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Explicit solution to optimal management of storage systems in renewable energy communities

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ABSTRACT

Renewable energy communities constitute one of the most promising tools to contribute to the green energy transition and to provide social, environmental, and economic benefits to their members. These goals are pursued through the cooperative efforts of the community participants to increase the local energy self-consumption. In this paper, the optimal operation of energy storage system in an incentive-based energy community is addressed. By exploiting the flexibility provided by the storage facilities, the problem of minimizing the community energy bill by taking advantage of incentives related to local self-consumption is formulated. Conditions for the convenience of using storage systems inside an energy community, and explicit optimal solutions for managing such systems are analytically derived. Robustness of the solution against uncertainty affecting consumption and generation profiles of the community members is investigated. Numerical simulations are provided to assess the performance and the feasibility of the proposed solution.

1. Introduction

The Net Zero plan of the European community aims at achieving climate neutrality by 2050 [1]. One of the most promising solutions relies on the paradigm of renewable energy communities (RECs). As defined by the European Union [2], an energy community is a legal entity where participation is established on a voluntary and open basis, with the primary purpose of ensuring environmental, social and economic benefits to its members and shareholders. These benefits may be obtained by providing auxiliary services through renewable generation facilities, energy storage systems and electric vehicles. Thus, a REC can contribute to reducing gas emissions, triggering renewable self-consumption mechanisms, and increasing environmental sustainability. Additionally, it allows citizens to play an active role in the energy transition, fostering virtuous behaviors and economic growth.

1.1. Related works

Generally, an energy community is characterized by a massive adoption of renewable resources, ensuring the self-consumption mechanism either through local markets [3,4], or by exploiting community incentives introduced by state or regional regulations [5,6]. The former approach is focused on establishing energy trading operations among all community members, where local prices are usually computed according to distributed-like approaches, including ADMM algorithms [7], and blockchain technologies [8]. The latter relies on incentives granted to the community based on the cooperation of its members [9], or on the community virtual self-consumption [10], defined as the minimum between the overall energy demand of the community and the total renewable energy generated within the REC.

In [11], it is shown that energy communities may help to provide environmental benefits and cost reductions for their users, leading to a higher social acceptability and an increase of community self-consumption [12]. Moreover, the optimal coordination of community members can provide lower operation costs [13]. Since acting towards the community generally leads to cheaper energy bills, fair redistribution schemes are employed to allow that every member can benefit by joining the community [14]. The economic benefits of RECs are also analyzed in the context of buildings [15], while in [16] it is shown that building performance in the community system can be improved by employing optimization routines. Nevertheless, the optimal allocation of renewable resources may allow to reduce investment costs while considering different aspects of the community system, such as electric mobility [17], and local conditions for the installation of photovoltaic or wind turbine facilities [18].

Energy storage systems can play a crucial role in achieving high levels of community self-resiliency. Actually, to enhance the community operation, they can be employed as a shared resource [19], or operated through coordinated efforts of private users [20]. However, their optimal operation must consider different aspects, including power flow

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modeling [21] and uncertainties in renewable generation or consumption profiles [22]. Concerning uncertainty-aware approaches, two-stage formulations [22,23] and robust approaches [24] are employed to handle uncertain variables. Also, risk-constrained procedures have been developed to satisfy grid requirements [25] and design a community storage facility [26].

1.2. Contribution

This paper is focused on incentive-based RECs, where the community is assumed to be composed of consumers, producers and prosumers, while the incentive provided to the REC is granted on the basis of the virtual self-consumption in each time period of the day. For instance, such framework complies with the Italian regulation, where the virtual self-consumption is computed on an hourly basis. We suppose that prosumers and producers can be equipped with energy storage units that can be used to provide flexibility to the community. To manage the energy flows inside the community, a central entity, called REC manager, is engaged to manage the energy storage systems. Using load and renewable generation profiles provided by REC members, it operates the storage systems to achieve the following objectives:

- minimize the energy bought from the grid by each prosumer;
- coordinate the storage systems to minimize the community energy cost.

Thus, the optimal control problem is formulated, and optimality conditions are derived to find the optimal storage schedule. The main contributions of the proposed study are:

- closed-form solution for the optimal charging/discharging strategy which makes the solution nicely scalable for RECs of arbitrary size, featuring negligible runtimes;
- optimization procedure useful for designing and building a REC, with reference to quantification of the economic convenience of exploiting storage systems depending on the level of uncertainty and the optimal sizing and management of storage systems of the REC;
- adaptation of the previous solutions to the case of uncertain load and generation profiles according to a worst-case approach.

To validate the effectiveness of the optimal solution obtained, numerical simulations involving renewable energy communities of large size are performed.

1.3. Novelty

Due to the high relevance to REC's design and operation, there are several recent works in the literature that address this topic. In fact, many research works are related to optimal management of *shared* storage systems inside RECs, like in [27], where the influence of the variability of power demand on the total operating and maintenance costs of a community is investigated through a multi-objective analysis. In [28], it is analytically proved that the optimal control of a shared storage system over a horizon of several days is equivalent to solving many optimization problems over a single day, thus reducing the overall complexity. In the recent work [29], authors address the problem of operating shared storage systems in a REC to maximize the overall self-consumption while preserving battery lifespan. To this purpose, a two-layer optimal control model composed of a planner and a real-time controller is proposed.

Differently from the above-mentioned contributions, in this paper it is assumed that no shared batteries are available, whereas prosumers and producers can be equipped with their own energy storage units that can be operated to provide flexibility to the community. In this context, the case of distributed storage systems is analyzed in [30], where a mixed-integer linear programming (MILP) problem is formulated to reduce the total costs of the aggregation for supply and storage, while maximizing profits and additional ancillary services provided to the grid. Optimal operation of storage systems in RECS under demand response programs is considered in [31,32], where the optimal battery management is provided through the solution of low-complexity MILP problems. A community energy management system under demand response programs is also considered in [33], where the devised energy management system operates on two levels: the community controller level and the consumer level. To reduce the overall expected cost, battery energy usage is optimized through a genetic algorithm. Contrary to the previously cited works, in this paper a closed-form solution for the optimization problem aimed at the management of energy storage systems is provided. So, the obtained management strategy turns out to be easily implementable in real applications. Moreover, explicit worst-case scenarios are derived to apply the storage optimal control law in a robust optimization framework.

1.4. Notation

For a given optimization problem, the superscript * denotes all the quantities related to the optimal solution S^*. Accordingly, all the variables related to a given feasible solution $\widetilde{S}$ are denoted with the superscript $\widetilde{\cdot}$.

A flowchart illustrating the main topics treated in each section of the paper is reported in Fig. 1.

2. Materials and methods

2.1. Problem formulation

The problem is formulated in a discrete time setting, where the storage operation is performed over one day, divided into T time indexes and represented by the set $\mathcal{T} = \{0, 1, \dots, T-1\}$. Consider a renewable energy community composed of a set $\mathcal{U}$ of members (or entities). For a given community member $u \in \mathcal{U}$, let $l_u(t)$ be the load between time t and $t+1$, whereas $r_u(t)$ be the renewable energy generated in the same time interval. In general, an entity can be a consumer, a producer or a prosumer. A consumer is characterized by its load ($l_u(t) \geq 0,\ r_u(t) = 0,\ \forall t$), a producer by the generated renewable energy ($l_u(t) = 0,\ r_u(t) \geq 0,\ \forall t$), while a prosumer involves both consumption and generation ($l_u(t) \geq 0,\ r_u(t) \geq 0,\ \forall t$).

Assume that a fraction of producers and prosumers are equipped with an energy storage system, and let them be gathered into the set $\mathcal{U}_s \subseteq \mathcal{U}$. Consumers are not considered in this set because it is assumed that a storage can contribute to the community only by using renewable sources (see for instance the Italian regulations [34]). Thus, 5 kinds of entities are considered, which represent all the possible combinations of load, generation and storage, as depicted in Fig. 2.

Let $\rho_u(t) = r_u(t) - l_u(t)$; for a given entity $u \in \mathcal{U}_s$ the maximum amount of energy that can be charged into the storage between t and $t+1$ is

$$\bar{e}_u(t) = \max\{\rho_u(t), 0\}. \tag{1}$$

Note that $\bar{e}_u(t)$ denotes the surplus between the generated and the consumed energy; the remaining generated energy is supposed to supply the entity load. In this paper, the storage capacity and maximum power rates are left as variables of the optimization problem. In fact, one of the targets of the proposed study is to provide information on the storage system size allowing for the minimization of the cost of the REC. This knowledge provide crucial information on how to build the REC structure and plan its operation. Note that such assumptions can be relaxed in practical applications by enforcing a posteriori feasible bounds on storage capacities and charging/discharging power rates, as reported in Section 3.

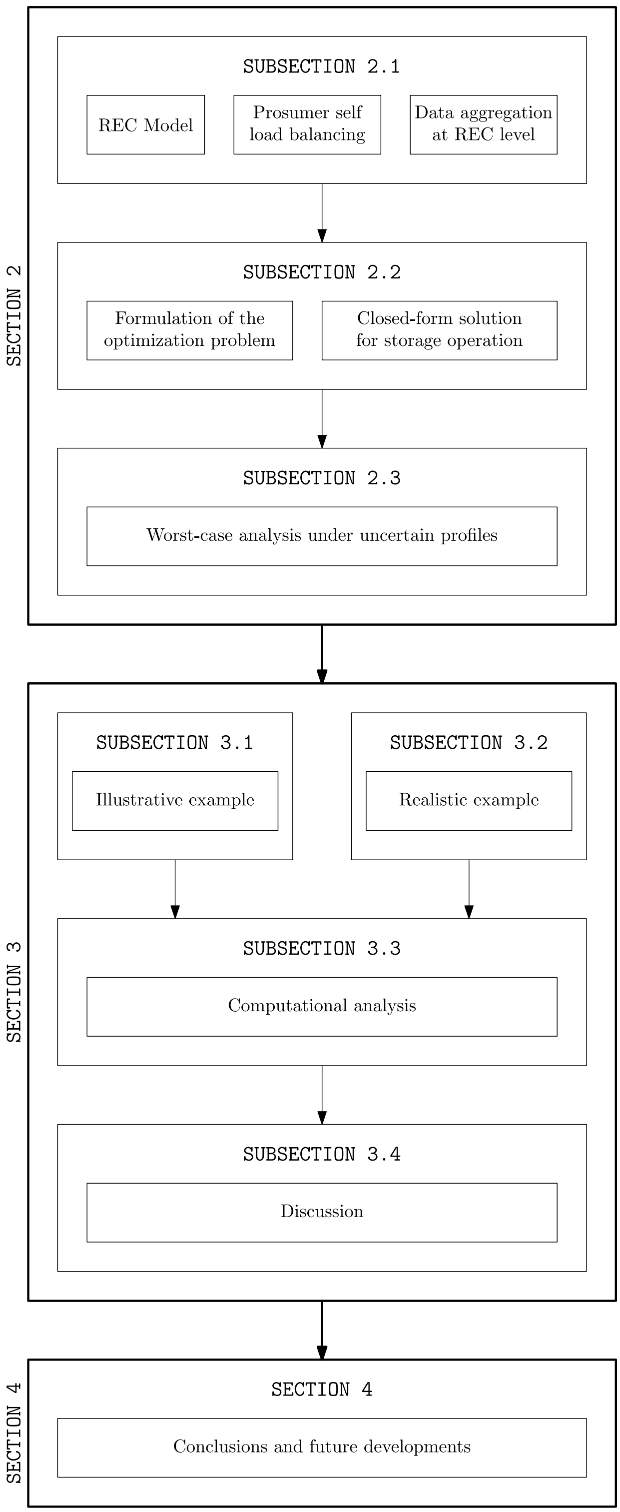

Fig. 1. Flowchart of paper structure.

Given an entity $u \in \mathcal{U}_s$, let $e_u^c(t)$ be the energy charged into the storage while $e_u^d(t)$ be the energy drawn from the storage between t and $t+1$. The energy stored at time $t+1$ is expressed as

$$s_u(t+1) = s_u(t) + \eta e_u^c(t) - \frac{1}{\eta} e_u^d(t), \tag{2}$$

where $\eta < 1$ denotes the battery efficiency. It is assumed that

$$s_u(0) = s_u(T) = 0. \tag{3}$$

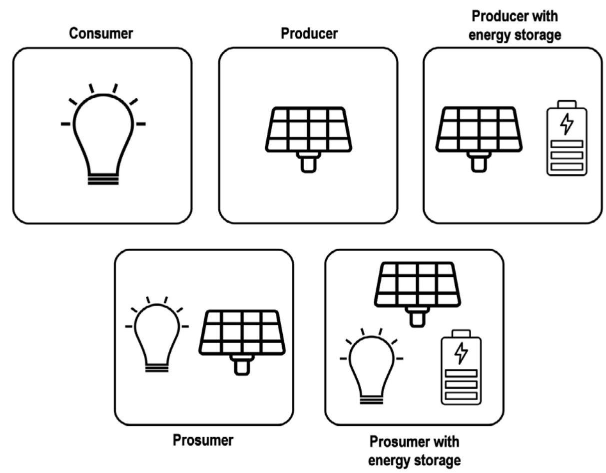

Fig. 2. Entity types in the considered community.

Clearly, $e_u^c(t)$ is a positive quantity and it is bounded by the surplus of generated energy, i.e.,

$$0 \leq e_u^c(t) \leq \bar{e}_u(t) \quad \forall t. \tag{4}$$

On the other hand, since $s_u(t) \geq 0$, $e_u^d(t)$ is positive and cannot exceed the stored energy, that is

$$0 \leq e_u^d(t) \leq \eta s_u(t) \quad \forall t. \tag{5}$$

Finally, to avoid situations of simultaneous charging and discharging, the following complementary constraint will be imposed

$$e_u^c(t) \cdot e_u^d(t) = 0 \quad \forall t. \tag{6}$$

In this paper, the storage devices will be used to fulfill two main purposes:

- optimize the prosumer operation by minimizing the amount of energy drawn from the grid;
- optimize the community operation by exploiting the incentive provided by the community self-consumption.

The formulation of the former task is reported in Section 2.1.1, while the latter is detailed in Section 2.1.2.

2.1.1. *Prosumer self load balancing*

To minimize the amount of electricity bought from the grid, it is assumed that prosumers first want to balance their own load by exploiting their storage facilities; after that, they operate them to increase the community self-consumption. To describe the former operation, hereafter named as *self load balancing*, for each prosumer $u \in \mathcal{U}_s$, the set of time indexes $\mathcal{T}$ is partitioned into two sets

$$\mathcal{R}_u^+ = \{t \in \mathcal{T} : \rho_u(t) \geq 0\},$$
$$\mathcal{R}_u^- = \{t \in \mathcal{T} : \rho_u(t) < 0\},$$

that define the time periods in which renewable energy surplus is available or not.

To provide self load balancing, the following control strategy is enforced

$$\hat{e}_u^c(t) = \begin{cases} \min\left\{ \bar{e}_u(t), -\frac{1}{\eta}\hat{s}_u(t) - \frac{1}{\eta^2}\sum_{\substack{\tau > t \\ \tau \in \mathcal{R}_u^-}} \rho_u(\tau) \right\} & \text{if } t \in \mathcal{R}_u^+, \\ 0 & \text{else,} \end{cases} \tag{7}$$

$$\hat{e}_u^d(t) = \begin{cases} \min\left\{ \eta \hat{s}_u(t), -\rho_u(t) \right\} & \text{if } t \in \mathcal{R}_u^-, \\ 0 & \text{else,} \end{cases} \tag{8}$$

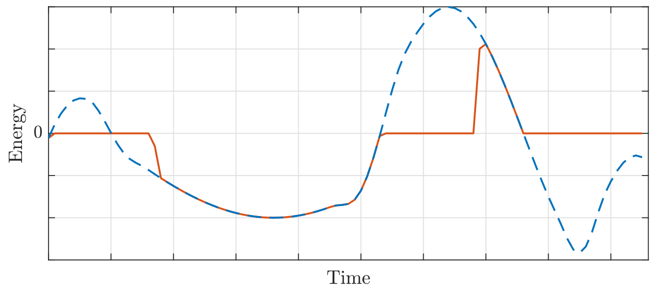

Fig. 3. Profile $\rho_u(t)$ (blue dashed) and the resulting profile $\rho'_u(t)$ (red), obtained by a given prosumer $u \in \mathcal{U}_s$ after applying the self load balancing operation.

$$\hat{s}_u(t+1) = \hat{s}_u(t) + \eta \hat{e}^c_u(t) - \frac{1}{\eta}\hat{e}^d_u(t), \tag{9}$$

$$\hat{s}_u(0) = 0, \tag{10}$$

where $\hat{e}^c_u(t)$, $\hat{e}^d_u(t)$, and $\hat{s}_u(t)$ denote the charging command, the discharging command and the energy stored during the self load balancing operation, respectively. In (7), the charging command is designed in a way such that the energy stored does not exceed future loads, while in (8), the discharging command is used to balance the load at each time step. In the next proposition, some useful properties of the self load balancing strategy are reported.

By applying the strategy reported in (7)–(10), for a given prosumer $u \in \mathcal{U}_s$, the original profile $\rho_u(t)$ changes. Such modified profile is denoted by $\rho'_u(t) = \rho_u(t) - \hat{e}^c_u(t) + \hat{e}^d_u(t)$. Then, the remaining renewable generation surplus may be further used by the storage system to provide additional flexibility to the community. An example of prosumer load balancing is shown in Fig. 3.

Note that this balancing procedure does not affect producers since they do not have loads, or consumers since they are not equipped with storage units. So, $\rho'_u(t) = \rho_u(t)$ for all $u \in \mathcal{U}_s$ except for prosumers equipped with storage systems.

Since the storage device has been used to change the prosumer profile, Eqs. (2)–(6) need be adapted accordingly to be used later for REC purposes. Specifically, variables and constraints for the community operation follow

$$s'_u(t+1) = s'_u(t) + \eta e^{c'}_u(t) - \frac{1}{\eta}e^{d'}_u(t) \qquad \forall u \in \mathcal{U}_s, \forall t, \tag{11}$$

$$s'_u(0) = s'_u(T) = 0, \tag{12}$$

$$0 \le e^{c'}_u(t) \le \overline{e}'_u(t) \qquad \forall u \in \mathcal{U}_s, \forall t, \tag{13}$$

$$0 \le e^{d'}_u(t) \le \eta s'_u(t) \qquad \forall u \in \mathcal{U}_s, \forall t, \tag{14}$$

$$e^{c'}_u(t) \cdot e^{d'}_u(t) = 0 \qquad \forall u \in \mathcal{U}_s, \forall t, \tag{15}$$

where

$$\overline{e}'_u(t) = \max\{\rho'_u(t), 0\} = \overline{e}_u(t) - \hat{e}^c_u(t) \quad \forall u \in \mathcal{U}_s, \forall t. \tag{16}$$

In (11)–(16), the *prime* symbol is used to denote variables involving quantities obtained after the self load balancing operation. So, $e^{c'}_u(t)$ and $e^{d'}_u(t)$ denote the charging/discharging commands of entity u to be applied for maximizing the incentive gained at community level, assuming the self load balancing operations have been accomplished. Therefore, it is mandatory to show that the storage operations for self load balancing and REC incentive maximization are compatible, i.e., their superposition satisfies (2)–(6). Such a statement is formalized in the following theorem.

Theorem 1. *Let $\hat{e}^c_u(t)$, $\hat{e}^d_u(t)$, $\hat{s}_u(t)$ in (7)–(10) and $e^{c'}_u(t)$, $e^{d'}_u(t)$, $s'_u(t)$ satisfying (11)–(15) be given. Then, the superimposed solution*

$$e^c_u(t) = \hat{e}^c_u(t) + e^{c'}_u(t)$$

$$e^d_u(t) = \hat{e}^d_u(t) + e^{d'}_u(t)$$

$$s_u(t) = \hat{s}_u(t) + s'_u(t)$$

satisfies (2)–(6), $\forall t \in \mathcal{T}$.

Proof. See Appendix A. □

Remark 1. Theorem 1 allows a prosumer to proceed sequentially: first, the storage commands needed to provide self load balancing are computed for the whole time horizon, allowing for the computation of the modified profile ρ'_u; then, the storage commands maximizing the REC incentive are computed on the basis of ρ'_u. The actual commands to be applied will be the sum of those obtained in the previous two steps, whose feasibility is guaranteed by Theorem 1.

2.1.2. *Community level aggregation*

This subsection is focused on the energy balance related to the overall community, once the self load balancing operations have been computed. At a given time t, the overall energy demand $L(t)$ required by all community entities is

$$L(t) = \sum_{u\in\mathcal{U}} \max\{-\rho'_u(t), 0\}, \tag{17}$$

whereas the surplus within the community $R(t)$ is

$$R(t) = \sum_{u\in\mathcal{U}} \max\{\rho'_u(t), 0\}. \tag{18}$$

After the self load balancing procedure, for each time step t, an entity u contributes to $L(t)$ if its load exceeds the generation, i.e., $\rho'_u(t) < 0$, while contributes to $R(t)$ if there is a generation surplus, i.e., $\rho'_u(t) > 0$. Therefore, the self-consumption at community level at time t is given by

$$A^0(t) = \min\{L(t), R(t)\}, \tag{19}$$

that represents the energy demand that is matched by the energy generation inside the community. Since monetary incentive will be granted to the REC on the basis of the community self-consumption, the aim is to operate the storage systems of its entities to increase such reward.

Concerning storage units, the stored energy at community level is

$$S(t) = \sum_{u\in\mathcal{U}_s} s'_u(t),$$

whereas the related charging and discharging energy commands are given by

$$E^c(t) = \sum_{u\in\mathcal{U}_s} e^{c'}_u(t),$$

$$E^d(t) = \sum_{u\in\mathcal{U}_s} e^{d'}_u(t).$$

Define

$$\overline{E}(t) = \sum_{u\in\mathcal{U}_s} \overline{e}'_u(t) = \sum_{u\in\mathcal{U}_s} \max\{\rho'_u(t), 0\}. \tag{20}$$

Then, at REC level, the constraints related to the storage can be rewritten as

$$S(t+1) = S(t) + \eta E^c(t) - \frac{1}{\eta}E^d(t) \qquad \forall t, \tag{21}$$

$$0 \le E^c(t) \le \overline{E}(t) \qquad \forall t, \tag{22}$$

$$0 \le E^d(t) \le \eta S(t) \qquad \forall t. \tag{23}$$

$$S(0) = S(T) = 0, \tag{24}$$

Since the storage systems are charged with energy produced by renewable generators, the net renewable energy injected into the grid at community level is obtained by the following energy balance equation

$$G(t) = R(t) - E^c(t) + E^d(t) \quad \forall t. \tag{25}$$

So, the REC self-consumption in the presence of storage at time t is defined as

$$A^s(t) = \min\{L(t), G(t)\} = \min\{L(t), R(t) - E^c(t) + E^d(t)\}. \tag{26}$$

Hereafter, the analysis will be done with quantities aggregated at community level, by employing expressions (17)–(26). In the following, it will be shown how to optimally operate the entity storage systems in order to minimize the REC overall cost.

2.2. Optimal storage operation

Let c^p and c^s be the energy purchase and selling prices, respectively. Moreover, let $k < c^p$ be the unitary incentive price for the self-consumed energy within the community. The objective of the community is to minimize its energy cost, which is defined as

$$\hat{J} = \sum_{\tau \in \mathcal{T}} \left(c^p L(\tau) - c^s G(\tau) - k A^s(\tau)\right). \tag{27}$$

Let $\mathcal{S}$ be the vector containing all the storage charging/discharging control signals at community level in $\mathcal{T}$, that is

$$\mathcal{S} = [E^c(0), \dots, E^c(T-1), E^d(0), \dots, E^d(T-1)].$$

Thus, the optimal storage schedule is the minimizer of the following optimization problem.

Problem 1.

$$\mathcal{S}^* = \arg\min_{\mathcal{S}} \hat{J} \quad \text{s.t. } (21) - (26).$$

It is worth recalling a lemma proved in [20], which states that $E^{c^*}(t)$ and $E^{d^*}(t)$ cannot be greater than 0 at the same time.

Lemma 1. *Let $E^{c^*}(t)$ and $E^{d^*}(t)$, $\forall t \in \mathcal{T}$ be the optimal charging and discharging control signals for Problem 1. Then,*

$$E^{c^*}(t) \cdot E^{d^*}(t) = 0, \quad \forall t \in \mathcal{T}. \tag{28}$$

Remark 2. It is worthwhile to highlight that, since the community storage systems are distributed among the community entities, $E^c(t)$ and $E^d(t)$ may be both non-zero for a generic feasible solution of Problem 1. However, thanks to Lemma 1 the optimal solution is such that the storage units are acting as a unique virtual storage of the community. In fact, the optimal solution provides a control signal which requires all the storage devices to be charged (or discharged) simultaneously, avoiding situations in which a storage system is charging when another one is discharging.

Since $S(0) = 0$, the explicit dynamics of the storage system can be written as

$$S(t) = \sum_{\tau \in \mathcal{T}} \left(\eta E^c(\tau) - \frac{1}{\eta} E^d(\tau)\right), \quad \forall t \in \{0, \dots, T\}. \tag{29}$$

So, constraint $S(T) = 0$ can be represented by

$$\eta^2 \sum_{\tau \in \mathcal{T}} E^c(\tau) = \sum_{\tau \in \mathcal{T}} E^d(\tau). \tag{30}$$

By (25) and (30), the following equation holds

$$\sum_{\tau \in \mathcal{T}} G(\tau) = \sum_{\tau \in \mathcal{T}} R(\tau) - \frac{1-\eta^2}{\eta^2} \sum_{\tau \in \mathcal{T}} E^d(\tau),$$

and hence the objective function (27) can be written as

$$\hat{J} = \sum_{\tau \in \mathcal{T}} \left(c^p L(\tau) - c^s R(\tau) + c^s \frac{1-\eta^2}{\eta^2} E^d(\tau) - k A^s(\tau)\right).$$

Define

$$\alpha = c^s \frac{1-\eta^2}{\eta^2} > 0. \tag{31}$$

Since neither the energy demand $L(t)$ nor the energy generation $R(t)$ depend on the decision variables, one can aim to minimize the following function

$$J = \sum_{\tau \in \mathcal{T}} \left(\alpha E^d(\tau) - k A^s(\tau)\right). \tag{32}$$

Therefore, the optimal solution of Problem 1 coincides with the optimal solution of the following problem.

Problem 2.

$$\mathcal{S}^* = \arg\min_{\mathcal{S}} \; J \quad \text{s.t. } (21) - (26).$$

To derive the optimal solution of the problem, it is convenient to focus on the value of k, i.e., on the unitary incentive on the REC self-consumed energy. Actually, depending on the value of k, it may be convenient or not to use the storage systems to optimize the community operation.

The following proposition derives conditions under which the trivial solution $E^d(t) = E^c(t) = 0, \ \forall t \in \mathcal{T}$ is optimal.

Proposition 1. *If $k \le \alpha$, then the optimal solution of Problem 2 is $E^d(t) = E^c(t) = 0, \ \forall t \in \mathcal{T}$.*

Proof. See Theorem 1 of [20]. □

Remark 3. Proposition 1 provides a threshold value on k as an indicator for the convenience of using the storage. In fact, if $k \le \alpha$ the best solution does not require the usage of storage systems to minimize the REC cost. For this reason, from now on, the case $k > \alpha$ is considered.

Theorem 2. *Consider Problem 2, and let $E^{c^*}(t)$ and $E^{d^*}(t)$ be the optimal charging and discharging control signals for $\forall t \in \mathcal{T}$. It holds that*

$$E^{c^*}(t) = 0 \quad \text{if } L(t) \ge R(t), \tag{33}$$

$$E^{d^*}(t) = 0 \quad \text{if } L(t) \le R(t), \tag{34}$$

$$E^{c^*}(t) \le R(t) - L(t) \quad \text{if } L(t) \le R(t), \tag{35}$$

$$E^{d^*}(t) \le L(t) - R(t) \quad \text{if } L(t) \ge R(t). \tag{36}$$

Proof. See Appendix B. □

Notice that Theorem 2 states that a candidate solution to Problem 2 must satisfy (33)–(34) that automatically enforces condition (28).

Corollary 1. *Let $\mathcal{S}^*$ be the optimal solution of Problem 2. Then,*

$$A^{s^*}(t) = A^0(t) + E^{d^*}(t) = \begin{cases} L(t) & \text{if } L(t) \le R(t) \\ R(t) + E^{d^*}(t) & \text{if } L(t) \ge R(t). \end{cases} \tag{37}$$

Proof. See Appendix C. □

Let the time indices be divided in two sets, depending on the fact that the REC load is less or greater than generation:

$$\mathcal{R}^+ = \{t \in \mathcal{T} : R(t) \ge L(t)\},$$
$$\mathcal{R}^- = \{t \in \mathcal{T} : R(t) < L(t)\}.$$

By Corollary 1, and by adding (33)–(36) to the constraints of Problem 2, the objective function in (32) can be rewritten as

$$J = \alpha \sum_{\tau \in \mathcal{T}} E^d(\tau) - k \sum_{\tau \in \mathcal{T}} \left(A^0(\tau) + E^d(\tau)\right) = -k \sum_{\tau \in \mathcal{T}} A^0(\tau) + (\alpha - k) \sum_{\tau \in \mathcal{R}^-} E^d(\tau),$$

where the last equality comes from (34).

Since $A^0(t)$ is known and constant, and $k > \alpha$, the optimal solution of Problem 2 coincides with the optimal solution of the following problem.

Problem 3.

$$S^* = \arg\max_{S} \sum_{t\in\mathcal{R}^-} E^d(t) \tag{38}$$

s.t.

$$S(t+1) = S(t) + \eta E^c(t) - \frac{1}{\eta}E^d(t) \qquad \forall t \in \mathcal{T}, \tag{39}$$

$$0 \le E^c(t) \le \overline{E}(t) \qquad \forall t \in \mathcal{T}, \tag{40}$$

$$0 \le E^d(t) \le \eta S(t) \qquad \forall t \in \mathcal{T}, \tag{41}$$

$$E^c(t) \le R(t) - L(t), \quad E^d(t) = 0 \qquad \forall t \in \mathcal{R}^+, \tag{42}$$

$$E^d(t) \le L(t) - R(t), \quad E^c(t) = 0 \qquad \forall t \in \mathcal{R}^-, \tag{43}$$

$$S(0) = S(T) = 0. \tag{44}$$

Note that the optimality conditions reported in Theorem 2 are summarized by constraints (42)–(43).

Since the objective function of Problem 3 involves only $E^d(t)$, it is apparent that the optimal solution maximizes the overall discharged energy. Operatively, at a given time $t \in \mathcal{R}^-$ the optimal control signal is to choose $E^d(t)$ as bigger as possible. Thanks to (41)–(43), the optimal solution results

$$E^{d^*}(t) = \min\{L(t) - R(t), \eta S(t)\}, \quad E^{c^*}(t) = 0, \quad \forall t \in \mathcal{R}^-. \tag{45}$$

On the contrary, when $t \in \mathcal{R}^+$ by (34) one has $E^d(t) = 0$. Note that the choice of the control variables at these time steps does not influence the objective function. Thus, the optimal charging control signal will be that maximizing the charging energy while satisfying the constraints.

By (30) it holds

$$\sum_{\tau=0}^{t-1} E^c(\tau) + E^c(t) + \sum_{\tau=t+1}^{T-1} E^c(\tau) = \sum_{\tau\in\mathcal{T}} \left(\frac{E^d(\tau)}{\eta^2}\right),$$

and hence

$$E^c(t) = \sum_{\tau=0}^{t-1}\left(\frac{E^d(\tau)}{\eta^2} - E^c(\tau)\right) + \sum_{\tau=t}^{T-1}\left(\frac{E^d(\tau)}{\eta^2}\right) - \sum_{\tau=t+1}^{T-1} E^c(\tau). \tag{46}$$

By (29), one has

$$-\frac{S(t)}{\eta} = \sum_{\tau=0}^{t-1}\left(\frac{E^d(\tau)}{\eta^2} - E^c(\tau)\right).$$

Since $E^c(t) \ge 0$, $\forall t$, from (46) it follows

$$E^c(t) \le -\frac{S(t)}{\eta} + \sum_{\tau=t}^{T-1}\left(\frac{E^d(\tau)}{\eta^2}\right).$$

Moreover, by (41) and (43) one has $0 \le E^d(t) \le L(t) - R(t)$, and then

$$E^c(t) \le -\frac{S(t)}{\eta} + \sum_{\tau\in\mathcal{R}^-,\tau>t} \frac{L(\tau) - R(\tau)}{\eta^2} \quad \forall t \in \mathcal{R}^+. \tag{47}$$

So, by (40), (42) and (47) the maximum, and thus the optimal, charging control signal is

$$E^{c^*}(t) = \min\left\{\overline{E}(t),\, R(t) - L(t),\, -\frac{S(t)}{\eta} + \sum_{\tau\in\mathcal{R}^-,\tau>t}\frac{L(t) - R(t)}{\eta^2}\right\}. \tag{48}$$

Remark 4. It is worthwhile to note that the optimal charging/discharging commands given in (45) and (48) represent the closed-form solution of Problem 1. This makes it possible to drastically reduce the computational burden of the storage optimal control law, thus allowing the procedure to be applied to large RECs, as shown in Section 3.

2.2.1. Distributed solution derivation

The solution of Problem 1 is obtained by using aggregated quantities of community storage systems, rather than exploiting charging/discharging commands of each single storage unit. Nevertheless, in the following, it will be shown how to retrieve a distributed optimal solution starting from the aggregated one. In particular, consider the quantities:

$$\gamma(t) = \begin{cases} \frac{E^{c^*}(t)}{\overline{E}(t)} & \text{if } E^{c^*}(t) > 0 \\ 0 & \text{otherwise} \end{cases} \qquad \forall t, \tag{49}$$

$$\delta(t) = \begin{cases} \frac{E^{d^*}(t)}{\eta S^*(t)} & \text{if } E^{d^*}(t) > 0 \\ 0 & \text{otherwise} \end{cases} \qquad \forall t, \tag{50}$$

that denote how close are the charging and discharging commands to their respective upper bounds.

Then, an optimal distributed solution is provided in the following proposition.

Proposition 2. *Let the optimal solution of Problem 1 be given, and let*

$$e_u^{c^*}(t) = \gamma(t)\overline{e}_u^{c'}(t) \qquad \forall u \in \mathcal{U}_s,\ \forall t, \tag{51}$$

$$e_u^{d^*}(t) = \eta\delta(t)s_u^*(t) \qquad \forall u \in \mathcal{U}_s,\ \forall t. \tag{52}$$

be the charging/discharging commands for each storage unit, where

$$s_u^*(t) = \sum_{\tau=0}^{t-1}\left(\eta e_u^{c^*}(\tau) - \frac{1}{\eta}e_u^{d^*}(\tau)\right) \qquad \forall u \in \mathcal{U}_s,\ \forall t, \tag{53}$$

denotes the energy stored at time t by applying (51) and (52). Then, (51) and (52) provide a distributed optimal solution of Problem 1.

Proof. See Theorem 2 of [20]. □

Therefore, the overall distributed storage operation can be obtained by using (7)–(9) that leads to the following equations

$$e_u^c(t) = e_u^{c^*}(t) + \hat{e}_u^c(t),$$
$$e_u^d(t) = e_u^{d^*}(t) + \hat{e}_u^d(t),$$
$$s_u(t) = s_u^*(t) + \hat{s}_u^c(t).$$

which, by Theorem 1, satisfy (2)–(6).

2.3. Optimal operation under uncertain profiles

The above results have been obtained assuming that $\rho_u'(t)$, $\forall u \in \mathcal{U}$ are known. However, in real settings, only forecasts of such quantities may be available, affected by a wide range of uncertainties. So, in this subsection, the uncertainty related to profiles $\rho_u'(t)$ is modeled and handled according to a robust framework. Specifically, an additive unknown-but-bounded (UBB) uncertainty $\xi_u(t) \in \left[0, \overline{\xi}_u(t)\right]$ is considered, where $\overline{\xi}_u(t)$ is a known upper bound. This bound is either known a priori on the basis of knowledge on the system, or it is estimated by historical data. So, the resulting perturbed entity profile can be modeled as

$$\omega_u(t) = \rho_u'(t) + \xi_u(t). \tag{54}$$

Remark 5. For ease of exposition, it is supposed that $\xi_u(t)$ is non-negative. However, it is possible to seamlessly adapt the overall reasoning to uncertainties that can take negative values. Such an adjustment is reported at the end of this subsection.

According to (54), all the aggregated quantities involving $\rho_u'(t)$ need be adapted to take into account the uncertainty $\xi_u(t)$ in their profiles. In particular, $L(t)$, $R(t)$ and $\overline{E}(t)$ defined in (17), (18) and (20), need be replaced with

$$L^{\xi}(t) = \sum_{u\in\mathcal{U}} \max\{0, -\omega_u(t)\}, \tag{55}$$

$$R^{\xi}(t) = \sum_{u\in\mathcal{U}} \max\{0, \omega_u(t)\}, \tag{56}$$

$$\overline{E}^{\xi}(t) = \sum_{u\in\mathcal{U}_s} \max\{0, \omega_u(t)\}. \tag{57}$$

In order to find the worst-case uncertainty realization, define the following uncertain variables

$$\Delta^L(t) = \sum_{u\in\mathcal{U}} \min\{\xi_u(t), -\rho'_u(t)\} - \min\{0, -\rho'_u(t)\}, \tag{58}$$

$$\Delta^R(t) = \sum_{u\in\mathcal{U}} \max\{\xi_u(t), -\rho'_u(t)\} - \max\{0, -\rho'_u(t)\}, \tag{59}$$

$$\Delta^E(t) = \sum_{u\in\mathcal{U}_s} \max\{\xi_u(t), -\rho'_u(t)\} - \max\{0, -\rho'_u(t)\}. \tag{60}$$

Note that, since $\xi_u(t) \geq 0$, one has that $\Delta^R(t) \geq 0$, $\Delta^L(t) \geq 0$ and $\Delta^E(t) \geq 0$.

Consider the community objective function expressed as in (27). By inserting the uncertainty in $L(t)$ and $G(t)$, the community cost is formulated as

$$\hat{J}^{\xi} = \sum_{\tau\in\mathcal{T}} c^p L^{\xi}(\tau) - c^s G^{\xi}(\tau) - k \min\{L^{\xi}(\tau), G^{\xi}(\tau)\}, \tag{61}$$

where

$$G^{\xi}(t) = R^{\xi}(t) + E^d(t) - E^c(t). \tag{62}$$

Concerning optimization constraints, (22) becomes

$$0 \leq E^c(t) \leq \overline{E}^{\xi}(t) \quad \forall t \in \mathcal{T}, \tag{63}$$

which depends on the uncertainty due to (57).

Let $\mathcal{E}$ be a vector collecting all the uncertain variables $\xi_u(t)$, $\forall t \in \mathcal{T}$, $\forall u \in \mathcal{U}$, then the robust optimization problem to be solved is as follows.

Problem 4.

$$[S^*, \mathcal{E}^*] = \arg\min_{S}\max_{\mathcal{E}} \quad \hat{J}^{\xi}$$
$$\text{s.t. } (21), (23), (24),$$
$$(61) - (63).$$

The worst-case realization of the uncertainty $\mathcal{E}^*$ in Problem 4 is reported in the following theorem.

Theorem 3. *Consider Problem 4. Then, the worst-case realization of the uncertainty is $\mathcal{E}^* = 0$, i.e., $\xi^*_u(t) = 0$, $\forall t \in \mathcal{T}$, $\forall u \in \mathcal{U}$.*

Proof. See Appendix D. □

Now, it is possible to generalize the reasoning to any kind of bounded uncertainty such that $\xi_u(t) \in [\underline{\xi}_u(t), \overline{\xi}_u(t)]$, where $\underline{\xi}_u(t) < \overline{\xi}_u(t)$, $\forall t \in \mathcal{T}$, $\forall u \in \mathcal{U}$.

Corollary 2. *Consider Problem 4 and assume that the uncertain variable is such that $\xi_u(t) \in [\underline{\xi}_u(t), \overline{\xi}_u(t)]$, where $\underline{\xi}_u(t) < \overline{\xi}_u(t)$, $\forall t \in \mathcal{T}$, $\forall u \in \mathcal{U}$. Then, the worst-case uncertainty is $\xi^*_u(t) = \underline{\xi}_u(t)$, $\forall t \in \mathcal{T}$, $\forall u \in \mathcal{U}$.*

Proof. See Appendix E. □

Remark 6. These above results are obtained by using profiles $\rho'_u(t)$, which represent the entity profile after performing the self load balancing operation. However, by (7), the lower $\overline{e}_u(t)$, the lower the energy charged. Therefore, the least amount of energy shifted to balance the prosumer load is related to the self load balancing operation applied to the profile $\rho_u(t) + \underline{\xi}_u(t)$.

3. Numerical results

To evaluate the performance of the proposed solution under different scenarios, two examples are provided: a simplified one and an example based on a realistic energy community. The former example involving three players only shows how the single load/generation profiles are combined together to provide the overall REC profile. On the other hand, the realistic example involving a community of 60 entities is aimed at assessing the performance of the proposed approach in a realistic scenario. Finally, the computational feasibility is validated by considering different community sizes. For all the examples, the optimization time horizon spans over 24 h with a sampling time of 5 min, i.e., $T = 288$. Both prosumers and producers are assumed to be equipped with storage units whose efficiency is set to $\eta = 0.9$. Grid prices are supposed to be $c^p = 0.35$ €/kWh and $c^s = 0.18$ €/kWh, whereas the incentive is set to $k = 0.12$ €/kWh. Note that, since $\alpha = c^s(1-\eta^2)/\eta^2 = 0.042$ €/kWh, Proposition 1 confirms that the incentive k is chosen to foster storage utilization.

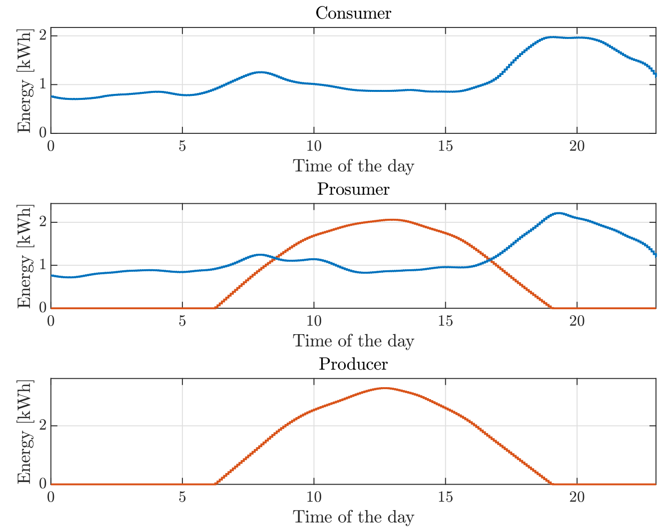

Fig. 4. Illustrative example. Load (blue) and generation (red) profiles of the community entities.

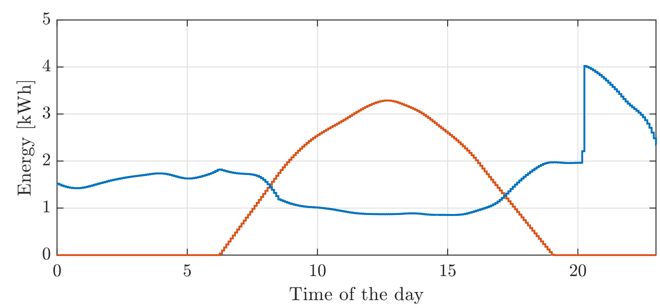

Fig. 5. Illustrative example. Load (blue) and generation (red) profiles at community level.

3.1. Illustrative example

In this example, a simplified community structure involving one consumer, one prosumer, and one producer is considered. The load and generation profiles of the community entities are depicted in Fig. 4. Then, by aggregating at community level, community load and generation are reported in Fig. 5.

These aggregated profiles show generation surplus during the mid hours of the day, and load excess in other periods. Note that the load peak occurring around 20:00 is due to the self load balancing operation performed by the prosumer.

By exploiting (45) and (48), the energy surplus is employed to balance the load during the last part of the day. This behavior is

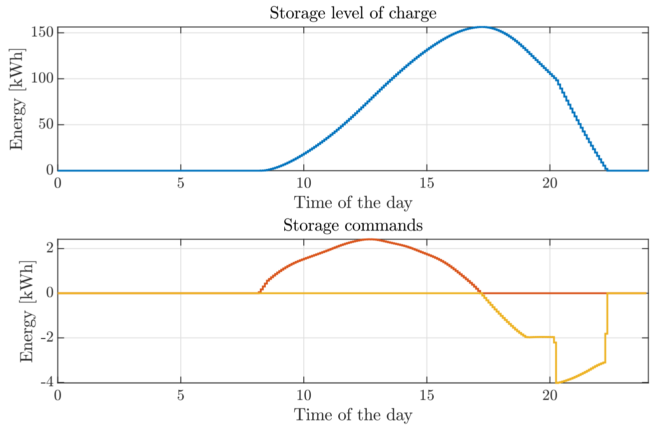

Fig. 6. Illustrative example. Storage level of charge (blue) and related charging (red) and discharging (yellow) control signals at community level.

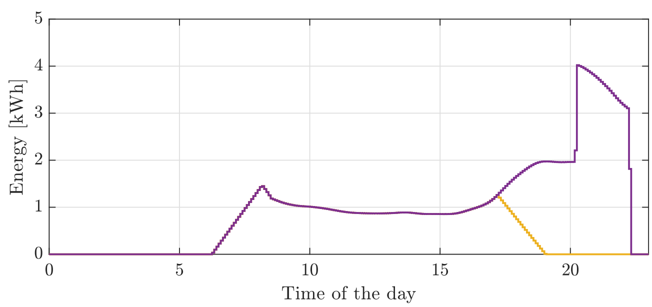

Fig. 7. Illustrative example. Self-consumption profiles $A^0(t)$ (yellow) and $A^{s^*}(t)$ (purple).

Table 1
Community cost and incentive.

	Illustrative example		Large-scale example	
	No storage	Optimal	No storage	Optimal
Cost [€]	90.45	79.51 (−12.1%)	19 901	18 065 (−9.2%)
Incentive [€]	16.21	33.93 (+109.3%)	6069	8901 (+46.7%)

evident by analyzing the storage dynamics and the related charging and discharging control signals in Fig. 6. Moreover, as stated by Corollary 1, the obtained storage schedule is such that the self-consumption $A^0(t)$ is maintained (enhanced) when the storage is charging (discharging), as illustrated in Fig. 7. Thus, the obtained profile closely follows the load until the storage is fully discharged. The optimal storage operation in this setup is capable of reducing the community cost of about 12%, while the incentive gained (and directly the community self-consumption) is more than doubled. Community costs and incentives concerning both setups are summarized in Table 1.

3.2. Realistic example

In this example, a large energy community is simulated over 10 days. The investigated setup involves:

- 30 consumers,
- 10 prosumers with storage system,
- 10 prosumers without storage system,
- 7 producers with storage system,
- 3 producers without storage system.

Load profiles are derived from real-world data involving both residential and commercial entities. For renewable generation, a unique profile obtained from real recordings of a PV plant is utilized. If a prosumer is considered, the corresponding generation profile is scaled according to the user load profile. On the other hand, for a producer, the peak power production is randomly set to 50, 75 or 100 kW. The normalized PV profile is depicted in Fig. 8.

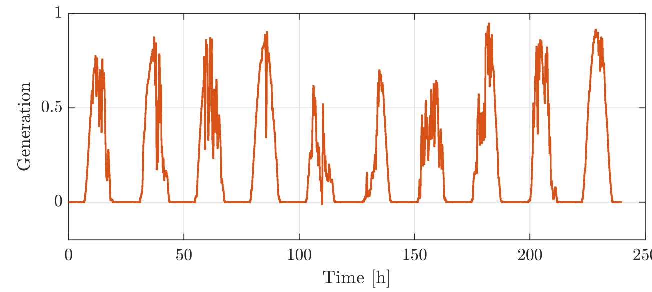

Fig. 8. Realistic example. Normalized generation profile in the simulation period.

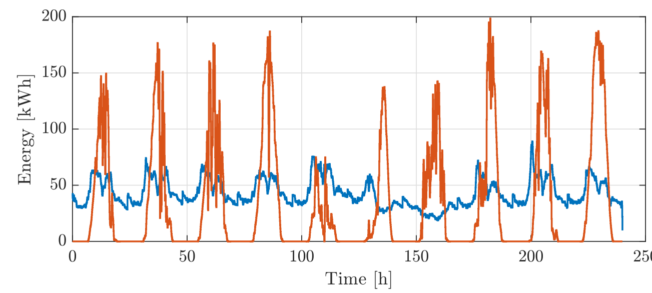

Fig. 9. Realistic example. Load (blue) and generation (red) profiles at community level.

To assess the performance of the proposed approach, first the optimal solution related to the deterministic setting is investigated, where no uncertainties are considered. Then, the sensitivity of the solution is evaluated according to different magnitudes of uncertainty.

3.2.1. Deterministic results

After performing the load balancing operation and aggregating at community level, $L(t)$ and $R(t)$ profiles are obtained. Time plots concerning these profiles are depicted in Fig. 9.

Similarly to the previous setup, for most of the days the generation is exceeding the energy demand of the community in the mid hours of the day. Therefore, for each day, the optimal strategy is such that the renewable energy generation is shifted by the storage facilities during evening hours. This behavior can be also noted in Fig. 10, where the optimal self-consumption profile $A^{s^*}(t)$ is compared with its counterpart $A^0(t)$. In fact, the storage facilities are operated so as to increase the community self-consumption during the last periods of the day. As shown in Table 1, the optimal solution is capable of reducing the community cost by about 9%, while the community self-consumption is increased by almost 47%.

Finally, the optimal solution involves storage quantities that are feasible for real-world settings. To support this statement, for each storage system, the maximum capacity $\bar{s}_u = \max_t s_u(t)$ and the maximum charging/discharging power $\bar{p}_u$ are analyzed, where $\bar{p}_u$ can be roughly approximated as

$$\bar{p}_u = \frac{1}{12} \max_t \left(e_u^c(t) + e_u^d(t) \right),$$

where $\frac{1}{12}$ is the sampling time expressed in hours.

Storage capacities are compared with the daily energy surplus (i.e., $\sum_{\tau \in T} \bar{e}_u(\tau)$), while power rates are validated by considering the ratio between $\bar{s}_u$ and $\bar{p}_u$.

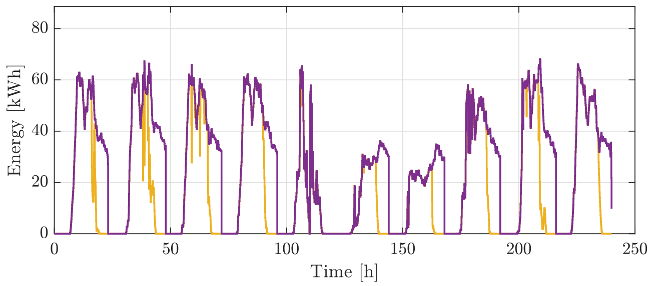

Fig. 10. Realistic example. Self-consumption profiles $A^0(t)$ (yellow) and $A^{s^*}(t)$ (purple).

In the investigated scenario, storage capacities are at most 3% higher than the average daily energy surplus, while they are smaller than 40% of the maximum daily surplus. Concerning power rates, the ratio between the capacity and the maximum charging/discharging power is always above 2 h, which is consistent with the state-of-the-art technologies [35].

3.2.2. Uncertainty analysis

To show how the problem solution is affected by uncertainty, the profile of each user is perturbed by a random variable having symmetric support. For each user $u \in \mathcal{U}$, the maximum perturbation is chosen according to $\alpha \max_t |\rho_u(t)|$, where α denotes a given uncertainty level. The nominal profile of a consumer, a prosumer, and a producer compared with different levels of uncertainty is shown in Fig. 11. Since the energy profiles of consumers and producers have fixed signs, their uncertain profiles are constrained to be negative or positive, respectively. Notice that the profile of the consumer reported in Fig. 11 refers to an entity which drastically reduces its consumption during weekend, like, e.g., an administrative office. Clearly, by Corollary 2, the worst-case uncertainty corresponds to its lowest value which is the lower bound of the uncertainty band for all the depicted cases. So, the self load balancing and the community operations are performed using the lowest profiles obtained for different values of α.

Performance is evaluated by analyzing the percentage of savings and the community self-consumption obtained by employing the worst-case scenarios with and without storage systems. The graphical tendency of savings for different values of α is depicted in Fig. 12. As expected, the optimization performance deteriorates when the uncertainty increases. In particular, for $\alpha \geq 0.17$ the potential savings drop below 1%, becoming negligible beyond $\alpha = 0.25$. Focusing on self-consumption, the energy self-consumed with and without storage at REC level, according to different magnitudes of uncertainty, is shown in Fig. 13. As one may notice, the role of the storage system becomes increasingly marginal as the uncertainty affecting community profiles grows.

3.3. Computational analysis

Regarding computational aspects, the overall procedure shows very small computation times for the considered problem size. The analysis has been carried out by considering communities involving 10, 100, 1000, and 10 000 entities. For each of these configurations, 40% of entities are supposed to be equipped with storage systems. Performance indicators investigated are: i) optimization time, defined as the time required to solve a single optimization problem instance; and ii) operation time, which represents the time needed to perform load balancing, optimize community operation, and generate a distributed optimal solution. The figure representing optimization and operation times as functions of the community size are reported in Fig. 14. Thanks to the closed-form solution for optimal storage operation derived in Section 2.2, even for the largest community size involving 10 000 entities, the average optimization time is below 600 μs, and also the operation time remains negligible compared to a daily time scale.[1]

3.4. Discussion

Simulation results show how storage facilities can contribute to the effective operation of an energy community. The illustrative example highlights the key aspects of the optimal solution, which exploits the energy surplus of the community to increase the self-consumption later in the day. On the other hand, the realistic example demonstrates the potential benefits by considering a community configuration that is closer to real-world settings. Moreover, the uncertainty analysis showed how the optimal solution behaves in more realistic settings. By optimally operating the storage facilities, both the reported examples show that it is possible to provide economic savings and substantially increase the community self-consumption. From the computational viewpoint, the devised technique shows high scalability with respect to the REC size. The proposed technical results provides further tools to analyze the role of energy storage systems in energy communities, by providing insights into their potential benefits.

4. Conclusions

In this work, the contribution of storage facilities for enhancing the REC operation is analyzed. The load/generation profiles of the community entities are merged on the basis of member typology. Then, the problem is formulated as a linear program where the objective is the minimization of the community cost by exploiting the storage units installed within the REC entities. To this purpose, the minimum value of the self-consumption incentive for triggering the storage operation is derived. Successively, necessary conditions for the optimal storage scheduling are devised. Specifically, the optimal storage operation turns out to be the energy schedule that maximizes the community self-consumption. Next, the overall procedure is adapted to an uncertain framework where load and generation profiles are affected by additive uncertainties.

Numerical results show a consistent cost reduction with respect to scenarios not involving storage units. Most notably, in both the considered examples, the community self-consumption is considerably increased when the proposed method is employed, leading to several environmental benefits. Focusing on the uncertain setup, the optimal storage operation is analyzed according to different levels of uncertainty. The optimal solution shows good performance for limited values of uncertainty, while the presence of a storage system is no longer useful when dealing with high perturbations.

Future research directions will be focused on adapting the optimal operation in real-time settings through receding horizon techniques, as well as enriching the proposed framework with more complex elements such as electric vehicles, shiftable loads, and demand response programs.

CRediT authorship contribution statement

Giovanni Gino Zanvettor: Writing – original draft, Software, Methodology, Conceptualization. **Marco Casini:** Writing – review & editing, Methodology, Funding acquisition, Conceptualization. **Antonio Vicino:** Supervision, Methodology, Investigation.

[1] Simulations have been run using MATLAB on an AMD Ryzen 9 9900x@4.4 GHz with 32 GB RAM.

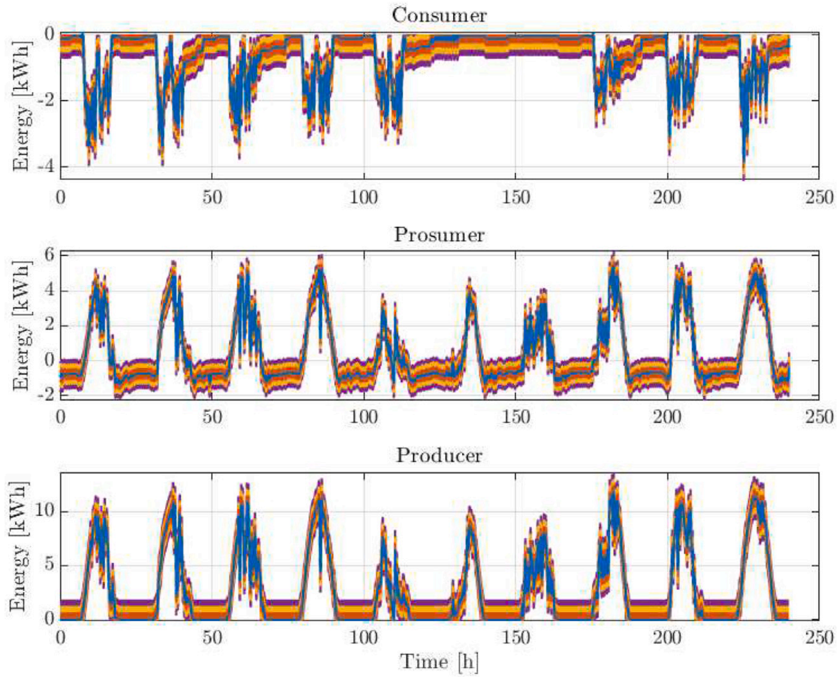

Fig. 11. Realistic example. Nominal profile $\rho_u(t)$ (blue) of different community entities compared with uncertainty levels of α amounting to 5% (orange), 10% (yellow), 15% (purple).

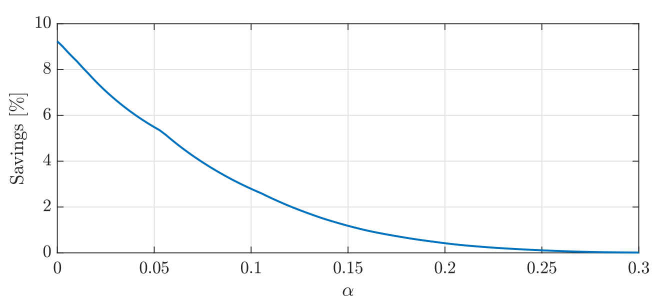

Fig. 12. Realistic example. Percentage of savings as a function of α.

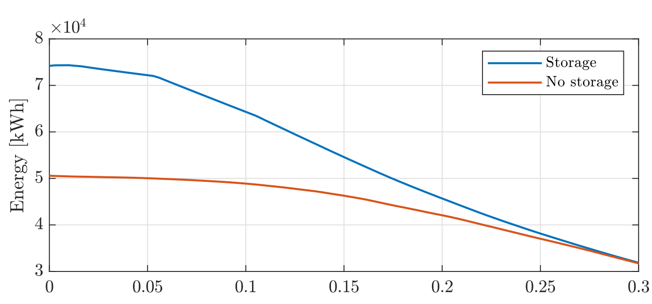

Fig. 13. Realistic example. Community self-consumption with and without storage systems as a function of α.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

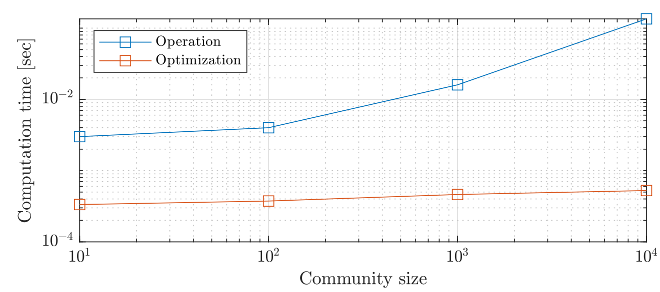

Fig. 14. Average optimization and operation time for different community sizes.

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Appendix A. Proof of Theorem 1

In order to prove Theorem 1, the following lemmas are introduced.

Lemma 2. *The self load balancing control strategy in (7)–(10) enjoys the following properties*

$$\hat{e}_u^c(t) \ge 0 \qquad \forall t \in \mathcal{T}, \tag{A.1}$$

$$\hat{e}_u^d(t) \ge 0 \qquad \forall t \in \mathcal{T}, \tag{A.2}$$

$$0 \le \hat{s}_u(t) \le -\frac{1}{\eta} \sum_{\substack{\tau \ge t \\ \tau \in \mathcal{R}_u^-}} \rho_u(\tau) \qquad \forall t \in \mathcal{T}, \tag{A.3}$$

$$\hat{s}_u(T) = 0. \tag{A.4}$$

Proof. The lemma is proven by induction. At time $t = 0$, by (10) one has $\hat{s}_u(0) = 0$ and so $\hat{e}^d_u(0) = 0$. Moreover,

$$\hat{e}^c_u(0) = \begin{cases} \min\left\{ \bar{e}_u(0), -\frac{1}{\eta^2} \sum_{\substack{\tau>0 \\ \tau\in\mathcal{R}^-_u}} \rho_u(\tau) \right\} & \text{if } \rho_u(0) \geq 0, \\ 0 & \text{else,} \end{cases}$$

which is a non-negative quantity.

Now, assuming that at a given time t

$$\hat{e}^c_u(t) \geq 0,$$
$$\hat{e}^d_u(t) \geq 0,$$
$$0 \leq \hat{s}_u(t) \leq -\frac{1}{\eta} \sum_{\substack{\tau\geq t \\ \tau\in\mathcal{R}^-_u}} \rho_u(\tau), \tag{A.5}$$

it will be shown that the set of inequalities still hold at time $t+1$.

Consider $\hat{s}_u(t+1)$. Since by (8) it holds that $\hat{e}^d_u(t) \leq \eta \hat{s}_u(t)$, by (9) one gets $\hat{s}_u(t+1) \geq 0$ and hence $\hat{e}^d_u(t+1) \geq 0$, which prove the left hand side of (A.2) and (A.3), respectively. To prove the right hand side of (A.3), suppose that $t \in \mathcal{R}^-_u$, then $\hat{e}^c_u(t) = 0$ and $\hat{e}^d_u(t) \geq 0$. By (8), the energy stored at time $t+1$ is

$$\hat{s}_u(t+1) = \hat{s}_u(t) - \frac{1}{\eta}\hat{e}^d_u(t) = \max\left\{ 0, \hat{s}_u(t) + \frac{1}{\eta}\rho_u(t) \right\}$$

and by (A.5) one gets

$$\hat{s}_u(t+1) \leq \max\left\{ 0, -\frac{1}{\eta} \sum_{\substack{\tau\geq t+1 \\ \tau\in\mathcal{R}^-_u}} \rho_u(\tau) \right\} = -\frac{1}{\eta} \sum_{\substack{\tau\geq t+1 \\ \tau\in\mathcal{R}^-_u}} \rho_u(\tau).$$

Now, consider the case $t \in \mathcal{R}^+_u$, which implies $\hat{e}^c_u(t) \geq 0$ and $\hat{e}^d_u(t) = 0$. Then, by (7) one has

$$0 \leq \hat{e}^c_u(t) \leq -\frac{1}{\eta}\hat{s}_u(t) - \frac{1}{\eta^2} \sum_{\substack{\tau>t \\ \tau\in\mathcal{R}^-_u}} \rho_u(\tau),$$

which implies, by (9)

$$\hat{s}_u(t+1) \leq -\frac{1}{\eta} \sum_{\substack{\tau>t \\ \tau\in\mathcal{R}^-_u}} \rho_u(\tau) = -\frac{1}{\eta} \sum_{\substack{\tau\geq t+1 \\ \tau\in\mathcal{R}^-_u}} \rho_u(\tau).$$

To prove (A.1), two cases are distinguished. If $t+1 \in \mathcal{R}^-_u$, it holds $\hat{e}^c_u(t+1) = 0$. If $t+1 \in \mathcal{R}^+_u$, by (A.3) one has

$$\hat{s}_u(t+1) \leq -\frac{1}{\eta} \sum_{\substack{\tau\geq t+1 \\ \tau\in\mathcal{R}^-_u}} \rho_u(\tau) = -\frac{1}{\eta} \sum_{\substack{\tau>t+1 \\ \tau\in\mathcal{R}^-_u}} \rho_u(\tau) \tag{A.6}$$

and hence,

$$-\frac{1}{\eta}\hat{s}_u(t+1) - \frac{1}{\eta^2} \sum_{\substack{\tau>t+1 \\ \tau\in\mathcal{R}^-_u}} \rho_u(\tau) \geq 0. \tag{A.7}$$

Due to (A.7) and since $\bar{e}_u(t) \geq 0$ for all $t \in \mathcal{T}$, from (7) it holds $\hat{e}^c_u(t+1) \geq 0$. Since $\nexists \tau \in \mathcal{R}^-_u$ such that $\tau \geq T$, (A.3) implies $\hat{s}(T) = 0$. □

Lemma 3. *Let $\hat{e}^c_u(t)$, $\hat{e}^d_u(t)$, $\hat{s}_u(t)$ in (7)–(10) be given. If $\bar{e}'_u(t) = 0,\ \forall t \in \mathcal{T}$ then $e^{d\prime}_u(t) = 0,\ \forall t \in \mathcal{T}$. Otherwise, let*

$$t^+_u = \min_{t:\bar{e}'_u(t)>0} t,$$

then

$$e^{d\prime}_u(t) = 0 \quad \forall t \leq t^+_u, t \in \mathcal{T} \tag{A.8}$$
$$\hat{e}^c_u(t) = 0 \quad \forall t > t^+_u, t \in \mathcal{T}. \tag{A.9}$$

Proof. Consider the case $\bar{e}'_u(t) = 0,\ \forall t \in \mathcal{T}$. By (13) it follows that for all $t \in \mathcal{T}$ one has $e^{c\prime}_u(t) = 0$, and hence also $s'_u(t) = 0$, which implies $e^{d\prime}_u(t) = 0$.

Assume now there exists t^+_u. Then, one has $\bar{e}'_u(t) = 0,\ \forall t < t^+_u$. By repeating the same reasoning as above, one gets $e^{d\prime}_u(t) = 0,\ \forall t < t^+_u$. Moreover, since $s'_u(t^+_u) = 0$ one has $e^{d\prime}_u(t^+_u) = 0$, which proves (A.8).

Since $\bar{e}'_u(t^+_u) > 0$, one has $\hat{e}^c_u(t^+_u) < \bar{e}_u(t^+_u)$ and by exploiting (7) one gets

$$\hat{e}^c_u(t^+_u) = -\frac{1}{\eta}\hat{s}_u(t^+_u) - \frac{1}{\eta^2} \sum_{\substack{\tau>t^+_u \\ \tau\in\mathcal{R}^-_u}} \rho_u(\tau). \tag{A.10}$$

Thus, by (9)

$$\hat{s}_u(t^+_u + 1) = -\frac{1}{\eta} \sum_{\substack{\tau>t^+_u \\ \tau\in\mathcal{R}^-_u}} \rho_u(\tau). \tag{A.11}$$

By employing (9), the stored energy at time T can be written as

$$\hat{s}_u(T) = \hat{s}_u(t^+_u + 1) + \sum_{\tau=t^+_u+1}^{T-1} \left(\eta\hat{e}^c_u(\tau) - \frac{1}{\eta}\hat{e}^d_u(\tau) \right). \tag{A.12}$$

Thanks to (A.4), and by combining (A.11)–(A.12), it must hold that

$$-\frac{1}{\eta} \sum_{\substack{\tau>t^+_u \\ \tau\in\mathcal{R}^-_u}} \rho_u(\tau) + \eta \sum_{\tau=t^+_u+1}^{T-1} \hat{e}^c_u(\tau) = \frac{1}{\eta} \sum_{\tau=t^+_u+1}^{T-1} \hat{e}^d_u(\tau) \leq -\frac{1}{\eta} \sum_{\substack{\tau>t^+_u \\ \tau\in\mathcal{R}^-_u}} \rho_u(\tau), \tag{A.13}$$

where the inequality in (A.13) holds if and only if $\hat{e}^c_u(t) = 0,\ \forall t > t^+_u$. □

Proof of Theorem 1. By (9) and (11) it holds that

$$\begin{aligned} s_u(t+1) &= \hat{s}_u(t+1) + s'_u(t+1) \\ &= \hat{s}_u(t) + s'_u(t) + \eta\left(\hat{e}^c_u(t) + e^{c\prime}_u(t)\right) - \frac{1}{\eta}\left(\hat{e}^c_u(t) + e^{d\prime}_u(t)\right) \\ &= s_u(t) + \eta e^c_u(t) - \frac{1}{\eta} e^d_u(t), \end{aligned}$$

and hence (2) holds. Moreover, by using (10), (12) and (A.4), Eq. (3) follows directly.

Concerning $\hat{e}^c_u(t)$ and $e^{c\prime}_u(t)$, by substituting (16) in (13), one has

$$\hat{e}^c_u(t) + e^{c\prime}_u(t) \leq \bar{e}_u(t).$$

By (A.1) one has $\hat{e}^c_u(t) \geq 0$, and then (4) holds.

By (8) and (A.2) one has $0 \leq \hat{e}^d_u(t) \leq \eta\hat{s}_u(t)$. Thus, by exploiting (14), it follows that

$$0 \leq \hat{e}^d_u(t) + e^{d\prime}_u(t) \leq \eta\hat{s}_u(t) + \eta s'_u(t)$$

and hence (5) holds.

In order to prove (6), notice that by (7) and (8) one has $\hat{e}^c_u(t)\cdot\hat{e}^d_u(t) = 0$, whereas (15) states $e^{c\prime}_u(t) \cdot e^{d\prime}_u(t) = 0$, for all $t \in \mathcal{T}$. So, it is sufficient to show that

$$\hat{e}^c_u(t) \cdot e^{d\prime}_u(t) = 0 \quad \forall t \in \mathcal{T}, \tag{A.14}$$

$$e^{c\prime}_u(t) \cdot \hat{e}^d_u(t) = 0 \quad \forall t \in \mathcal{T}. \tag{A.15}$$

Condition (A.14) follows directly from Lemma 3. If $t \in \mathcal{R}^+_u$, by (8) one has $\hat{e}^d_u(t) = 0$ and hence $e^{c\prime}_u(t)\cdot\hat{e}^d_u(t) = 0$. Consider $t \in \mathcal{R}^-_u$, that is $\rho_u(t) < 0$. By (1) and (7) one has $\bar{e}_u(t) = 0$ and $\hat{e}^c_u(t) = 0$, respectively. Then, by (16), $\bar{e}'_u(t) = 0$ which, by (13), implies $e^{c\prime}_u(t) = 0$. Thus, (A.15) holds $\forall t \in \mathcal{T}$. □

Appendix B. Proof of Theorem 2

To prove Theorem 2, the following lemmas are introduced.

Lemma 4. *Let $E^{c^*}(t)$ and $E^{d^*}(t)$, $\forall t \in \mathcal{T}$ be the optimal charging and discharging control signals for Problem 2. Let $t_1 \in \mathcal{T}$ be such that $E^{c^*}(t_1) > 0$ and set $t_2 = \min\{t > t_1 : E^{d^*}(t) > 0\}$. Let $0 < \varepsilon < \min\{E^{c^*}(t_1), \frac{1}{\eta^2}E^{d^*}(t_2)\}$, and let $\widetilde{S}$ be a charging/discharging policy composed by the following control signals:*

$$\widetilde{E}^c(t_1) = E^{c^*}(t_1) - \varepsilon, \qquad \widetilde{E}^c(t) = E^{c^*}(t),\ \forall t \neq t_1,$$

$$\widetilde{E}^d(t_2) = E^{d^*}(t_2) - \eta^2\varepsilon, \qquad \widetilde{E}^d(t) = E^{d^*}(t),\ \forall t \neq t_2,$$

Then, $\widetilde{S}$ is a feasible solution for Problem 2.

Proof. First, notice that t_2 always exists. In fact, since $S^*(t_1+1) > 0$ and $S^*(T) = 0$, there exists a time $t = t_1+1, \dots, T-1$ where the storage is discharged.

Now, the feasibility of $\widetilde{S}$ is proved. For any time $t < t_1$ the two solutions are identical. At time t_1 one has $0 < \widetilde{E}^c(t_1) < E^{c^*}(t_1)$ and by Lemma 1 $\widetilde{E}^d(t_1) = E^{d^*}(t_1) = 0$. So, the charging control signal at t_1 is feasible. For any time $t_1 < t < t_2$ both solutions involve the same charging control signals, while $\widetilde{E}^d(t) = E^{d^*}(t) = 0$. At time t_2, the storage is discharged by $\widetilde{E}^d(t_2)$. In order to be feasible, it must be guaranteed $\widetilde{E}^d(t_2) \leq \eta\widetilde{S}(t_2)$, according to (23). Since $\widetilde{E}^d(t) = 0$ for $t = t_1+1, \dots, t_2-1$, one has

$$\widetilde{S}(t_2) = S^*(t_1) + \eta\widetilde{E}^c(t_1) + \eta\sum_{\tau=t_1+1}^{t_2-1} E^{c^*}(\tau) = S^*(t_1) + \eta\sum_{\tau=t_1}^{t_2-1} E^{c^*}(\tau) - \eta\varepsilon.$$

So,

$$\begin{aligned}\widetilde{S}(t_2+1) &= S^*(t_1) + \eta\sum_{\tau=t_1}^{t_2-1} E^{c^*}(\tau) - \eta\varepsilon - \frac{1}{\eta}\widetilde{E}^d(t_2)\\ &= S^*(t_1) + \eta\sum_{\tau=t_1}^{t_2-1} E^{c^*}(\tau) - \eta\varepsilon - \frac{1}{\eta}(E^{d^*}(t_2) - \eta^2\varepsilon)\\ &= S^*(t_1) + \eta\sum_{\tau=t_1}^{t_2-1} E^{c^*}(\tau) - \frac{1}{\eta}E^{d^*}(t_2) = S^*(t_2+1).\end{aligned}$$

Since $\widetilde{S}(t_2+1) = S^*(t_2+1)$, and because the two solutions are identical from time t_2+1 onwards, $\widetilde{S}$ is a feasible solution for Problem 2. □

Lemma 5. *Let $E^{c^*}(t)$ and $E^{d^*}(t)$, $\forall t \in \mathcal{T}$ be the optimal charging and discharging control signals for Problem 2. Let $t_2 \in \mathcal{T}$ be such that $E^{d^*}(t_2) > 0$ and set $t_1 = \max\{t < t_2 : E^{c^*}(t) > 0\}$. Let $0 < \varepsilon < \min\{\eta^2 E^{c^*}(t_1), E^{d^*}(t_2)\}$ and let $\widetilde{S}$ be a charging/discharging policy composed by the following control signals:*

$$\widetilde{E}^c(t_1) = E^{c^*}(t_1) - \frac{1}{\eta^2}\varepsilon, \qquad \widetilde{E}^c(t) = E^{c^*}(t),\ \forall t \neq t_1,$$
$$\widetilde{E}^d(t_2) = E^{d^*}(t_2) - \varepsilon, \qquad \widetilde{E}^d(t) = E^{d^*}(t),\ \forall t \neq t_2,$$

Then, $\widetilde{S}$ is a feasible solution for Problem 2.

Proof. The proof follows the same reasoning as that of Lemma 4. □

Proof of Theorem 2. First, (33) is proven. Let $t_1 \in \mathcal{T}$ be such that $L(t_1) \geq R(t_1)$. By contradiction assume $E^{c^*}(t_1) > 0$. Let $\widetilde{S}$ be defined as in Lemma 4, so it is a feasible solution for Problem 2. Let J^* be the optimal cost of Problem 2, while $\widetilde{J}$ be the cost related to $\widetilde{S}$. Notice that J^* and $\widetilde{J}$ differ only in the terms depending on t_1 and t_2. Moreover, by Lemma 1, $E^{d^*}(t_1) = E^{c^*}(t_2) = \widetilde{E}^d(t_1) = \widetilde{E}^c(t_2) = 0$. Thus,

$$J^* - \widetilde{J} = -k\left(A^{s^*}(t_1) - \widetilde{A}^s(t_1)\right) + \alpha\left(E^{d^*}(t_2) - \widetilde{E}^d(t_2)\right) - k\left(A^{s^*}(t_2) - \widetilde{A}^s(t_2)\right).$$

Since $L(t_1) \geq R(t_1)$, by (26) one has

$$\begin{aligned}&A^{s^*}(t_1) = R(t_1) - E^{c^*}(t_1),\\ &\widetilde{A}^s(t_1) = R(t_1) - \widetilde{E}^c(t_1) = R(t_1) - E^{c^*}(t_1) + \varepsilon.\end{aligned}$$

and hence

$$J^* - \widetilde{J} = k\varepsilon + \alpha\eta^2\varepsilon - k\left(A^{s^*}(t_2) - \widetilde{A}^s(t_2)\right).$$

By definition

$$\begin{aligned}\widetilde{A}^s(t_2) &= \min\{L(t_2), R(t_2) + \widetilde{E}^d(t_2)\} = \min\{L(t_2), R(t_2) + E^{d^*}(t_2) - \eta^2\varepsilon\}\\ &= \min\{L(t_2) + \eta^2\varepsilon, R(t_2) + E^{d^*}(t_2)\} - \eta^2\varepsilon \geq A^{s^*}(t_2) - \eta^2\varepsilon.\end{aligned}$$

Then,

$$J^* - \widetilde{J} \geq k\varepsilon + \alpha\eta^2\varepsilon - k\eta^2\varepsilon = k\varepsilon(1-\eta^2) + \alpha\eta^2\varepsilon > 0.$$

Since $J^* > \widetilde{J}$ a contradiction occurs. To prove (34), a specular reasoning can be repeated by exploiting Lemma 5.

Now, (35) is proven. Let $t_1 \in \mathcal{T}$ be such that $L(t_1) \leq R(t_1)$. By contradiction assume $E^{c^*}(t_1) > R(t_1) - L(t_1)$. Let $\widetilde{S}$ be defined as in Lemma 4 and assume $0 < \varepsilon < \min\{E^{c^*}(t_1), \frac{1}{\eta^2}E^{d^*}(t_2), E^{c^*}(t_1) - R(t_1) + L(t_1)\}$. Since $R(t_1) \geq L(t_1)$ one has $E^{c^*}(t_1) > E^{c^*}(t_1) - R(t_1) + L(t_1)$. So, choosing ε such that $0 < \varepsilon < \min\{\frac{1}{\eta^2}E^{d^*}(t_2), E^{c^*}(t_1) - R(t_1) + L(t_1)\}$ leads to a feasible solution to Problem 2.

Let J^* be the optimal cost of Problem 2, while $\widetilde{J}$ be the cost related to $\widetilde{S}$. Notice that J^* and $\widetilde{J}$ differ only in the terms depending on t_1 and t_2. Moreover, by Lemma 1, $E^{d^*}(t_1) = E^{c^*}(t_2) = \widetilde{E}^d(t_1) = \widetilde{E}^c(t_2) = 0$. Thus,

$$J^* - \widetilde{J} = -k\left(A^{s^*}(t_1) - \widetilde{A}^s(t_1)\right) + \alpha\left(E^{d^*}(t_2) - \widetilde{E}^d(t_2)\right) - k\left(A^{s^*}(t_2) - \widetilde{A}^s(t_2)\right).$$

Since $E^{c^*}(t_1) > R(t_1) - L(t_1)$ one has

$$\begin{aligned}A^{s^*}(t_1) &= \min\{L(t_1), R(t_1) - E^{c^*}(t_1)\}\\ &= \min\{L(t_1) - R(t_1), -E^{c^*}(t_1)\} + R(t_1) = R(t_1) - E^{c^*}(t_1).\end{aligned}$$

Recalling that $\varepsilon < E^{c^*}(t_1) - R(t_1) + L(t_1)$

$$\begin{aligned}\widetilde{A}^s(t_1) &= \min\{L(t_1), R(t_1) - \widetilde{E}^c(t_1)\} = \min\{L(t_1) - R(t_1), -E^{c^*}(t_1) + \varepsilon\} + R(t_1)\\ &= \min\{E^{c^*}(t_1) - R(t_1) + L(t_1), \varepsilon\} + R(t_1) - E^{c^*}(t_1) = R(t_1) - E^{c^*}(t_1) + \varepsilon.\end{aligned}$$

Thus $A^{s^*}(t_1) - \widetilde{A}^s(t_1) = -\varepsilon$ and hence

$$J^* - \widetilde{J} = k\varepsilon + \alpha\eta^2\varepsilon - k\left(A^{s^*}(t_2) - \widetilde{A}^s(t_2)\right).$$

By definition

$$\begin{aligned}\widetilde{A}^s(t_2) &= \min\{L(t_2), R(t_2) + \widetilde{E}^d(t_2)\} = \min\{L(t_2), R(t_2) + E^{d^*}(t_2) - \eta^2\varepsilon\}\\ &= \min\{L(t_2) + \eta^2\varepsilon, R(t_2) + E^{d^*}(t_2)\} - \eta^2\varepsilon \geq A^{s^*}(t_2) - \eta^2\varepsilon.\end{aligned}$$

Then,

$$J^* - \widetilde{J} \geq k\varepsilon + \alpha\eta^2\varepsilon - k\eta^2\varepsilon = k\varepsilon(1-\eta^2) + \alpha\eta^2\varepsilon > 0.$$

Since $J^* > \widetilde{J}$ a contradiction occurs. To prove (36), a specular reasoning can be repeated by exploiting Lemma 5 and by setting $\varepsilon < E^{d^*}(t_2) - L(t_2) + R(t_2)$. □

Appendix C. Proof of Corollary 1

Proof. Consider the case $L(t) \leq R(t)$. By (34), $E^{d^*}(t) = 0$, and by (35), one has $E^{c^*}(t) \leq R(t) - L(t)$. Thus,

$$A^{s^*}(t) = \min\{L(t), R(t) - E^{c^*}(t)\} = L(t).$$

Analyze the case $L(t) \geq R(t)$. By (33), $E^{c^*}(t) = 0$ and hence

$$A^{s^*}(t) = \min\{L(t), R(t) + E^{d^*}(t)\}.$$

By (36), $E^{d^*}(t) \leq L(t) - R(t)$ and hence $A^{s^*}(t) = R(t) + E^{d^*}(t)$. □

Appendix D. Proof of Theorem 3

In order to prove Theorem 3, the following lemma is introduced.

Lemma 6. *Let $L^\xi(t)$, $R^\xi(t)$ and $\overline{E}^\xi(t)$ be defined as in (55)–(57). Then, they can be written as*

$$L^\xi(t) = L(t) - \Delta^L(t), \tag{D.1}$$
$$R^\xi(t) = R(t) + \Delta^R(t), \tag{D.2}$$
$$\overline{E}^\xi(t) = \overline{E}(t) + \Delta^E(t), \tag{D.3}$$

where $\Delta^L(t)$, $\Delta^R(t)$ and $\Delta^E(t)$ are defined as in (58)–(60).

Proof. To prove (D.1), by exploiting (54) it is possible to write (55) as

$$L^\xi(t) = \sum_{u\in\mathcal{U}} \max\{0, -\omega_u(t)\} = \sum_{u\in\mathcal{U}} \max\{0, -\rho_u'(t) - \xi_u(t)\}$$

$$= \sum_{u\in\mathcal{U}} \max\{0, -\rho'_u(t)\} + \max\{-\xi_u(t), \rho'_u(t)\} + \min\{0, -\rho'_u(t)\}$$
$$= \sum_{u\in\mathcal{U}} \max\{0, -\rho'_u(t)\} - \min\{\xi_u(t), -\rho'_u(t)\} + \min\{0, -\rho'_u(t)\}.$$

Concerning (D.2), a similar reasoning can be made. In particular, it is possible to write

$$R^{\xi}(t) = \sum_{u\in\mathcal{U}} \max\{0, \omega_u(t)\} = \sum_{u\in\mathcal{U}} \max\{0, \rho'_u(t) + \xi_u(t)\}$$
$$= \sum_{u\in\mathcal{U}} \max\{0, \rho'_u(t)\} + \max\{\xi_u(t), -\rho'_u(t)\} + \min\{0, \rho'_u(t)\}$$
$$= \sum_{u\in\mathcal{U}} \max\{0, \rho'_u(t)\} + \max\{\xi_u(t), -\rho'_u(t)\} - \max\{0, -\rho'_u(t)\}.$$

By (17) and (58), the aggregated load under uncertainty can be written as

$$L^{\xi}(t) = L(t) - \Delta^L(t),$$

while, by (18) and (59), it holds that

$$R^{\xi}(t) = R(t) + \Delta^R(t).$$

Finally, the derivation of (D.3) follows the same reasoning of (D.2) applied to the set $\mathcal{U}_s$. □

Proof of Theorem 3. By exploiting Lemma 6 and (62), it is possible to write (61) as

$$\hat{J}^{\xi} = \sum_{\tau\in\mathcal{T}} c^p L(\tau) - c^p \Delta^L(t) - c^s R(t) - c^s \Delta^R(t) - c^s E^d(t) +$$
$$c^s E^c(t) - k \min\{L(t) - \Delta^L(t),\ R(t) + \Delta^R(t) + E^d(t) - E^c(t)\}.$$

Note that, $R(t)$, $L(t)$, $E^d(t)$ and $E^c(t)$ do not involve any uncertainty. So, the optimal solution $\mathcal{E}^*$ will be the same achieved by removing the terms independent on the uncertainty from $\hat{J}^{\xi}$. Then, one has

$$\arg\max_{\mathcal{E}} \hat{J}^{\xi} = \arg\max_{\mathcal{E}} J^{\xi}$$

where

$$J^{\xi} = \sum_{\tau\in\mathcal{T}} (k - c_p)\Delta^L(t) - c^s \Delta^R(t) - k \min\{L(t),\ R(t) + \Delta^L(t) + \Delta^R(t) + E^d(t) - E^c(t)\}.$$

Note that, since $k < c_p$, the coefficients multiplying the uncertain terms $\Delta^L(t)$ and $\Delta^R(t)$ are negative. Thus, the highest value of J^{ξ} is attained for the lowest values of $\Delta^L(t)$ and $\Delta^R(t)$, i.e., for $\Delta^L(t) = \Delta^R(t) = 0,\ \forall t$. Moreover, by exploiting (58) and (59), it holds that

$$\Delta^L(t) = \Delta^R(t) = 0,\ \forall t \implies \xi_u(t) = 0,\ \forall t,\ \forall u \in \mathcal{U}.$$

Finally, by using (60), constraint (63) can be written as

$$0 \le E^c(t) \le \overline{E}(t) + \Delta^E(t) \quad \forall t \in \mathcal{T}.$$

Since $\Delta^E(t) \ge 0,\ \forall t \in \mathcal{T}$, the worst-case realization of the uncertainty is the one that provides the smallest feasible set. Such a condition is obtained when $\Delta^E(t) = 0,\ \forall t \in \mathcal{T}$. Then, the worst-case realization is again attained by imposing $\xi_u(t) = 0,\ \forall t \in \mathcal{T},\ \forall u \in \mathcal{U}_s$. □

Appendix E. Proof of Corollary 2

Proof. By (54), for a given $u \in \mathcal{U}$ and $t \in \mathcal{T}$, the perturbed profile is

$$\omega_u(t) = \rho'_u(t) + \xi_u(t).$$

Define the new variables

$$\phi_u(t) = \rho'_u(t) + \underline{\xi}_u(t),$$
$$\psi_u(t) = \xi_u(t) - \underline{\xi}_u(t).$$

Clearly, $\psi_u(t) \in [0, \overline{\xi}_u(t) - \underline{\xi}_u(t)]$, and $\omega_u(t) = \phi_u(t) + \psi_u(t)$. By Theorem 3, one has that $\psi^*_u(t) = 0$ leads to the worst-case realization of the uncertainty. Therefore, the associated perturbed profile is equal to

$$\omega^*_u(t) = \phi_u(t) = \rho'_u(t) + \underline{\xi}_u(t),$$

where the worst-case uncertainty corresponds to $\xi^*_u(t) = \underline{\xi}_u(t)$. □

Data availability

Data will be made available on request.

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