A Family of Switching Pursuit Strategies for a Multi-Pursuer Single-Evader Game

Marco Casini, Andrea Garulli

Abstract

A new family of pursuit strategies is introduced for a multi-pursuer single-evader game. By exploiting the optimal solution of the game involving two pursuers, conditions are derived under which the multi-pursuer game becomes equivalent to the two-pursuer one. This opens the possibility of designing a number of pursuit strategies in which the pursuers first try to enforce the satisfaction of the aforementioned condition and then switch to a two-pursuer game as soon as it is verified. The contribution is useful in two ways. First, new winning pursuit strategies can be devised starting from simple plans, such as pure pursuit. Moreover, the performance of existing pursuit strategies, like those based on Voronoi partitions, can be significantly improved by resorting to the corresponding switching version.

Index Terms

Pursuit-evasion games; multi-agent systems; autonomous agents.

I. Introduction

Pursuit-evasion games provide a general framework for a number of problems in the broad research area of multi-agent systems, with applications in mobile robotics [1], [2], optimal robot control [3], space operations [4], [5], predator-prey interactions in living species [6], and many other fields. In recent years, there has been a growing interest towards games involving either multiple pursuers [7], [8], [9], multiple evaders [10], or both [11], with a wide variety of alternative problem formulations and solutions.

The standard way to tackle pursuit-evasion problems is to formulate them as differential games [12], [13]. However, finding an optimal solution within this setting is often challenging, especially when multiple agents are involved. A notable exception is that of the game

M. Casini and A. Garulli are with the Dipartimento di Ingegneria dell'Informazione e Scienze Matematiche, Università di Siena, via Roma 56, 53100 Siena, Italy. E-mail: marco.casini@unisi.it, andrea.garulli@unisi.it.

involving two pursuers and one evader, moving in simple motion within a two-dimensional environment, for which a minimum-time solution is available [14], [15], [16]. Conversely, for the three-pursuer single-evader game, only suboptimal solutions have been proposed [17]. Moreover, it has been observed that optimal strategies must involve switching among different linear sub-paths [18], [19]. For games involving more than three pursuers, a minimum-time open-loop solution is proposed in [20], showing that all agents have to travel along linear paths. However, a closed-loop solution of the differential game has not been found yet.

Motivated by the above observations, a new family of pursuit strategies is proposed in this paper, for a multi-pursuer single-evader game in which planar agents move in simple motion with equal maximum speed. The main idea is to derive and exploit conditions under which the multi-pursuer game becomes equivalent to a game involving only two pursuers, and then devise strategies for the pursuer team in order to enforce such conditions and switch to a two-pursuer game as soon as they are verified. The resulting switching pursuit strategies, albeit suboptimal, always reduce the capture time with respect to the corresponding strategies without switching. Therefore, they are able to improve the performance of existing pursuit techniques and allow one to design new successful multi-pursuer strategies.

The contribution of the paper is threefold: i) all the agents configurations in which capture of the evader can be achieved are characterized, thus providing a complete solution of the game of kind; ii) conditions are derived under which the minimum-time pursuit strategy boils down to that of a two-pursuer single-evader game; iii) by exploiting the aforementioned conditions, new families of switching pursuit strategies guaranteeing capture are devised. In particular, very simple pursuit strategies (like, e.g., pure pursuit), which in general may end up with the evader escape, are transformed into winning pursuit strategies by switching to a suitable two-pursuer game. Moreover, it is shown via numerical simulations that the switching mechanism can remarkably reduce the capture time of techniques that are guaranteed to achieve capture, like the one based on Voronoi partitions proposed in [7].

The paper is organized as follows. The considered multi-pursuer single-evader game is formulated in Section II, which provides also a review of the optimal solution of the two-pursuer game. The solution of the game of kind and the conditions for switching to the two-pursuer game are derived in Section III. New switching pursuit strategies are proposed in Section IV and analyzed via numerical simulations in Section V. Finally, some concluding remarks are given in Section VI. Due to lack of space, the proofs of the results are omitted and can be found in [21].

A. *Notation and definitions*

Given a vector V, its transpose is denoted by V', while $\|V\|$ is its Euclidean norm. For $V, W \in \mathbb{R}^2$, we denote by $\overline{VW}$ the segment with V and W as endpoints. $\mathcal{C}(P, r) = \{Q \in \mathbb{R}^2 \colon \|Q - P\| \le r\}$ denotes a circle centered in $P \in \mathbb{R}^2$ with radius r. For a closed set $\mathcal{A}$, $\partial\mathcal{A}$ is the boundary of $\mathcal{A}$, while $\mathbb{H}\{\mathcal{A}\}$ denotes the convex hull of $\mathcal{A}$. The interior of $\mathcal{A}$ is represented by $\mathtt{int}\{\mathcal{A}\} = \mathcal{A} \backslash \partial\mathcal{A}$.

II. Problem formulation

In this paper, we consider a *multi-pursuer one-evader game* (hereafter referred to as MP1EG) where p pursuers aim at capturing one evader. The positions of the pursuers and the evader at time t are denoted by $P_k(t) \in \mathbb{R}^2, k = 1, \dots p$, and $E(t) \in \mathbb{R}^2$, respectively. The players move in the plane in simple motion, that is

$$\begin{cases} \dot{E}(t) = v_E(t)\,, \\ \dot{P}_k(t) = v_{P_k}(t)\,,\; k = 1, \dots, p \end{cases}$$

where $v_E(t), v_{P_k}(t) \in \mathbb{R}^2$ are the velocity vectors of the players. It is assumed that all agents have the same maximum velocity, which w.l.o.g. is set to 1, that is $\|v_E(t)\| \le 1$, $\|v_{P_k}(t)\| \le 1,\; k = 1, \dots, p$, $\forall t \ge 0$. At each time t, $v_E(t)$ and $v_{P_k}(t)$ are computed as functions of the state of the game $\xi(t) = [E'(t)\; P_1'(t)\; \dots\; P_p'(t)]'$. Notice that the agents do not have information on the velocities of the other players. The *strategies* of the agents are functions mapping the current state into their chosen velocity vector, $v_E = \phi(\xi)$, $v_{P_k} = \psi_k(\xi)$ (dependence on time is omitted when it is clear from the context).

The aim of the pursuers is to capture the evader, while the evader tries to avoid capture or to protract the game as long as possible. Capture occurs when the distance between at least one pursuer and the evader is equal to the radius of capture $r > 0$. Hereafter, only initial conditions such that

$$\|P_k(0) - E(0)\| > r, \quad k = 1, \dots, p \tag{1}$$

will be considered. A *winning pursuit strategy* is a strategy adopted by the pursuers which guarantees that capture occurs in finite time whatever is the strategy adopted by the evader. The capture time T is such that

$$\|P_k(T) - E(T)\| = r, \text{ for some } k \in \{1, \dots, p\}. \tag{2}$$

A *minimum time pursuit strategy* is a strategy $\psi = [\psi_1, \ldots, \psi_p]$ which is a solution of the min-max problem

$$\min_{\psi} \max_{\phi} \; T. \tag{3}$$

For brevity, we will refer to functions ψ and ϕ solving problem (3) as *optimal strategies*. To the best of our knowledge, (3) is still an open problem for $p \geq 3$. Conversely, for $p = 2$ the problem has been fully solved [14], [15], [16]. In the following, the solution of the *two-pursuer one-evader game* (`2P1EG`) given in [16] is recalled. It will play a key role in the definition of the class of winning strategies for the `MP1EG` proposed in this work.

A. Minimum-time solution of the two-pursuer one-evader game

Consider two pursuers P_1, P_2 trying to capture one evader. Let us define the region

$$\mathcal{D}_{12} = \texttt{int}\left\{\mathbb{H}\left\{\mathcal{C}(P_1, r) \cup \mathcal{C}(P_2, r)\right\}\right\}. \tag{4}$$

First, a solution to the game of kind is provided [16].

Proposition 1: There exists a winning pursuit strategy for the `2P1EG` if and only if $E \in \mathcal{D}_{12}$. □

Hence, set $\mathcal{D}_{12}$ is the *2-pursuer capture region* associated to P_1 and P_2; if the evader belongs to this region it will be captured by the pursuers in finite time. An example is depicted in Fig. 1. To simplify the treatment, w.l.o.g. we will refer to the reference frame reported in Fig. 1, where the origin is the midpoint of the pursuers, and they lie on the x-axis, i.e., $P_1 = [-d, 0]'$, $P_2 = [d, 0]'$, $E = [x, y]'$. Let $E \in \mathcal{D}_{12} \backslash (\mathcal{C}(P_1, r) \cup \mathcal{C}(P_2, r))$ and define

$$T_{12} = \frac{\kappa r + |y|\sqrt{\kappa^2 - 4x^2(r^2 - y^2)}}{2(r^2 - y^2)} \tag{5}$$

where $\kappa = d^2 - x^2 + y^2 - r^2$. Now, let us define the point H_{12} satisfying

$$\|E - H_{12}\| = \|P_1 - H_{12}\| - r = \|P_2 - H_{12}\| - r = T_{12}. \tag{6}$$

Within the considered reference frame, one has $H_{12} = [0,\, h_{12}]'$ where

$$h_{12} = \begin{cases} y + \text{sign}(y)\sqrt{T_{12}^2 - x^2} & \text{if } y \neq 0 \\ \sqrt{T_{12}^2 - x^2} & \text{if } y = 0. \end{cases}$$

The following result provides a solution to the minimum-time `2P1EG` [16].

Proposition 2: Let $p = 2$. The strategies solving problem (3) require each player to go along a straight line to the point H_{12} defined by (6). Then, the evader will be captured in H_{12}, at time T_{12} given by (5). □

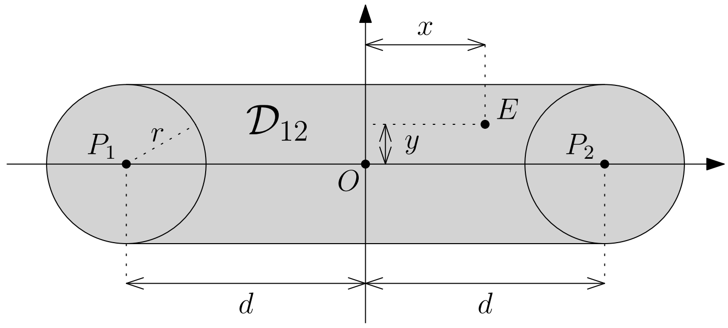


Fig. 1. The 2-pursuer capture region for the 2P1EG.

III. Multi-pursuer games: game of kind and switching conditions

In this section, the solution of the game of kind for the MP1EG is presented. Then, a condition is derived under which the MP1EG boils down to a 2P1EG.

For a pair of pursuers P_i, P_j, let $\mathcal{D}_{ij}$ be the corresponding 2-pursuer capture region, given by (4). If $E \in \mathcal{D}_{ij}$, let us denote by ψ^{ij} the minimum-time pursuit strategy for the 2P1EG played by P_i and P_j, and by ϕ^{ij} the corresponding optimal evader strategy. Moreover, we denote by T_{ij} the time needed by P_i and P_j to capture E when the agents P_i, P_j and E play the optimal 2P1EG strategies. The capture time T_{ij} and capture point H_{ij} are computed according to (5)-(6), for each pair i, j.

Let us define the convex hull of the pursuer locations

$$\mathcal{P} = \mathbb{H}\{P_1, P_2, \ldots, P_p\} \tag{7}$$

and introduce the *multi-pursuer capture region*, defined as

$$\mathcal{M} = \mathtt{int}\left\{\mathbb{H}\left\{\bigcup_{i=1}^{p} \mathcal{C}(P_i, r)\right\}\right\}. \tag{8}$$

The following result provides the solution of the game of kind for the MP1EG.

Theorem 1: If $E \in \mathcal{M}$ in (8), then there exists a winning pursuit strategy. On the contrary, if $E \notin \mathcal{M}$, then the evader can avoid capture indefinitely. □

Fig. 2 shows an example in which the evader does not belong to the capture region $\mathcal{M}$, and then it can escape from the pursuers by moving straight in the direction v.

The next theorem reports conditions under which the MP1EG reduces to the 2P1EG, in the sense that only two pursuers can provide capture in minimum time, irrespectively of the strategy played by the other pursuers.

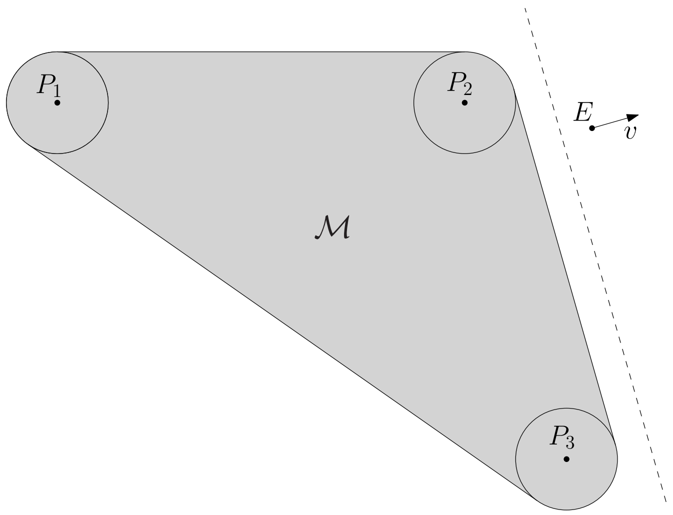


Fig. 2. Example of multi-pursuer capture region $\mathcal{M}$ for $p = 3$. Since $E \notin \mathcal{M}$, the evader can avoid capture by moving along the direction v.

Theorem 2: Let $E \in \mathcal{D}_{ij}$ for some $i, j \in \{1, 2, \dots, p\}$. If

$$\|H_{ij} - P_k\| \geq T_{ij} + r, \quad k = 1, \dots, p \tag{9}$$

then the optimal MP1EG strategies for P_i, P_j, E are ψ^{ij} and ϕ^{ij}, respectively, guaranteeing capture at time T_{ij}. The strategies adopted by the other pursuers are irrelevant to the game duration. □

Notice that, if P_i, P_j and E play their optimal 2P1EG strategies, one has $\|H_{ij} - P_i\| = \|H_{ij} - P_j\| = T_{ij} + r$ and so (9) can be rewritten as

$$\|H_{ij} - P_k\| \geq \|H_{ij} - P_i\|, \quad k = 1, \dots, p. \tag{10}$$

Such a condition states that if there is no pursuer closer to H_{ij} than P_i (and P_j), then (9) holds and the optimal MP1EG strategies for P_i, P_j, E are ψ^{ij} and ϕ^{ij}. This means that the other pursuers cannot improve the capture time even if they go straight to H_{ij}. As a direct consequence of Theorem 2, if there exist two distinct pairs of pursuers (P_i, P_j) and (P_h, P_l) satisfying the conditions of Theorem 2, then $T_{ij} = T_{hl}$.

Now, assume that condition (9) or (10) is satisfied by a pair of pursuers P_i, P_j at a certain time t. Theorem 2 states that the strategies of these two pursuers must switch to ψ^{ij} in order to be optimal from time t onwards, irrespectively of what the other pursuers will do. Similarly, the evader must switch to strategy ϕ^{ij} in order to maximize its survival time. This fact will be exploited in the next section to devise a new family of winning pursuit strategies for the MP1EG.

IV. Switching pursuit strategies

In this section, the concept of *switching pursuit strategy* is introduced. The aim of such strategies is to guarantee that condition (9) holds at some finite time, and hence capture is assured by switching to the appropriate `2P1EG` strategies.

Definition 1: A pursuit strategy is a *potential switching strategy* if there exists a pair of pursuers P_i, P_j for which condition (9) holds at some finite time, for any possible strategy of the evader. A potential switching strategy is referred to as *switching strategy* if P_i, P_j play the optimal `2P1EG` strategy as soon as condition (9) holds. □

By the previous definition, a switching strategy leads to the evader capture in finite time, and so it is a winning strategy. In fact, from the switching time onwards the pursuers will play the optimal `2P1EG` strategy, guaranteeing the evader capture.

It is easy to see that for every winning strategy, capture is achieved simultaneously by at least two pursuers. The following result holds.

Theorem 3: Let the pursuers adopt a winning strategy and assume that capture occurs at time $\bar{t}$. Then, at time $\bar{t}$ condition (9) is satisfied. □

Theorem 3 states that for a winning strategy condition (9) must be satisfied *at least* when capture occurs. However, this turns out to be a very special case, as in most games such a condition is satisfied long before capture occurs (it may be satisfied even when the game starts). Since Theorem 2 guarantees that when condition (9) holds it is always advantageous for the pursuers to switch to the `2P1EG` strategy, it can be concluded that every winning strategy is a potential switching strategy and its capture time cannot be increased by switching.

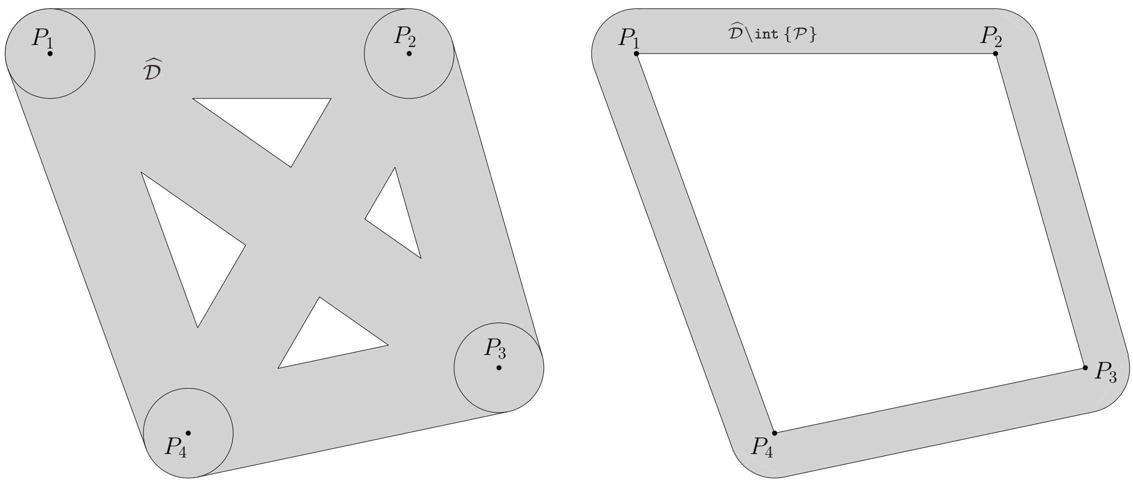
In general, it is not easy to design pursuit strategies that guarantee satisfaction of condition (9). Therefore, it is useful to derive sufficient conditions under which (9) holds, that the pursuers can enforce whatever is the strategy played by the evader. Hereafter, two such conditions are presented.

Let us define the union of all the 2-pursuer capture regions as

$$\widehat{\mathcal{D}} = \bigcup_{i,j \in \{1,\dots,p\}} \mathcal{D}_{ij}.$$

An example of set $\widehat{\mathcal{D}}$ is shown in Fig. 3-(a), while the corresponding set $\widehat{\mathcal{D}} \backslash \mathtt{int}\,\{\mathcal{P}\}$ is depicted in Fig. 3-(b). The next theorem provides a sufficient condition for switching to `2P1EG`.

Theorem 4: If $E \in \widehat{\mathcal{D}} \backslash \mathtt{int}\,\{\mathcal{P}\}$, then there exists a pair of pursuers P_i, P_j for which condition (9) holds. □



(a) (b)

Fig. 3. (a) Set $\widehat{D}$ denoting the union of all the 2-pursuer capture regions; (b) set $\widehat{\mathcal{D}}\backslash\mathtt{int}\,\{\mathcal{P}\}$.

Notice that, in particular, Theorem 4 guarantees the satisfaction of (9) whenever $E \in \partial\mathcal{P}$. Moreover, Theorem 4 suggests that if the pursuers play a strategy which steadily reduces the size (in some sense) of $\mathcal{P}$ over time, then (9) is eventually assured. Similarly, the following result guarantees the satisfaction of (9) whenever the pursuers are sufficiently close to each other.

Theorem 5: Let us consider a pursuit strategy such that

$$\max_{i,j=1,\dots,p} \|P_i(\bar{t}) - P_j(\bar{t})\| \leq \sqrt{3}r \tag{11}$$

at a certain time $\bar{t}$. Then, there exist a time $\hat{t} < \bar{t}$ and a pair of pursuers P_i, P_j for which condition (9) holds at $\hat{t}$. Hence, the considered pursuit strategy is a potential switching strategy. □

Notice that condition (11) in Theorem 5 is much simpler than (9), since it does not involve the evader position. Therefore, it can be effectively employed in designing families of winning switching strategies, starting from very simple pursuit strategies that would not be successful without switching. An example is that of a *Fixed-Point Pursuit Strategy (FPPS)*, in which each pursuer P_k goes straight towards a fixed point $M \in \mathbb{R}^2$, with speed $\|v_{P_k}(t)\| \geq \varepsilon$, for some $0 < \varepsilon \leq 1$, and stops once it reaches M.

Theorem 6: Any FPPS is a potential switching strategy. □

An example of FPPS is the *centroid-based* strategy, where all pursuers move with the same velocity towards the center of the minimum-radius circle which includes them.

Another simple pursuit strategy is the so-called *pure pursuit*. Such a strategy is defined as the one in which all pursuers always point towards the evader at maximum velocity, i.e.

$$v_{P_k}(t) = \frac{E(t) - P_k(t)}{\|E(t) - P_k(t)\|}, \quad k = 1, \ldots, p. \tag{12}$$

It is well known that in general pure pursuit does not guarantee capture of the evader. However, it becomes a winning strategy if switching to `2P1EG` is adopted.

Theorem 7: The pure pursuit strategy is a potential switching strategy. □

Theorems 6 and 7 are just simple examples of winning pursuit strategies based on switching; it is apparent that many other switching strategies can be devised by exploiting the geometric conditions in Theorems 4 and 5.

V. Numerical simulations

In this section, some simulations are reported to show the improvement (in terms of capture time) when a pursuit switching strategy is adopted w.r.t. a non-switching one. Three pursuer strategies are considered: pure pursuit strategy (PPS), centroid-based strategy (CS), and the strategy proposed in [7], referred to as Voronoi-based strategy (VS), in which the pursuers aim at minimizing the area of the evader Voronoi cell. All these strategies are simulated both in their original form (non-switching) and in switching form, i.e. by applying the optimal `2P1EG` strategy when condition of Theorem 2 holds. Switching strategies are denoted by the prefix “S-”, e.g., S-PPS denotes the switching pure pursuit strategy.

In the following, simulated games will be graphically illustrated. In the figures, a square denotes the initial condition of a player and a dot its final position. Blue and red lines represent the trajectories of the pursuers and the evader, respectively. Capture occurs when at least one pursuer reaches the circle of radius r centered at the evader, drawn in dashed black. If the evader manages to avoid capture, the simulation is stopped as soon as it exits from the capture region $\mathcal{M}$.

Example 1: Three pursuers are initially located at the vertices of an equilateral triangle centered at the origin with sides of length 20, that is $P_1(0) = [-10, -10/\sqrt{3}]'$, $P_2(0) = [10, -10/\sqrt{3}]'$, $P_3(0) = [0, 20/\sqrt{3}]'$. The capture radius is $r = 1$ and the maximum speed of the agents is fixed to 1. The evader moves downwards with speed 1, i.e., $v_E(t) = [0, -1]'$, independently of the position of the pursuers.

Let the pursuers play PPS, and let the evader initial position be $E(0) = [0,\ 0]'$. In Fig. 4 (left), the trajectories of the players are depicted until time $t = 5.59$, when the evader goes outside the capture region $\mathcal{M}$. So, the evader can indefinitely escape by going downwards.

A different situation occurs when all the players switch to the `2P1EG` strategy as soon as condition (9) is satisfied. In fact, thanks to Theorem 7, capture in finite time is assured if $E(0) \in \mathcal{M}(0)$. This game is reported in Fig. 4 (right). At time $t = 4.47$, condition (9) holds and capture occurs at time $t = 20.64$.

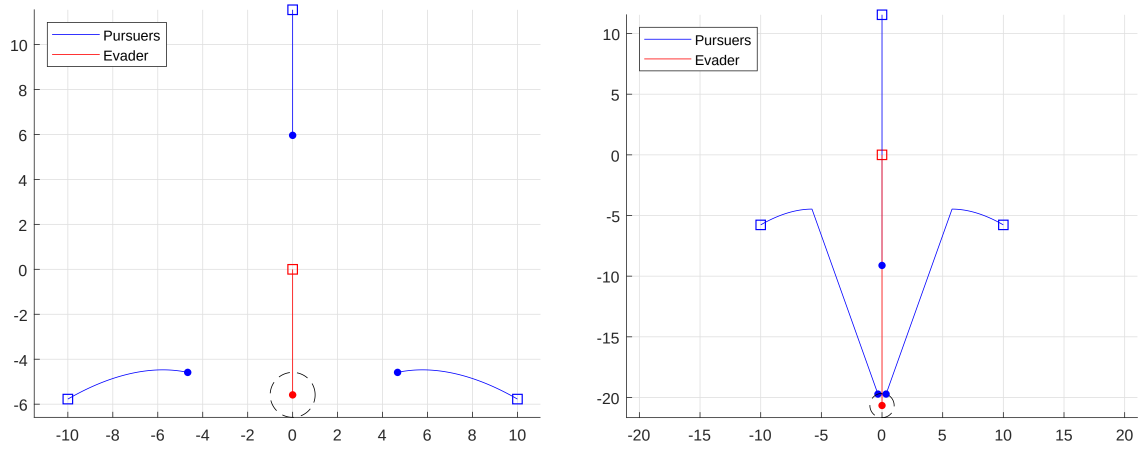


Fig. 4. Example 1. Pursuers play PPS (left) and S-PPS (right).

Example 2: Under the same setting of Example 1, the pursuers play CS, i.e., they point towards their circumcenter, that is towards the origin. Without switching, the evader can escape, see Fig. 5 (left). In fact, at $t = 4.52$ it holds $E(t) \notin \mathcal{M}(t)$, and the evader can avoid capture by going downwards. Assuming that the pursuers play S-CS, capture occurs at time $t = 25.57$, as shown in Fig. 5 (right).

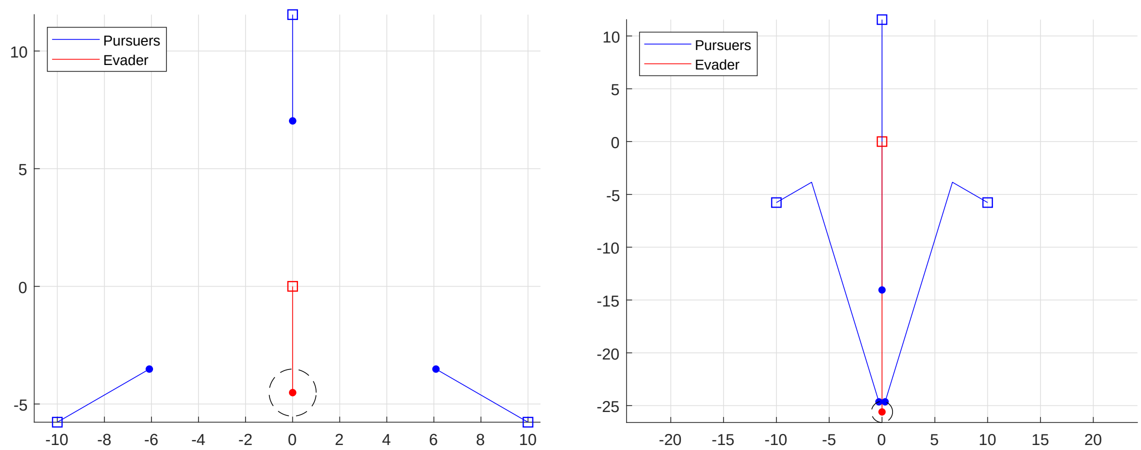


Fig. 5. Example 2. Pursuers play CS (left) and S-CS (right).

Example 3: Consider the same setting as in Examples 1 and 2, and let the pursuers play VS. In [7], it has been proved that capture occurs in finite time independently on the strategy adopted by the evader. The result of such a game is reported in Fig. 6 (left), in which capture occurs at time $t = 16.98$. If the pursuers play S-VS, when $E \simeq [0, -9]'$ condition (9) occurs and the agents start playing the corresponding optimal `2P1EG` strategy. The evader changes its direction and starts moving upwards. Capture occurs simultaneously by the three pursuer at $t = 14.35$. Hence, S-VS is able to achieve capture in a time about 15% less than VS.

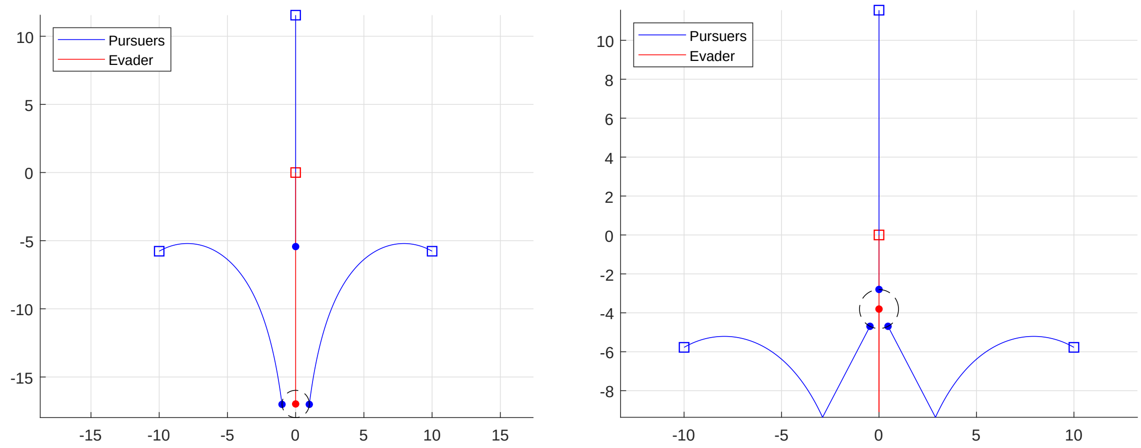


Fig. 6. Example 3, $E(0) = [0, 0]'$. Pursuers play VS (left) and S-VS (right).

It is worthwhile to note that the improvement of S-VS w.r.t. VS can be much larger, even for simple games involving 3 pursuers. For instance, under the same setting, let us set the initial evader position to $E(0) = [0, -5.5]'$. The capture time required by VS turns out to be $t = 169.28$, while that needed by S-VS is $t = 96.37$ (see Fig. 7). In this case, switching to the `2P1EG` strategy leads to a capture time reduction of about 43%.

Example 4: Let us now consider games involving more than three pursuers. The initial position of the evader is $E(0) = [0, 0]'$, while the initial position of the pursuers is randomly chosen on a circumference centered at the origin with a radius R_p. The capture radius is fixed to $r = 1$, and the maximum speed of the players is set to 1. We compare the capture time obtained when pursuers play the Voronoi-based strategy (namely, T_{VS}), with that obtained when playing the switching version S-VS ($T_{S\text{-}VS}$). In particular, we focus on the ratio between these two quantities, i.e.,

$$\rho = \frac{T_{VS}}{T_{S\text{-}VS}}.$$

In all games, the evader strategy is to point towards the farthest vertex of its Voronoi cell

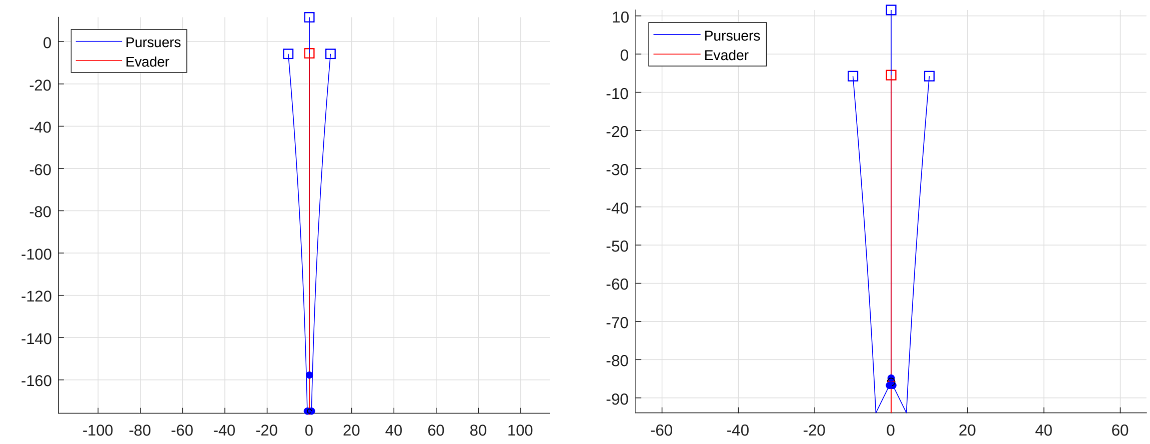


Fig. 7. Example 3, $E(0) = [0, -5.5]'$. Pursuers play VS (left) and S-VS (right).

at each time t. A total of $N = 100$ games are played for any combination of the number of pursuers $p \in \{5, 8\}$, and of the radius of their initial condition $R_p \in \{10, 25, 50, 75, 100\}$. The average and maximum values of ρ are reported in Table 4. Notice that, in general, the average capture time when pursuers use the switching strategy is much less than that obtained by adopting the original Voronoi-based strategy, with a maximum ratio ρ which is greater than 1.6. Thus, it is apparent that the capture time can be significantly reduced if pursuers switch to $\texttt{2P1EG}$ once condition (9) occurs.

TABLE I

EXAMPLE 4. MEAN AND MAXIMUM VALUE OF ρ FOR DIFFERENT GAME CONFIGURATIONS (OVER 100 GAMES)

	$p = 5$		$p = 8$	
	$\mathrm{mean}(\rho)$	$\max(\rho)$	$\mathrm{mean}(\rho)$	$\max(\rho)$
$R_p = 10$	1.180	1.405	1.074	1.320
$R_p = 25$	1.262	1.376	1.145	1.341
$R_p = 50$	1.317	1.488	1.193	1.346
$R_p = 75$	1.346	1.546	1.219	1.579
$R_p = 100$	1.365	1.587	1.236	1.620

VI. CONCLUSIONS

A new family of pursuit strategies for a multi-pursuer single-evader game has been introduced. The main property that has been exploited is the fact that such a game must eventually boil down to a two-pursuer single-evader game, for which a minimum-time solution exists. Hence, conditions for the switching between the two game settings have been derived, allowing one to define new winning pursuit strategies and to improve the performance of existing ones. Future developments concern the investigation of pursuit strategies achieving the switching condition in such a way that the overall capture time is minimized. The extension of the proposed approach to the case of multi-pursuer multi-evader games will also be addressed.

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