has to choose which vehicle should be assigned to each request. If one considers a deterministic setup, then it is possible to formulate a mixed-integer linear program to compute the optimal assignment [26]. On the other hand, adopting the same kind of approach in a stochastic framework is intractable, since one has to compute $\overline{E}_{h_i^v}(\epsilon_1)$ and $\underline{E}_{h_i^v}(\epsilon_2)$ for all the possible request-to-vehicle assignments. Clearly, this is a combinatorial problem that becomes prohibitive to solve even for a small number of EVs and requests. Thus, in this section we present a heuristic solution to the problem of assigning an EV to each request.

First, the requests are sorted by their pickup time and they are processed sequentially in that order. Then, the proposed heuristic builds the final assignment incrementally for all h, v. At a given iteration, a new request h is assigned to a vehicle v. In doing so, it must be guaranteed that the resulting assignment is a feasible assignment. Specifically, two conditions must be met. First, the request h must not overlap in time with the previous requests already assigned to vehicle v. Second, there must exist a feasible charging profile $P_v(t)$ such that conditions (4), (7), (21) and (22) are satisfied.

In general, at a given iteration there exist multiple EVs that can be assigned to request h without violating any of the above conditions (candidate vehicles). In this case, the proposed method attempts to assign h to the most promising candidate vehicle, by maximizing the time that a vehicle has at its disposal to recharge its battery before request h begins. For each candidate vehicle, the *availability time* is computed as the maximum of the delivery times of the requests already assigned to that vehicle. Then, request h is assigned to the candidate vehicle having the earliest availability time. Intuitively, this heuristic tries to maximize the time between two consecutive requests that must be served by the same vehicle. This choice aims at introducing more flexibility in the charging schedule problem to be solved afterwards, thus obtaining better results.

However, checking that (4), (7), (21) and (22) are feasible for the vehicle v with a given request set $\mathcal{H}_v$ requires the solution of a linear program. To speed up such computation, in the next section, necessary and sufficient conditions to quickly check the feasibility of a given assignment will be introduced.

4.1. Assignment feasibility

Suppose that a request-to-vehicle assignment is given. The feasibility of the assignment is defined as follows.

Definition 1. A request-to-vehicle assignment is said feasible if there exists an EV power schedule such that (4), (7), (21) and (22) are satisfied.

Since no coupling constraints between vehicles are present, the conditions establishing the feasibility of the assignment will be provided by considering a given vehicle v, and then one can extend the reasoning for the whole set of vehicles.

Proposition 1. *If $0 \leq \lambda_{h_i} \leq \Delta_T\eta_v\overline{P}_v\tau_{h_i}$, where $\tau_{h_i} = t^p_{h_i} - t^d_{h_{i-1}}$, $t^d_0 = 0$, then there exist $P_v(t)$ satisfying (4) and (7).*

Proof. Define

$$P_v(t) = \begin{cases} \lambda_{h_i}/\tau_{h_i} & \text{if } t = t^d_{h_{i-1}},\dots,t^p_{h_i}-1 \\ 0 & \text{else.}\end{cases}$$

Clearly, $P_v(t)$ satisfies (7). Moreover, from $0 \leq \lambda_{h_i} \leq \Delta_T\eta_v\overline{P}_v\tau_{h_i}$, $P_v(t)$ satisfies (4) as well. □

From the feasibility point of view, constraints (4), (7), (21) and (22) are equivalent to the following set of inequalities:

$$\lambda_{h_i} \geq 0 \tag{25}$$
$$\lambda_{h_i} \leq \Delta_T\eta_v\overline{P}_v\tau_{h_i} \tag{26}$$
$$\sum_{j=1}^{i}\lambda_{h_j} + S_v(0) - \gamma_{h_i} \geq \overline{E}_{h_i}(\epsilon_1) \tag{27}$$
$$\sum_{j=1}^{i}\lambda_{h_j} + S_v(0) - \overline{S}_v \leq \underline{E}_{h_i}(\epsilon_2) \tag{28}$$

where (27) and (28) coincide with (21),(22) and are rewritten here for convenience. From (15), the previous constraints can be rewritten as:

$$\lambda_{h_i} \geq 0 \tag{29}$$
$$\lambda_{h_i} \leq \Delta_T\eta_v\overline{P}_v\tau_{h_i} \tag{30}$$
$$\lambda_{h_i} \geq \overline{E}_{h_i}(\epsilon_1) - \sum_{j=1}^{i-1}\lambda_{h_j} - S_v(0) + \hat{E}_{h_i} \tag{31}$$
$$\sum_{j=1}^{i-1}\lambda_{h_j} \geq \overline{E}_{h_i}(\epsilon_1) - S_v(0) \tag{32}$$
$$\lambda_{h_i} \leq \underline{E}_{h_i}(\epsilon_2) - \sum_{j=1}^{i-1}\lambda_{h_j} - S_v(0) + \overline{S}_v, \tag{33}$$

where (27) is equivalent to (31)–(32), and (28) is equivalent to (33). It is worthwhile to note that if there exists a sequence of λ_{h_i} satisfying (29)–(33), then according to Proposition 1 there exists a power schedule satisfying (4), (7), (21) and (22). Now, let us introduce the following quantities:

$$x_{h_i} = \sum_{j=1}^{i}\lambda_{h_j} \qquad i\in\{1,\dots,m_v+1\} \tag{34}$$
$$c_{h_{i-1}} = \Delta_T\eta_v\overline{P}_v\tau_{h_i} \qquad i\in\{1,\dots,m_v+1\} \tag{35}$$
$$\underline{x}_{h_i} = \max\left\{\overline{E}_{h_i}(\epsilon_1) - S_v(0) + \hat{E}_{h_i},\ \overline{E}_{h_{i+1}}(\epsilon_1) - S_v(0)\right\} \qquad i\in\{1,\dots,m_v\} \tag{36}$$
$$\overline{x}_{h_i} = \underline{E}_{h_i}(\epsilon_2) - S_v(0) + \overline{S}_v \qquad i\in\{1,\dots,m_v+1\} \tag{37}$$
$$x_{h_0} = \underline{x}_{h_0} = \overline{x}_{h_0} = 0 \tag{38}$$
$$\underline{x}_{h_{m_v+1}} = \overline{E}_{h_{m_v+1}}(\epsilon_1) - S_v(0) + \hat{E}_{h_{m_v+1}}. \tag{39}$$

The above quantities are instrumental to streamline the mathematical derivations. In particular, for the battery of vehicle v, x_{h_i} represents the energy charged up to time $t^p_{h_i}$ and $c_{h_{i-1}}$ denotes the maximum energy that can be charged from $t^d_{h_{i-1}}$ to $t^p_{h_i}$, while $\underline{x}_{h_i}$ and $\overline{x}_{h_i}$ are lower and upper bounds to x_{h_i}, respectively. Using the quantities defined in (34)–(39), the system of inequalities (29)–(33) can be reformulated as stated in the following lemma.

Lemma 1. *Let c_{h_i}, $\underline{x}_{h_i}$, $\overline{x}_{h_i}$ and x_{h_i} be defined as in (34)–(39), then (29)–(33) can be written in the following equivalent form:*

$$\underline{x}_{h_i} \leq x_{h_i} \leq \overline{x}_{h_i} \qquad \forall i\in\{0,\dots,m_v+1\} \tag{40}$$
$$x_{h_i} \leq x_{h_{i+1}} \leq x_{h_i} + c_{h_i} \qquad \forall i\in\{0,\dots,m_v\}. \tag{41}$$

The proof of Lemma 1 is reported in the appendix. In order to certify the feasibility of a given assignment, let us first introduce the next Lemma.

Lemma 2. *Consider the system of inequalities:*

$$\underline{x}_i \le x_i \le \overline{x}_i \qquad \forall i \in \{1,\dots,m\}$$
$$x_i \le x_{i+1} \le x_i + c_i \qquad \forall i \in \{1,\dots,m-1\},$$

where $\underline{x}_i$, $\overline{x}_i$, $c_i \ge 0$ and $m \ge 1$ are given parameters, and define the quantities:

$$\underline{w}_i = \max\left\{\max_{1\le k\le i} \underline{x}_k, \quad \underline{w}_{i+1} - c_i\right\} \tag{42}$$

$$\overline{w}_i = \min\left\{\min_{i\le k\le m} \overline{x}_k, \quad \overline{w}_{i-1} + c_{i-1}\right\}, \tag{43}$$

where $\overline{w}_1 = \min_{1\le k\le m} \overline{x}_k$ and $\underline{w}_m = \max_{1\le k\le m} \underline{x}_k$. Then, the system has nonempty intersection if and only if

$$\underline{w}_i \le \overline{w}_i \quad \forall i \in \{1,\dots,m\}.$$

The proof of Lemma 2 is given in the appendix. Thanks to Lemma 2, necessary and sufficient conditions to check the feasibility of a given assignment are provided by the following theorem.

Theorem 1. *Let an assignment $\hat{z}_{h,v}$ be given and consider the parameters defined in (34)–(39). Moreover, define*

$$\underline{w}_{h_i} = \max\left\{\max_{0\le k\le i} \underline{x}_{h_k}, \quad \underline{w}_{h_{i+1}} - c_{h_i}\right\}$$

$$\overline{w}_{h_i} = \min\left\{\min_{i\le k\le m_v+1} \overline{x}_{h_k}, \quad \overline{w}_{h_{i-1}} + c_{h_{i-1}}\right\},$$

where $\overline{w}_{h_1} = \min_{0\le k\le m_v+1} \overline{x}_{h_k}$ and $\underline{w}_{h_{m_v+1}} = \max_{0\le k\le m_v+1} \underline{x}_{h_k}$. Then, the assignment is feasible if and only if

$$\underline{w}_{h_i} \le \overline{w}_{h_i} \quad \forall i \in \{0,\dots,m_v+1\},\ \forall v \in \mathcal{V}.$$

Proof of Theorem 1. Since there is no constraint that couples EV charging, in order to check the feasibility of the assignment it is sufficient to check that (29)–(33) are satisfied for each vehicle separately. Thanks to Lemma 1, one can write the constraints for each EV in the form (40)–(41). Then, note that the constraint structure (40)–(41) fits the one introduced in Lemma 2. Hence, application of Lemma 2 concludes the proof. □

Remark 1. Theorem 1 allows one to efficiently check the feasibility of a new request. In fact, instead of solving an LP problem every time that a new request is assigned, the problem feasibility can be checked by exploiting an algorithm whose complexity grows linearly with respect to the total number of requests.

5. Numerical results

To assess the performance of the proposed approach, three setups are considered. First, the fulfillment of chance-constraints is validated by considering a system configuration where the charging station is equipped with only one vehicle. Second, the sensitivity of the assignment procedure is evaluated in a scenario involving different request-to-vehicle ratios. Third, the role of ε to trade-off cost performance and constraint violation is investigated in a setting where the number of requests and the number of vehicles are fixed. For all the simulations, parameters concerning the EV charging process are chosen as estimated average values of current technologies. Specifically, EV battery capacity is $\overline{S}_v = 50$ kWh, battery efficiency is $\eta_v = 0.9$, and maximum charging power is $\overline{P}_v = 22$ kW. The sampling time is set to 10 min and the peak price is set to 0.15 €/kW, while the electricity prices are taken from the Italian electricity market [27]. Concerning probability levels, values $\varepsilon_1 = \varepsilon_2 = \varepsilon/2$ and $\beta = 0.01$ are chosen. For each request h, the distribution of E_h is supposed to be a truncated Gaussian with mean $3.3\Delta_T(t_h^d - t_h^p)$ kWh and standard deviation set at 10% of its mean.

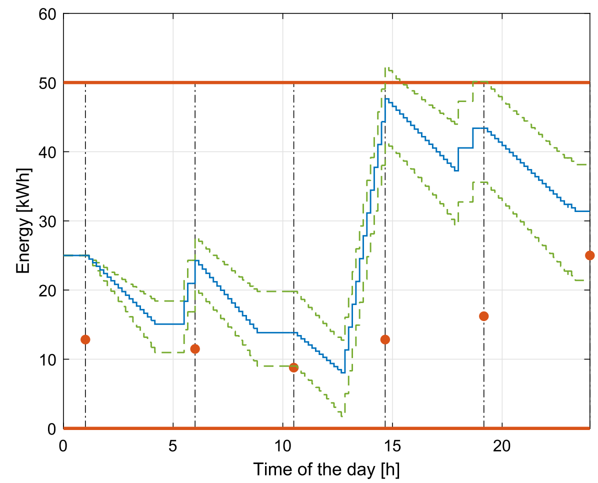

Fig. 1. Battery energy level profile for a simulation which satisfies all the constraints (blue solid), envelope of all the battery energy level profiles over $N = 10^4$ simulations (green dashed), capacity constraints (red solid), and request constraints (red dots). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

5.1. Chance-constraint validation

In this test, ε is set to 0.1 and five requests arranged in ten different configurations are designed. To generate the configurations, the pickup time of a request is supposed to be uniformly distributed over the set of values ranging from 3 to 12 time steps after the delivery time of the previous request, whereas the request length is sampled from a uniform distribution ranging from 2 hours to 4 hours. For each configuration, the EV charging profiles are computed a priori by solving Problem 1. Then, $N = 10^4$ simulations are run by applying the predetermined EV charging profiles under different realizations of energy demand for each request. The performance of the proposed approach is finally assessed by evaluating, for each request, the percentage of simulations in which the bounds on the battery energy level at pickup or delivery time are violated (percentage of violation). The largest percentage of violation over the considered requests and configurations is 5.49%, well below the 10% probability level of violation set by ε.

Figs. 1 and 2 refer to one of the configurations of the test. In both figures, the black dot-dashed lines represent the pickup times of the five requests, the red solid lines represent the energy bounds in (3), and the red dots represent the estimated energy required by each request, denoted $\hat{E}_{h_i}$ for request h_i. In Fig. 1, the blue solid line is a simulation for which the battery energy level stays always within the capacity bounds and is capable of satisfying all the requests and the terminal constraint (5). The figure also shows (green dashed lines) the maximum and minimum battery energy level for each time instant $t \in \mathcal{T}$ over the N simulations. Consistently with the probabilistic setting of this work, aiming at satisfying the constraints on the battery energy level with high probability, it happens that constraints are violated in a limited number of simulations, in particular around 3PM and at the end of the day. Two of these simulations are shown in Fig. 2. The magenta dashed line is the simulation which features the worst violation of the battery capacity limit (around 3PM), while the blue solid line is the simulation with the worst violation of the terminal constraint (5). In both cases, however, violations can be considered acceptable.

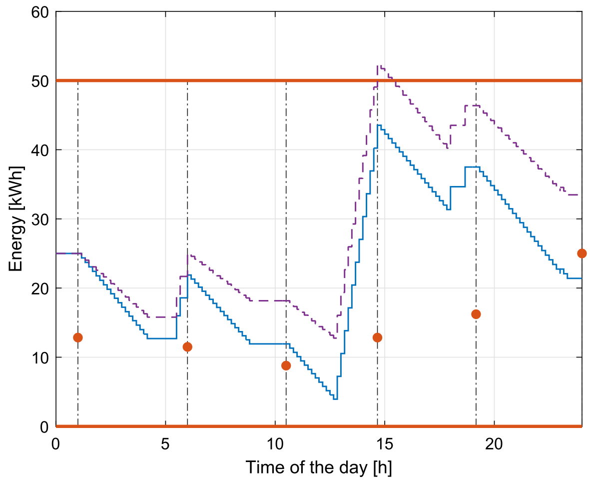

Fig. 2. Battery energy level profiles for a simulation which violates the terminal constraint (blue solid) and for another which violates the capacity constraint (magenta dashed), capacity constraints (red solid), and request constraints (red dots). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

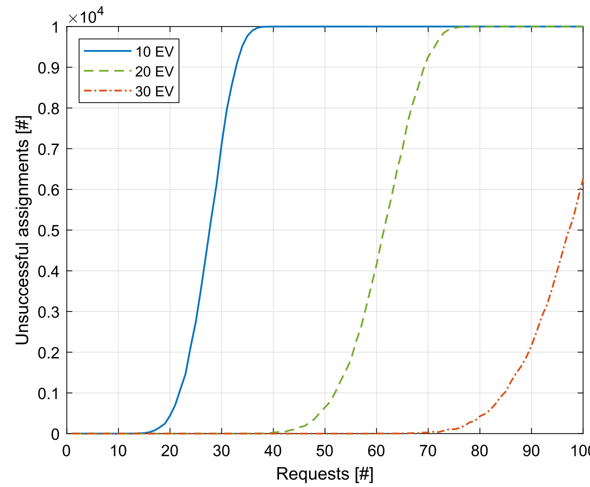

Fig. 3. Number of infeasible assignments out of $M = 10^4$ scenarios versus the number of requests for 10, 20, 30 vehicles.

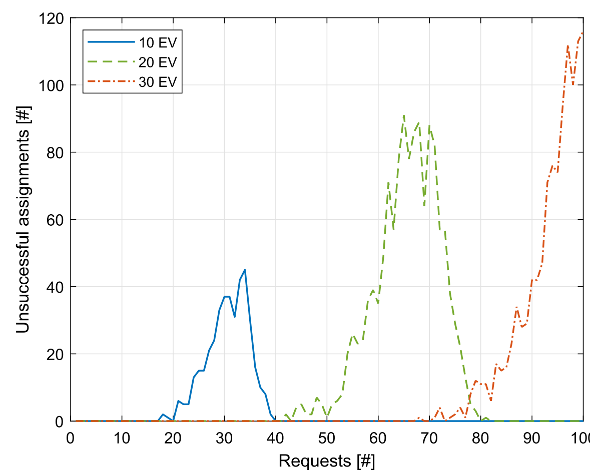

Fig. 4. Number of infeasible assignments out of the filtered scenarios versus the number of requests for 10, 20, 30 vehicles.

Table 1
Fraction of infeasible assignments.

# EVs \ # Requests	10	20	30	40	50
10	0	0.04	0.71	1	1
20	0	0	0	0.01	0.06
30	0	0	0	0	0
40	0	0	0	0	0
50	0	0	0	0	0

5.2. Assignment feasibility

To assess the performance of the assignment procedure, several setups with different request-to-vehicle ratios are analyzed. In particular, combinations of number of requests ranging from 0 to 100 and number of vehicles in the set $\{10, 20, \dots, 50\}$ are considered. For a fixed number of requests, $M = 10^4$ different request scenarios are generated. This is done by drawing the pickup time of each request (expressed in time steps) from a discrete uniform distribution over the set $\{0, 1, \dots, 102\}$, whereas the request duration is fixed and equal to 4 hours.

reports the fraction of times that the heuristic procedure of Section 4 returns an infeasible assignment (checked via Theorem 1) for several combinations of number of requests and numbers of vehicles. Plots of the number of infeasible assignments out of $M = 10^4$ scenarios versus the number of requests are shown in Fig. 3 for three different values of the number of vehicles. As expected, when the number of vehicles is greater than or equal to the number of requests, all the assignments are trivially feasible.

In the setup of this section, the proposed heuristic procedure may not provide a feasible assignment for two reasons. It may happen either that the heuristic procedure cannot find a solution even though at least one exists, or the request set does not admit a solution (e.g., when the number of requests that must be simultaneously satisfied is greater than the available number of EVs). In order to properly assess the performance of the proposed heuristic procedure, scenarios where the number of simultaneous requests exceeds the number of vehicles are excluded from the analysis. The remaining scenarios are referred to as *filtered scenarios*. Plots of the number of infeasible assignments out of the filtered scenarios versus the number of requests are shown in Fig. 4 for three different values of the number of vehicles. It can be observed that the number of infeasible assignments starts decreasing when the number of requests is approximately three times the number of EVs. This occurs because, for a large number of requests, the probability of generating a scenario where the number of simultaneous requests exceeds the number of available vehicles approaches 1. Hence, the number of filtered scenarios (and consequently the number of infeasible assignments) decreases to 0. Nevertheless, the heuristic procedure shows good assignment performance among the filtered scenarios. This can be observed in Fig. 5, showing the fraction of infeasible assignments to the total number of filtered scenarios versus the number of requests. This fraction represents a probability estimate of not finding a feasible assignment for request sets compatible with the number of vehicles (i.e., whenever the number of simultaneous requests does not exceed the number of EVs). It is worth noting that, as long as the number of requests is approximately less than three times the number of vehicles, the probability mentioned above is below 5%.

5.3. Sensitivity with respect to ε

The performance of the proposed chance-constrained optimization is evaluated for different values of the probability level ε. A setup with 20 vehicles and 50 requests is considered. Request sets are generated as described in Section 5.2, until one is found, for which the assignment returned by the heuristic procedure is feasible. After the assignment, several EV charging profiles are computed by solving Problem 1 for

Table 2
Performance indicators for the solutions provided by the stochastic and the deterministic optimization.

	Largest percentage of violation [%]	Largest violation [kWh]	Cost [€]
Stochastic ($\varepsilon = 0.01$)	0.68%	3.90	87.97
Stochastic ($\varepsilon = 0.05$)	2.92%	5.30	84.76
Stochastic ($\varepsilon = 0.10$)	5.70%	6.02	83.13
Stochastic ($\varepsilon = 0.15$)	8.36%	6.48	82.07
Deterministic	99.10%	9.77	73.91

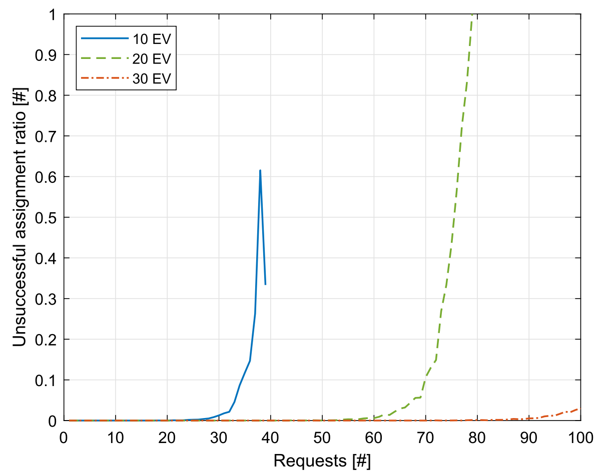

Fig. 5. Fraction of infeasible assignments to the total number of filtered scenarios versus the number of requests for 10, 20, 30 vehicles.

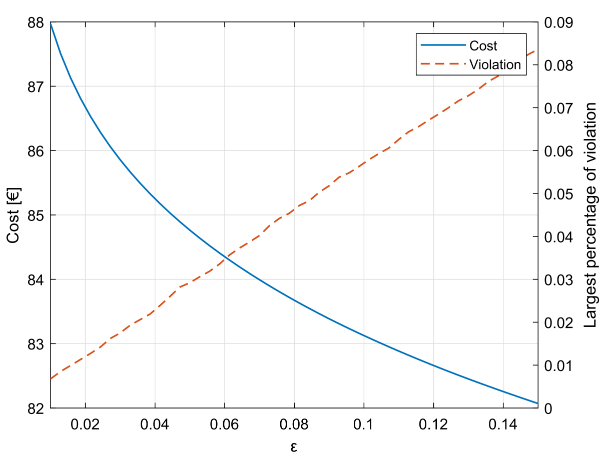

Fig. 6. Charging station cost and largest percentage of violation versus ε.

different values of ε. This provides the cost (23) as a function of ε. Then, as in Section 5.1, $N = 10^4$ simulations are run by applying the predetermined EV charging profiles under different realizations of energy demand for each request. This makes it possible to evaluate the largest percentage of violation and the largest constraint violation (see Section 5.1) as a function of ε. All these values are reported in Table 2 for ε ranging from 0.01 to 0.15. The same table also shows the results obtained with the EV charging profiles returned by the deterministic optimization formulated in Section 3.1, where the energy required by each request is assumed to be the expected value of the related distribution.

Since no stochastic guarantee is provided, it can be observed that the deterministic solution leads to an overall cost which is lower than that of the stochastic solution for all the considered values of ε. However, the solution provided by the stochastic optimization outperforms the one provided by the deterministic optimization in terms of feasibility. Indeed, even considering $\varepsilon = 0.15$, the largest percentage of violation is still much smaller than the one obtained with the solution of the deterministic problem. Moreover, the largest constraint violation is 6.48 kWh for the stochastic solution, whereas it is 9.77 kWh for the deterministic solution. Finally, Fig. 6 shows the plots of the cost (blue solid) and the largest percentage of violation (red dashed) versus ε. It is worthwhile to note that the latter grows almost linearly with ε. On the other hand, the operation cost features high sensitivity for small values of ε, while its trend tends to be almost linear when $\varepsilon \geq 0.07$.

6. Conclusions

In this paper, we considered the optimal management of a fleet of EVs used within a rental system. The decision problem is concerned with assigning rental requests to vehicles, and determining the EV charging policies so that the battery of each vehicle has sufficient charge to fulfill each request assigned to it. The charging process occurs when vehicles are returned to the charging station, during inactivity intervals between consecutive requests. Since the battery energy level when a vehicle returns to the station is uncertain, a stochastic formulation was adopted. The objective is to minimize the electricity cost, while guaranteeing that the energy demand of the served requests is met with a prescribed probability level. To cope with the mixed-integer probabilistic nature of the assignment problem, a two-step suboptimal approach was proposed. In the first step, an efficient heuristic procedure accomplishes the request-to-vehicle assignment. Then, for a given assignment, the EV charging policies are determined by solving a relaxed chance-constrained problem, thus obtaining a computationally tractable approach which involves the solution of a linear program. The good performance of the proposed procedures was assessed by means of extensive numerical results. In particular, the results in Section 5.1 showed that the relaxed chance-constrained problem typically provides EV battery charging profiles that are feasible for the original problem with joint chance-constraints, with only occasional and modest violations of the bounds on the battery energy level. In Section 5.2 it was shown that, for the simulated characteristics of the requests, the heuristic assignment procedure was able to provide feasible assignments in a very high percentage of instances for a number of requests at least up to the double of the number of vehicles. Finally, Section 5.3 showed that, in the presence of stochastic energy demand, the EV battery charging profiles obtained with the stochastic procedure outperform those obtained with the deterministic procedure in terms of fulfillment of constraints, with an acceptable loss in terms of cost for electricity. The deterministic procedure considers each request as characterized by the expected value of the corresponding energy demand distribution.

The results of this paper provided useful insights into the impact of demand uncertainty, combined with the peculiar characteristics of EVs, on the optimal management of EV fleets within rental and sharing systems. Future work will focus on introducing and managing other sources of uncertainty in the problem at hand (e.g., uncertain pickup and delivery times), as well as considering different types of rental services (e.g., one-way services, where the station of pickup and that of delivery do not coincide), and mixed fleets of heterogeneous vehicles.

CRediT authorship contribution statement

Giovanni Gino Zanvettor: Writing – original draft, Software, Conceptualization. **Marco Casini:** Writing – review & editing, Methodology, Conceptualization. **Antonio Giannitrapani:** Methodology, Investigation, Conceptualization. **Simone Paoletti:** Writing – review & editing, Investigation, Conceptualization. **Antonio Vicino:** Supervision, Methodology, Investigation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A

Proof of Lemma 1. By (34), one can write (29)–(33) as

$$x_{h_i} \geq x_{h_{i-1}} \tag{44}$$

$$x_{h_i} \leq \Delta_T \eta_v \overline{P}_v \tau_{h_i} + x_{h_{i-1}} \tag{45}$$

$$x_{h_i} \geq \overline{E}_{h_i}(\varepsilon_1) - S_v(0) + \hat{E}_{h_i} \tag{46}$$

$$x_{h_{i-1}} \geq \overline{E}_{h_i}(\varepsilon_1) - S_v(0) \tag{47}$$

$$x_{h_i} \leq \underline{E}_{h_i}(\varepsilon_2) - S_v(0) + \overline{S}_v. \tag{48}$$

Then, by (35)–(39), inequalities (44)–(48) become

$$\underline{x}_{h_i} \leq x_{h_i} \leq \overline{x}_{h_i} \qquad \forall i \in \{0, \dots, m_v + 1\}$$

$$x_{h_i} \leq x_{h_{i+1}} \leq x_{h_i} + c_{h_i} \qquad \forall i \in \{0, \dots, m_v\}. \quad \square$$

Proof of Lemma 2. It is easy to note that if (40)–(41) admit a solution x_i^* with $i \in \{1, \dots, m\}$, then the following inequalities are satisfied

$$x_i^* \geq \underline{x}_k \qquad \forall k \in \{1, \dots, i\} \tag{49}$$

$$x_i^* \leq \overline{x}_k \qquad \forall k \in \{i, \dots, m\} \tag{50}$$

$$x_i^* \leq x_k^* + \sum_{j=1}^{i-k} c_{i-j} \qquad \forall k \in \{1, \dots, i-1\} \tag{51}$$

$$x_i^* \leq \overline{x}_k + \sum_{j=1}^{i-k} c_{i-j} \qquad \forall k \in \{1, \dots, i-1\} \tag{52}$$

$$x_i^* \geq x_k^* - \sum_{j=0}^{k-i-1} c_{i+j} \qquad \forall k \in \{i+1, \dots, m\} \tag{53}$$

$$x_i^* \geq \underline{x}_k - \sum_{j=0}^{k-i-1} c_{i+j} \qquad \forall k \in \{i+1, \dots, m\}. \tag{54}$$

Let us prove the necessity. Suppose that there exist a solution x_i^* such that (40)–(41) are satisfied, then we want to prove that

$$\underline{w}_i \leq \overline{w}_i \quad \forall i \in \{1, \dots, m\}.$$

By recursively applying the definition of $\underline{w}_i$ and $\overline{w}_i$ in (42)–(43), the explicit expression of $\underline{w}_i$ and $\overline{w}_i$ can be written as

$$\underline{w}_i = \max\left\{ \max_{1\leq k\leq i} \underline{x}_k, \quad \max_{i<k\leq m} \left\{ \underline{x}_k - \sum_{j=0}^{k-i-1} c_{i+j} \right\} \right\} \tag{55}$$

$$\overline{w}_i = \min\left\{ \min_{i\leq k\leq m} \overline{x}_k, \quad \min_{1\leq k<i} \left\{ \overline{x}_k + \sum_{j=1}^{i-k} c_{i-j} \right\} \right\}. \tag{56}$$

Thus, by considering a feasible solution x_i^* with $i \in \{1, \dots, m\}$, we have that (49)–(54) are satisfied and hence

$$\max_{1\leq k\leq i} \{\underline{x}_k\} \leq x_i^* \leq \min_{i\leq k\leq m} \{\overline{x}_k\}$$

$$\max_{i<k\leq m} \left\{ \underline{x}_k - \sum_{j=0}^{k-i-1} c_{i+j} \right\} \leq x_i^* \leq \min_{1\leq k<i} \left\{ \overline{x}_k + \sum_{j=1}^{i-k} c_{i-j} \right\}$$

which, from (55)–(56) proves that $\underline{w}_i \leq \overline{w}_i$, $\forall i \in \{1, \dots, m\}$.

Now we want to prove that if $\underline{w}_i \leq \overline{w}_i$ for $i \in \{1, \dots, m\}$, then (40)–(41) are satisfied.

Let us consider a candidate solution $x_i^* = \overline{w}_i$, we want to show that it is also a feasible solution. By exploiting the definition in (56), the following inequalities hold

$$\overline{w}_i \leq \overline{w}_{i-1} + c_{i-1} \tag{57}$$

$$\overline{w}_i \leq \min_{i\leq k\leq m} \{\overline{x}_k\} \leq \overline{x}_i. \tag{58}$$

So, by considering x_i^*, the right hand sides of (40)–(41) are satisfied. Thus, the following inequalities need to be checked

$$\overline{w}_i \geq \overline{w}_{i-1} \tag{59}$$

$$\overline{w}_i \geq \underline{x}_i. \tag{60}$$

First, focus on $\overline{w}_i \geq \overline{w}_{i-1}$. We have that if $\overline{w}_i = \overline{w}_{i-1} + c_{i-1}$ then (59) is satisfied since $c_{i-1} \geq 0$. On the other hand, if $\overline{w}_i = \min_{i\leq k\leq m} \overline{x}_k$ then, by exploiting (58), the we have

$$\min_{i\leq k\leq m} \overline{x}_k \geq \min_{i\leq k\leq m-1} \overline{x}_k \geq \overline{w}_{i-1}.$$

Finally, we have to show that the inequality in (60) is also true. In particular, we can write

$$\overline{w}_i \geq \underline{w}_i \geq \underline{x}_i,$$

which concludes the proof. $\square$

Data availability

Data will be made available on request.

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