Demand-Response in Building Heating Systems: a Model Predictive Control Approach

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Abstract

In this paper we consider the problem of optimizing the operation of a building heating system under the hypothesis that the building is included as an active consumer in a demand response program. Demand response requests to the building operational system come from an external market player or a grid operator. Requests assume the form of price-volume signals specifying a maximum volume of energy to be consumed during a given time slot and a monetary reward assigned to the participant in case it fulfills the conditions. A receding horizon control approach is adopted for the minimization of the energy bill, by exploiting a simplified model of the building. Since the resulting optimization problem is a mixed integer linear program which turns out to be manageable only for buildings with very few zones, a heuristics is devised to make the algorithm applicable to realistic size problems as well. The derived control law is tested on the realistic simulator EnergyPlus to evaluate pros and cons of the proposed algorithm. The performance of the suboptimal control law is evaluated on small- and large-scale test cases.

Keywords: Energy Management Systems, Model Predictive Control, Building Heating Systems, Demand Response, Mathematical Modeling, Optimization.

1. Introduction

Building energy consumption, both commercial and residential, represents almost 40% of the global energy produced worldwide. About 50% of this huge amount of energy is consumed for heating, ventilation and air conditioning (HVAC). HVAC plants, especially the oldest ones, are operated through simple rule-based strategies, which actuate the system in feedforward at a centralized level, while local thermal control is made through standard thermostatic devices. The need to improve these out-of-date techniques has stimulated research for several years, with the aim of reducing consumption and improving comfort through the design of more appropriate and sophisticated feedback

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control laws exploiting real-time information from the several components of the buildings. Most of the proposed approaches are based on Model Predictive Control (MPC), because of its attractive features, ranging from the possibility of handling constraints on numerous variables involved to optimizing economical objectives in a time-varying context (see e.g., [1, 2, 3, 4, 5] and references therein). The recent paper [6] provides a comprehensive framework based on MPC and co-simulation for real time control of the energy management system of a building.

An important issue which is gaining much attention in the recent literature on electricity systems and building thermal control concerns Demand Response (DR). The concept of DR has been introduced several years ago in the literature on smart energy grids [7, 8, 9]. Recently, a complete commercial and technical architecture has been developed in the European project ADDRESS ([10, 11, 12]). The relevance of this concept to the design of efficient energy management systems is testified by several recent papers, like e.g, [13, 14]. The key idea is that the end users play an active role in the electricity system by adjusting their consumption patterns according to dynamic energy pricing policies enforced by the players involved in energy markets. DR participation does not take place on an individual basis, but rather via the aggregation of a community of individual consumers, possibly represented by an intermediary subject, the *aggregator*. The aggregator's main objective is to provide value by employing the *flexibility* of the consumption load profile of individual consumers. Its basic role is to collect certain amounts of energy over specified time intervals, i.e., the energy saved by consumers accepting the aggregator's offers. This energy can be used for several purposes. For instance, the DSO (Distribution System Operator) may ask an aggregator to enforce energy reduction in a given load area over a given time interval if an overload is foreseen in that area, in order to counteract possible network unbalancing. Efficient grid management, in turn, contributes to overall reduction of carbon dioxide production [15]. A further reason for the aggregator to collect energy is that options related to reprofiling of the load curve in specific load areas of the distribution system, can be sold on the market. An aggregator has a pool of subscribers (end users), and is able to send them *price-volume signals* in order to affect their consumption pattern. These signals are typically sent once or twice a day and specify a monetary reward (price) if power consumption, during certain hours of the day, is below or above specified thresholds (volume) [12].

In this context, building operators can be considered as excellent candidates for demand flexibility, as they might find it convenient to schedule certain tasks in order to obtain a reward. In particular, the possibility of shifting HVAC electric loads according to a smart strategy is crucial to participation in DR services. In [16], the impact of buildings in DR programs on the electricity market is modeled through an agent-based simulation platform, and it is shown how different levels of DR penetration affect the market prices. An approach for allocating the requested energy among heterogeneous devices in buildings depending of DR requests and electricity price has been reported in [17].

In the present paper, we propose an optimization approach based on Model Predictive Control (MPC) for allowing the temperature control system of large buildings to participate in a DR program. The idea of exploiting MPC for supervisory control of building energy management systems traces back to the late eighties [18], even if the intrinsic computational burden of the approach prevented realistic applications until a few years ago. Participation of buildings in DR programs has been recently addressed in [19], where a pricing policy has been proposed for offering real time regulation services, and in [20], where an optimization framework based on genetic algorithms is provided for dealing with a DR case study for the heating of an office building in Canada. In order to rapidly reply to DR requests, a fast chiller power demand response control strategy for commercial buildings is introduced in [21], with the aim of maintaining internal thermal comfort by regulating the chilled water flow distribution under the condition of insufficient cooling supply. A controller for HVAC systems able to curtail peak load while maintaining reasonable thermal comfort has been introduced in [22, 23], where the set-point temperature of a building is changed whenever the retail price is higher than customers preset price.

The novelty introduced in this paper with respect to the work mentioned above is the integration of price-volume signals provided by an aggregator into the temperature regulation system, and the development of a cost-optimal control strategy involving low computational complexity for DR-enabled large-scale buildings. On the basis of external price-volume signals, the optimizer analyses whether it is convenient for the building to honour the corresponding DR requests. The objective is the minimization of the energy bill. Since the complexity of the overall optimization problem is intractable even for buildings of modest dimension, a suitable heuristic search strategy based on problem decoupling is devised in order to make the computational burden acceptable without significant loss of accuracy. The approach is independent of the particular heating technology adopted and it can be easily generalized to cooling management as well.

The proposed technique is validated using EnergyPlus [24] as a realistic physical modelling simulator. The MPC optimal control problem is solved on the basis of an identified linear model of the building. The control law is tested on the linear model and on the physical model simulator for comparison purposes. In this sense, the present contribution is in the spirit of [25, 26, 27], where a different objective is considered.
It is shown that a decoupling approach which decomposes regulation of the different zones of the building into independent problems provides reliable results in the absence of DR participation. Since the presence of price-volume signals makes the computational burden of the optimization grow exponentially with the number of zones, a decoupled heuristic relaxation of the problem is devised. The obtained results are compared to the optimal solution on a three-zone building equipped with underfloor electric heaters. A test case involving a large-scale building equipped with a heat pump heating system is also worked out.

The paper is organized as follows. In Section 2 we formulate the problem specifying the objective to be optimized. In Section 3 we describe the proposed control algorithm. Section 4 reports the experimental results obtained on a small-scale and a large-scale test case, together with a discussion of the results obtained. Finally, conclusions are drawn in Section 5.

2. Problem formulation

2.1. Notation and nomenclature

In this paper, $\mathbb{R}$ denotes the real space, $\mathcal{B} = \{0, 1\}$ is the binary set, $x \in \mathbb{R}$ is a real scalar, $\mathbf{X} = \{x_{i,j}\}$, $i = 1, \ldots n$, $j = 1, \ldots m$ is a real $n \times m$ matrix and $\mathbf{X}'$ denotes its transpose, $\mathbf{x} = [x_1 x_2 \ldots x_m]'$ is an $m-$dimensional real vector,

$\mathcal{X}$ is a set, $\mathcal{X}^m$ is the cartesian product of m sets identical to $\mathcal{X}$. The notation $\mathcal{X}\backslash\{x\}$ indicates a set $\mathcal{X}$ without its element x. We denote the set of discrete time indices as $\mathcal{K} = \{0, 1, \dots\}$ and the generic time index as $k \in \mathcal{K}$, a time interval is denoted as $\mathcal{I}(k, \lambda) = [k, k + \lambda) \subseteq \mathcal{K}$.

2.2. System model

This paper focuses on a building with a centralized heating system (e.g., a government building) composed of m zones $\mathrm{Z}_1, \dots, \mathrm{Z}_m$ equipped with electrical heating devices, e.g., electrical radiant floors or heat pumps. Assume that each zone is equipped with a temperature sensor connected to the centralized controller and that each heater can be independently switched on or off by the control unit. Assume that the control system operates in discrete-time and denote the sampling period with Δ_s. Moreover, define:

- $u_i(k) \in \mathcal{B}$: heater status $\{0 = \text{ inactive}, 1 = \text{ active}\}$ at time k for Z_i,
- w_i: heater energy consumption [kWh per sampling period] for Z_i,
- $T_i(k)$: indoor temperature [$°C$] at time k, measured by the sensor for Z_i,
- $\mathcal{C}_i(k) = [\underline{T}_i(k), \overline{T}_i(k)]$: thermal comfort range for Z_i at time k,
- $\mathbf{u}(k) = [u_1(k) \dots u_m(k)]' \in \mathcal{B}^m$
- $\mathbf{T}(k) = [T_1(k) \dots T_m(k)]' \in \mathbb{R}^m$.
- $p(k)$: electricity price at time k, or forecast thereof.

Other than on the heater statuses $\mathbf{u}(k)$, indoor temperatures $\mathbf{T}(k)$ may depend on exogenous variables like outdoor temperature, solar radiation, indoor lights and appliances, human occupancy, etc. For a given building, available measurements or forecasts of some or all of these variables are collected in a vector $\mathbf{e}(k) = [e_1(k) \dots e_m(k)]'$. Hence, the temperature dynamics can be modeled in regressive form as

$$\mathbf{T}(k+1) = \mathbf{F}(\mathbf{\Phi}(k)), \tag{1}$$

where the regression matrix $\mathbf{\Phi}(k)$ is given by

$$\mathbf{\Phi}(k) = [\mathbf{T}(k) \ \dots \ \mathbf{T}(k - k_{\mathbf{T}}) \ \ \mathbf{u}(k) \ \dots \ \mathbf{u}(k - k_{\mathbf{u}}) \ \ \mathbf{e}(k) \ \dots \ \mathbf{e}(k - k_{\mathbf{e}})]', \tag{2}$$

being $k_{\mathbf{T}}, k_{\mathbf{u}}, k_{\mathbf{e}}$ suitable nonnegative integers that define the model order, and being $\mathbf{F}(\cdot)$ some (possibly nonlinear and time-varying) function.

Given the definitions above, thermal comfort at time k is guaranteed whenever $\mathbf{T}(k) \in \mathcal{C}(k)$ where

$$\mathcal{C}(k) = \{\mathbf{T}(k) \ : \ T_i(k) \in \mathcal{C}_i(k) \ \ \forall i = 1, \dots, m\}, \tag{3}$$

while the overall building consumption within the k-th time step can be computed as

$$q(k) = \sum_{i=1}^{m} w_i u_i(k). \tag{4}$$

Let us consider a generic time horizon $\mathcal{I}(k,\lambda)$, and let $Q(k,\lambda)$ denote the total consumption within $\mathcal{I}(k,\lambda)$, i.e.,

$$Q(k,\lambda) = \sum_{l=k}^{k+\lambda-1} q(l). \tag{5}$$

The total expected cost of energy in the interval $\mathcal{I}(k,\lambda)$ is therefore given by

$$C(k,\lambda) = \sum_{l=k}^{k+\lambda-1} p(l)q(l). \tag{6}$$

2.3. Demand-Response model

For the purpose of this work, a standard model of a DR program is employed. A DR program is a sequence of DR requests $\mathcal{R}_j$, each involving a time horizon $\mathcal{I}(h_j,\mu_j)$, a total energy bound S_j, and a monetary reward R_j. A single request $\mathcal{R}_j$ is said to be *fulfilled* if the total building consumption within $\mathcal{I}(h_j,\mu_j)$, i.e., $Q(h_j,\mu_j)$, is no higher than the prescribed threshold S_j, and in this case a monetary reward R_j is granted to the building operator.

Definition 1. *A DR program $\mathcal{P}$ is a sequence of DR requests $\mathcal{R}_j$, $j = 1,2,\dots,$ where $\mathcal{R}_j$ is the set*

$$\mathcal{R}_j = \{\mathcal{I}(h_j,\mu_j), S_j, R_j\}, \tag{7}$$

being

$$\mathcal{I}(h_j,\mu_j) \subseteq \mathcal{K}, \quad \mathcal{I}(h_{j_1},\mu_{j_1}) \cap \mathcal{I}(h_{j_2},\mu_{j_2}) = \emptyset, \quad \forall j_1 \neq j_2. \tag{8}$$

The request $\mathcal{R}_j$ is fulfilled if and only if

$$Q(h_j,\mu_j) \leq S_j. \tag{9}$$

For any given time horizon $\mathcal{I}(k,\lambda)$, let

$$\mathcal{P}(k,\lambda) = \{\mathcal{R}_j : \mathcal{I}(h_j,\mu_j) \subseteq \mathcal{I}(k,\lambda)\}, \tag{10}$$

be the set of DR requests that occur within the time horizon. Moreover, define $\mathcal{J}(k,\lambda)$ as the set of indices identifying such DR requests, i.e.,

$$\mathcal{J}(k,\lambda) = \{j : \mathcal{R}_j \in \mathcal{P}(k,\lambda)\}. \tag{11}$$

For each request $\mathcal{R}_j$, introduce a binary variable $\gamma_j \in \mathcal{B}$ defined as

$$\gamma_j = \begin{cases} 1 & \text{if } \mathcal{R}_j \text{ is fulfilled} \\ 0 & \text{otherwise.} \end{cases} \tag{12}$$

The overall expected cost of operation of the building heating system within the time horizon $\mathcal{I}(k,\lambda)$ under the DR program $\mathcal{P}$, is therefore given by

$$C^{\mathcal{P}}(k,\lambda) = C(k,\lambda) - \sum_{j\in\mathcal{J}(k,\lambda)} \gamma_j R_j, \tag{13}$$

i.e., the expected cost of energy minus the total reward for the fulfilled DR requests.

2.4. Optimal heating operation problem

Our goal is to devise a control algorithm for the thermal heating system of each zone in order to minimize the building electricity bill under a DR program $\mathcal{P}$, while preserving comfort constraints.

Consider a time horizon $\mathcal{I}(k,\lambda)$ and collect heater activation status variables for all zones within $\mathcal{I}(k,\lambda)$ in the following $\lambda \times m$ binary matrix:

$$\mathbf{U}(k,\lambda) = \begin{bmatrix} \mathbf{u}(k)' \\ \vdots \\ \mathbf{u}(k+\lambda-1)' \end{bmatrix} \in \mathcal{B}^{\lambda\times m}. \tag{14}$$

Assuming $\mathbf{e}(k)$ (or a forecast thereof) is available, the above problem can be formulated as a mixed-integer program as follows.

Problem 1. *Optimal heating control under DR program $\mathcal{P}$.*

$$\begin{cases} \mathbf{U}^*(k,\lambda) = \arg \min\limits_{\substack{\mathbf{U}(k,\lambda) \\ \gamma_j \,:\, j\in\mathcal{J}(k,\lambda)}} C(k,\lambda) - \sum\limits_{j\in\mathcal{J}(k,\lambda)} \gamma_j R_j \\ \text{s.t.:} \\ Q(h_j,\mu_j) \le \gamma_j S_j + (1-\gamma_j)\mu_j \sum_{i=1}^m w_i \quad \forall j \in \mathcal{J}(k,\lambda) & \text{(a)} \\ \gamma_j \in \mathcal{B} \qquad \forall j \in \mathcal{J}(k,\lambda) & \text{(b)} \\ \mathbf{T}(l+1) = \mathbf{F}(\mathbf{\Phi}(l)) & \text{(c)} \\ \mathbf{T}(l) \in \mathcal{C}(l) \quad \forall l \in \mathcal{I}(k,\lambda) & \text{(d)} \\ \mathbf{U}(k,\lambda) \in \mathcal{B}^{\lambda\times m} & \text{(e)} \end{cases} \tag{15}$$

In (15), (a) and (b) represent the DR constraints, i.e., $Q(h_j,\mu_j) \le S_j$ for each fulfilled $\mathcal{R}_j$, constraints (c) represent the temperature dynamics, (d) are the comfort constraints, while (e) forces the on/off heater behaviour. Note that if the map $\mathbf{F}(\cdot)$ is linear, then Problem 1 is a mixed-integer linear program (MILP).

3. Sub-Optimal Control Algorithm

It is known that temperature dynamics in a given zone depends on heater status, environmental temperature, solar radiation, internal lights and appliances, occupancy, and temperature of neighboring zones. A complex model which takes into account all the above mentioned aspects is not conceivable for Problem 1 due to the unacceptable computational burden. In particular, if the temperature variables $T_i(k)$ and the binary decision variables $u_i(k)$ and γ_j are fully coupled via the constraints in (15), then the computational complexity scales exponentially with m, thus making the approach totally unfeasible except for very small-scale problems. In order to overcome this limitation, we will derive sub-optimal solutions by suitably decoupling Problem 1 into m smaller problems. To this purpose, the first step is to obtain a decoupled linear regressive building model. Therefore, we enforce the following assumption, which boils down to neglecting thermal flow between zones.

Assumption 1. *The temperature dynamics of each zone* Z_i *is given by*

$$T_i(k+1) = \mathbf{\Phi}_i'(k)\Theta_i, \quad i = 1, \dots, m \tag{16}$$

where Θ_i*,* $i = 1, \dots, m$ *are parameter vectors of suitable dimension, and* $\mathbf{\Phi}_i(k)$ *is the* i*-th column of the regressor matrix* $\mathbf{\Phi}(k)$*.*

The above assumption is supported by the following arguments:

- if the zones are homogeneous and/or insulation is properly designed, as it happens in office or government buildings, the heat transfer between neighboring zones is really small;
- numerical simulations show good performance of a decoupled identified model of a well-established test case (see Section 4);
- possible discrepancies between the model and the real building can be compensated at each iteration by using sensor information and a receding horizon approach.

The need for a receding horizon strategy is further supported by the observation that optimizing over a long time horizon, i.e., one or more days, is quite unreliable. Indeed, satisfying the comfort constraints requires accurate prediction of the indoor temperature of each zone. Such predictions degrade with time due to a number of reasons, like model inaccuracies and lack of reliable weather forecasts. Moreover, long-term energy price forecasts may not be available.

It is worth noticing that if the control signals $u_i(k)$ were continuous rather than binary, then (15) would still be a MILP due to the presence of binary DR decision variables γ_j. However, the computational complexity would be drastically reduced since the number of DR events in each instance of the problem is typically small.

3.1. Problem decoupling

In view of Assumption 1, let $\mathbf{U}_i(k,\lambda)$ be the i-th column of $\mathbf{U}(k,\lambda)$, and define the following quantities:

$$q_i(k) = w_i u_i(k), \tag{17}$$

i.e., the energy consumption of zone Z_i in the time slot k, and

$$Q_i(k,\lambda) = \sum_{l=k}^{k+\lambda-1} q_i(l), \quad C_i(k,\lambda) = \sum_{l=k}^{k+\lambda-1} p(l) q_i(l), \tag{18}$$

which amount to total consumption and total cost for Z_i in $\mathcal{I}(k,\lambda)$, respectively.

Even when using a decoupled building model, it is apparent that Problem 1 cannot be split into m independent MILPs, one for each zone Z_i, with overall cost function equal to the sum of m marginal costs. Indeed, the decision variables of all zones are coupled through constraint (a) in (15), and the DR component in the cost function itself is a coupling term. In order to overcome this limitation, in the sequel we introduce a decoupled optimization problem leading to sub-optimal solutions to Problem 1. To this purpose, for each time step k, and for each $\mathcal{R}_j \in \mathcal{P}(k,\lambda)$, we introduce the following matrices of real parameters

$$\mathbf{S} = \{S_{j,i} : i = 1,\ldots,m,\ j \in \mathcal{J}(k,\lambda)\}, \tag{19}$$

$$\mathbf{R} = \{R_{j,i} : i = 1,\ldots,m,\ j \in \mathcal{J}(k,\lambda)\} \tag{20}$$

where $S_j = \sum_{i=1}^m S_{j,i}$ and $R_j = \sum_{i=1}^m R_{j,i}$, respectively, and we define

$$C_i^{\mathcal{P}}(k,\lambda) = C_i(k,\lambda) - \sum_{j\in\mathcal{J}(k,\lambda)} \gamma_{j,i} R_{j,i}. \tag{21}$$

For each $i = 1,\ldots m$, let us consider the following decoupled MILP:

Problem 2.

$$\begin{cases} \mathbf{U}_i^*(k,\lambda) = \arg \min\limits_{\substack{\mathbf{U}_i(k,\lambda) \\ \gamma_{j,i}\,:\, j\in\mathcal{J}(k,\lambda)}} C_i(k,\lambda) - \sum\limits_{j\in\mathcal{J}(k,\lambda)} \gamma_{j,i} R_{j,i} \\ \text{s.t.:} \\ \left.\begin{array}{l} Q_i(h_j,\mu_j) \le \gamma_{j,i} S_{j,i} + (1-\gamma_{j,i})\mu_j w_i \qquad \forall j \in \mathcal{J}(k,\lambda) \\ \gamma_{j,i} \in \mathcal{B} \qquad \forall j \in \mathcal{J}(k,\lambda) \end{array}\right. \\ \left.\begin{array}{l} T_i(l+1) = \mathbf{\Phi}_i'(l)\Theta_i \\ T_i(l) \in \mathcal{C}_i(l) \qquad\qquad \forall l \in \mathcal{I}(k,\lambda) \\ \mathbf{U}_i(k,\lambda) \in \mathcal{B}^{\lambda\times 1} \end{array}\right. \end{cases}. \tag{22}$$

which depends on the particular choice of $\mathbf{S}$ and $\mathbf{R}$. Let $C_i^{\mathcal{P}*}(k,\lambda)$ be the optimal solution of Problem 2. It is not difficult to see that for any choice of the set of parameters $\mathbf{S}$ and $\mathbf{R}$, the function

$$\overline{C^{\mathcal{P}}}(k,\lambda) = \sum_{i=1}^{m} C_i^{\mathcal{P}*}(k,\lambda) \tag{23}$$

is an upper bound for the optimal cost $C^{\mathcal{P}*}(k,\lambda)$ of Problem 1. Therefore, it makes sense to look for the values of $\mathbf{S}$ and $\mathbf{R}$ yielding a solution of the m problems (22) corresponding to the tightest upper bound, i.e., to find

$$\begin{cases} \min\limits_{\mathbf{S},\mathbf{R}} \overline{C^{\mathcal{P}}}(k,\lambda) \\ \text{s.t.:} \\ S_j = \sum_{i=1}^{m} S_{j,i}, \quad R_j = \sum_{i=1}^{m} R_{j,i} \quad \forall j \in \mathcal{J}(k,\lambda) \end{cases} . \tag{24}$$

This can be achieved by applying a local constrained minimization algorithm, which involves the solution of m MILPs of the form (22) at each step, or via some heuristics. A possible heuristic approach is presented in the following subsection.

3.2. Heuristic approach to Problem 2

The heuristics proposed here to assign the parameters $S_{j,i}$ for each zone Z_i and for each DR request $\mathcal{R}_j \in \mathcal{P}(k,\lambda)$ in Problem 2, can be summarized in the following stages:

1. Set the energy price $p(k)$ equal to some big value M for all $k \in \mathcal{I}(h_j,\mu_j)$ $\forall j \in \mathcal{J}(k,\lambda)$, and moreover set $\gamma_{j,i} = 0$ $\forall j \in \mathcal{J}(k,\lambda)$, $\forall i = 1,\ldots,m$. Then solve Problem 2 for each zone Z_i and compute

$$\tau_{j,i} = \sum_{k \in \mathcal{I}(h_j,\mu_j)} u_i(k) \quad \forall j \in \mathcal{J}(k,\lambda), \quad \forall i = 1,\ldots,m \tag{25}$$

   This step boils down to evaluating the minimum possibile heater activation time $\tau_{j,i}$ for each zone Z_i within the DR request intervals $\mathcal{I}(h_j,\mu_j)$ in order to satisfy the comfort constraints. The corresponding total energy needed by all zones during $\mathcal{I}(h_j,\mu_j)$ is given by

$$e_j = \sum_{i=1}^{m} \tau_{j,i} w_i \tag{26}$$

2. If for some j, the quantity e_j exceeds S_j, then there is no feasible solution to Problem 2 such that the DR request $\mathcal{R}_j$ is satisfied, i.e., under no circumstances can the request $\mathcal{R}_j$ be met without violating comfort constraints. Therefore, for all j such that $e_j > S_j$, we set $\mathcal{J}(k,\lambda) \leftarrow \mathcal{J}(k,\lambda)\backslash\{j\}$, i.e., we discard $\mathcal{R}_j$. Otherwise, we set $S_{j,i} = \tau_{j,i} w_i$ $\forall i = 1,\ldots,m$. Moreover, the quantity $S_j - e_j$, i.e., the estimated amount of excess energy still allowed by $\mathcal{R}_j$ with respect to the minimum possible consumption, is split among all zones according to weights proportional to the corresponding heater power ratings, i.e., we further set

$$S_{j,i} \leftarrow S_{j,i} + (S_j - e_j) w_i / \sum_{l=1}^{m} w_l. \tag{27}$$

3. The parameters $R_{j,i}$ are simply assigned by splitting R_j according to weights proportional to the power ratings, i.e.,

$$R_{j,i} = R_j w_i / \sum_{l=1}^{m} w_l. \tag{28}$$

4. We solve Problem 2 with the newly assigned $S_{j,i}$ and $\mathcal{J}(k,\lambda)$.

This heuristics can be performed at each step of a receding horizon implementation, as shown below.

3.3. Receding horizon implementation

Problem 1, as well as the decoupled version in Problem 2 combined with the heuristics just presented, does not lend itself to the standard MPC implementation, that is,

(i) acquire measurements and/or forecasts of relevant variables,

(ii) optimize the cost over $\mathcal{I}(k,\lambda)$ for fixed λ, and

(iii) apply the optimal control action $\mathbf{u}^*(k)$ (see Fig. 1).

Indeed, this basic implementation does not take into account DR requests that partially overlap in time with the moving interval $\mathcal{I}(k,\lambda)$. This issue can be overcome by performing the following further actions right before running the optimization (ii):

(i') adapt the horizon length λ dynamically in a way such that an integer number of DR requests falls inside $\mathcal{I}(k,\lambda)$, keeping λ greater or equal to a fixed minimum horizon length λ_{min} (see Fig. 2),

(i") as long as there exists a DR request $\mathcal{R}_j$ such that $h_j = k-1$, set $h_j = k$ and reduce μ_j by one, subtracting the consumption $q(k-1)$ [resp. $w_i u_i(k-1)$] from S_j [resp. $S_{j,i}$].

A schematic of the control architecture is depicted in Fig. 3 and the overall algorithm is synthesized in Table 1.

4. Test cases

In order to validate the proposed approach, both a small-scale and a large-scale test case involving different heating technologies are developed in this section.

4.1. Three-zone case

Let us consider an office building located in Milan (Italy) composed of three zones equipped with radiant floor heating systems. The building characteristics have been taken from an example provided in EnergyPlus and are reported in Tables 2 and 3 Hereafter, we assume the EnergyPlus model as the true (real) building, and the sampling time is set to $\Delta_s = 10$ minutes.

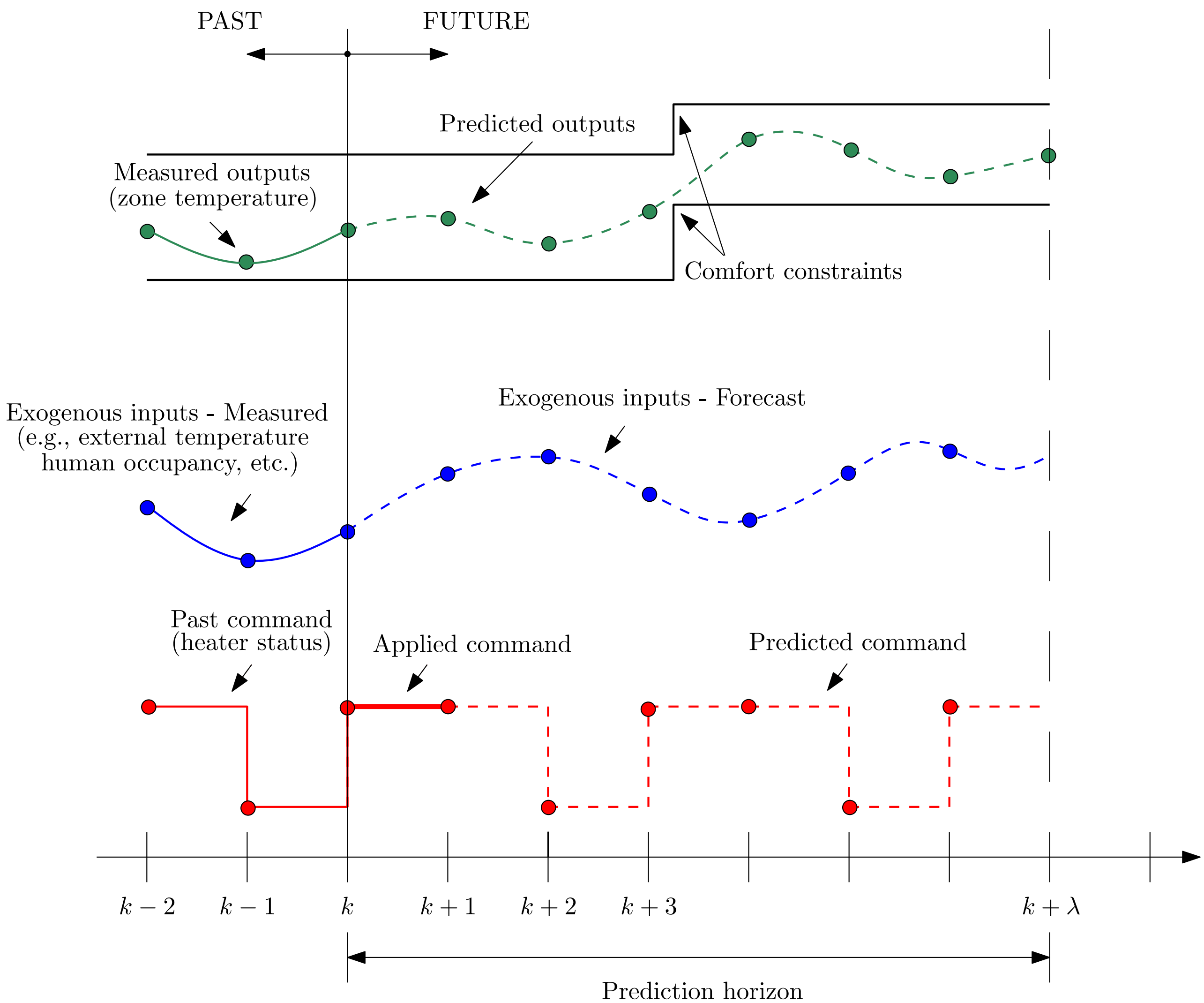

Figure 1: Time evolution of the system variables in standard MPC approach.

4.1.1. Modelling and identification

As stated in Section 3, the proposed control technique is based on decoupled linear time invariant models of the building zones.

The ARX family has been chosen to model the thermal behavior of zones. For a given zone, the input signals are assumed to be heater command, outdoor temperature, solar irradiance and internal heat gain (lights, appliances, human occupancy), while the output is the indoor air temperature.

We propose the following model for zone Z_i:

$$T_i(k+1) = \mathbf{\Phi}_i'(k)\Theta_i \tag{29}$$

Control Algorithm

for *each time step* $k = 0, 1, \ldots,$ **do**

  for *each zone* $i = 1, \ldots, m,$ **do**

    if $k + \lambda_{min}$ *falls inside the interval* $\mathcal{I}(h_j, \mu_j)$ *of the DR request* $\mathcal{R}_j$ **then**

      adapt the horizon length λ as in step i');

    end

    if $h_j = k - 1$ *for some* j **then**

      modify the parameters of the DR request $\mathcal{R}_j$ as in step i");

    end

    acquire regressor data $T_i(k)$ and $e_i(l),\ l = k, \ldots, k + \lambda - 1$;

    solve Problem 2 for the optimal command sequence $\mathbf{U}_i^*(k, \lambda)$;

    actuate the optimal command action $u_i^*(k)$;

  end

end

Table 1: Control Algorithm

being $\Theta_i = [\theta_{i,1} \ldots \theta_{i,12}]'$, and

$$\begin{aligned} \mathbf{\Phi}_i(k) = [T_i(k)\ T_i(k-1)\ u_i(k)\ u_i(k-1)\ u_i(k-2) \\ L(k)\ L(k-1)\ L(k-2)\ T_o(k+1|k)\ T_o(k) \\ I_o(k+1|k)\ I_o(k)]' \end{aligned} \tag{30}$$

in which $T_o(k)$ is the outdoor (environment) temperature, $I_o(k)$ is the external illuminance and $L(k)$ denotes the internal heat gain. $T_o(k+1|k)$ and $I_o(k+1|k)$ denote forecasts of $T_o(k)$ and $I_o(k)$ available at time k, respectively.

An identification experiment has been conducted to find suitable values for the parameter vectors Θ_i. The identification is performed over two weeks, while validation is done over three different days. The tool used to estimate the parameters of the model from the experiment data is the System Identification Toolbox [28] of Matlab.

For the identification phase, the input signals have been chosen as follows:

- the heater command $u(k)$ is a pseudo-random binary sequence (PRBS),
- outdoor temperature $T_o(k)$ and illuminance $I_o(k)$ are the temperature and illuminance in Milan in January taken from historic time series,
- the internal gain $L(k)$ is a binary signal equal to 1 during the working hours (8:00-18:00) and 0 elsewhere.

Choosing the input as a PRBS in the identification phase is standard practice [29] as it allows for persistent excitation of the plant at least in a sufficiently wide frequency range, thus improving the accuracy of the identified

Table 2: Three-zone case - Building features

Building Component		**Value**
Weather and Location		Milan
Floor Area [m^2]		130.1
Floor [#]		1
Zone [#]		3
Window to wall ratio [%]		0.07
Solar Transmittance		0.9
Solar Reflectance		0.031
Internal Loads	Occupant [#]	10
	Lighting [W/m^2]	1.7
	Equipment [W/m^2]	12.46
Heating: Electric Low Temperature Radiant System	Power range [kW]	$[8;12]$
	Throttling range [$\Delta°C$]	2

model. In Figure 4, a comparison between the real indoor temperature (computed by EnergyPlus) and the 24-step ahead prediction of model (29) in the validation days for zone Z_1 is reported. We evaluate the performance of the estimated model by using the *Best Fit* index (FIT) according to the definition in [29, 30]. Roughly speaking, the FIT measures the percentage of signal energy explained by the model. The FIT for all zones turns out to be above 83%. The values of the identified model parameters are reported in Table 4.

4.1.2. Experiment setup and discussion

The proposed optimization method has been validated on a three-day experiment. In such days, a DR program consisting of 5 DR requests is assumed, as reported in Table 5.

Heater power ratings for the three zones are set to 12, 8, and 8 kW, respectively. For all zones, the upper comfort bound is set to $\overline{T}_i = 22°C$ throughout the day, while the lower bound is set to $\underline{T}_i = 20°C$ from 8:00 to 18:00 and $\underline{T}_i = 16°C$ elsewhen. The energy cost profile has been taken from the Italian Electricity Market.

Measured indoor temperature and heater command computed by the proposed method applied to the EnergyPlus physical model are depicted in Fig. 5 for zone 1. The overall cost at the computed solution for the three zones is 26.17 €.

Not surprisingly, looking at Fig. 5 (top), one may observe how the controller tries to preheat a zone before each DR request in order to reduce the power consumption during such a period still maintaining control comfort. To reduce energy cost, the controller also preheats a given zone just before an energy price peak. This fact is easily observable in Fig. 5 (bottom), where it is shown that the heater is mainly switched off during on-peak periods. The other two zones exhibit similar behaviours.

Table 3: Three-zone case - Building construction materials (name/thickness [mm])

	External Walls	**Internal Walls**	**Windows**
Outside Layer	Cement plaster/25	Gypsum board/10	Generic Clear/3
Layer 2	Concrete block/100	Clay tile/20	
Layer 3	Gypsum board/10	Gypsum board/10	

	Floor	**Roof**
Outside Layer	Dried sand and gravel/100	Slag or stone/10
Layer 2	Expanded polystyrene/50	Felt and membrane/10
Layer 3	Gypsum concrete/12	Dense insulation/25
Layer 4	Radiant panels/10	HW concrete/50
Layer 4	Gypsum concrete/19	
Layer 5	Tile/2	

In Fig. 6, the internal temperature of zone 1 computed on the identified model is reported for both the solutions achieved through the heuristics and the exact optimal algorithm. The zone temperature obtained by both algorithms is quite similar in general. The main differences are due to the different problem solved: while the optimal algorithm computes the solution of a coupled problem (i.e., a problem involving all the building zones), the proposed heuristics works on to the decoupled building, i.e., solves one (small) optimization problem for each zone. Although in some places the zone temperature obtained by the two control strategies is different (see e.g., around hour 62 in the reported simulation), being the building composed of three rooms, temperature differences among zones may compensate giving rise to similar total energy costs.

Table 6 summarizes the results obtained by running both the proposed heuristics and the optimal algorithm on both the real system (EnergyPlus) and the identified model.

By comparing the overall cost obtained by the optimal and the proposed suboptimal control laws, both implemented on the identified model (see Table 6), we observe that the cost provided by the heuristic suboptimal control is 0.4% higher than that of the optimal control. Unfortunately, as far as we know, it is not possible to obtain an expression for the optimality gap in the general case. The computation time for the suboptimal control law is approximately 2-3 orders of magnitude smaller than that needed for the optimal solution. Actually, as expected, the computational burden of the optimal algorithm, which grows exponentially with the number of zones, leads to intractable problems even when just a few zones, e.g., 4 or 5, are involved. In fact, the computational burden scales in a linear fashion for the heuristic suboptimal algorithm, thus allowing for an efficient solution of large-scale problems, even with hundreds of zones. In addition, it is worthwhile to notice from Figure 6, that the behaviour of the controlled variable is very similar for the two alternative control laws.

Concerning the performance obtained in this scenario, it turns out that the proposed suboptimal control law behaves even better than the optimal one (see Table 6). Such a behaviour is essentially due to noise and modeling

Table 4: Three-zone case - Identified model parameters

	Z_1	Z_2	Z_3
$\theta_{1,i}$	1.307	1.264	1.279
$\theta_{2,i}$	-0.3134	-0.2723	-0.2867
$\theta_{3,i}$	0.7528	0.7575	0.7728
$\theta_{4,i}$	-0.2219	-0.2056	-0.1899
$\theta_{5,i}$	-0.1362	-0.1325	-0.1417
$\theta_{6,i}$	1.06	1.241	1.067
$\theta_{7,i}$	-1.15	-1.283	-1.107
$\theta_{8,i}$	0.1265	0.08702	0.07887
$\theta_{9,i}$	-0.05124	-0.07911	-0.07638
$\theta_{10,i}$	0.0562	0.08536	0.08228
$\theta_{11,i}$	-2.654e-06	-8.778e-06	-3.274e-06
$\theta_{12,i}$	5.263e-06	1.074e-05	4.649e-06

Table 5: Three-zone case - DR program

	h_j	μ_j	S_j $[kWh]$	R_j [€]
$\mathcal{R}_1$	67	5	3.9	0.60
$\mathcal{R}_2$	101	3	3.4	0.45
$\mathcal{R}3$	179	6	4.3	0.75
$\mathcal{R}_4$	325	6	4.2	0.20
$\mathcal{R}_5$	365	6	4.4	0.85

errors, and of course this is neither true nor predictable in general. However, by performing further simulations on different data and identified models, the overall costs of both control laws still remain very close.

One additional observation concerns the quality of the adopted simplified model. Several identification trials performed on data generated in different conditions by EnergyPlus simulations, invariably show that the decoupled model performs very satisfactorily on validation data. This fact is confirmed quite neatly by the cost achieved by the control law designed on the identified model and applied to the EnergyPlus simulator. The data reported in Table 6 show that the degradation of performance is acceptable, being about 3% for the optimal control law and less than 1% for the proposed heuristics.

4.2. Large-scale case

To evaluate the behavior of the proposed approach on a more realistic scenario, a 5-floor building composed of 20 zones for each floor has been modelled through DesignBuilder, a software tool for developing building models to be

Table 6: Three-zone case - Results

	Real [EP]		Simulated [Model]	
	Heur.	Opt.	Heur.	Opt.
Overall cost [€]	26.17	26.86	26.13	26.03
Fulfilled DR reqs	4	4	5	5
DR reward [€]	2.65	2.25	2.85	2.85

used in EnergyPlus simulations. The map of the first floor and a rendered image of the whole building are reported in Fig. 7. The building is equipped with heat pump heating systems and its main characteristics are reported in Tables 7 and 8. The building is still assumed to be located in Milan and the sampling time for simulation is set to 10 minutes.

Table 7: Large-scale case - Building features

Building Component		**Value**
Weather and Location		Milan
Floor Area [m^2]		1530
Floor [#]		5
Zone [#]		100
Window to wall ratio [%]		30
Solar Transmittance		0.84
Solar Reflectance		0.075
Internal Loads	Occupants [#]	100
	Lighting [W/m^2]	2.36
	Equipment [W/m^2]	1.39
Heating: Fan Coil Unit	Power range [kW]	[3; 9]
	Supply Humidity Ratio [$kgWater/kgDryAir$]	0.0156
	Supply Air Temperature [$°C$]	35

4.2.1. Modelling and identification

To apply the proposed heuristics for energy cost reduction, an ARX model for each zone has been modelled in a similar way to that used in the three-zone example. A PRBS signal has been used as input for estimating the ARX parameters; three days have been used for estimation and three for validation. In Fig. 8, a comparison between the real output and the 24-step ahead prediction for a given zone is reported. The average FIT for all zones is over 70%.

4.2.2. Experiment setup and discussion

A simulation of three days has been performed to evaluate the proposed heuristics. Five DR requests have been scheduled in that period as reported in Table 9. Comfort profiles and energy cost are set as in Subsection 4.1.2.

Table 8: Large-scale case - Building construction materials (name/thickness [mm])

	External Walls	**Internal Walls**
Outside Layer	Brickwork/100	Gypsum board/25
Layer 2	Extruded polystyrene/80	Air/10
Layer 3	Concrete block/100	Gypsum board/25
Layer 4	Gypsum plaster/13	

	Floor	**Roof**	**Windows**
Outside Layer	Cast concrete/100	Asphalt/10	Generic Clear/3
Layer 2	Tile/2	Glass wool/140	Air/13
Layer 3		Air/200	Generic Clear/3
Layer 4		Gypsum board/13	

Table 9: Large-scale case - DR program

	h_j	μ_j	S_j $[kWh]$	R_j [€]
$\mathcal{R}_1$	67	5	32	6.00
$\mathcal{R}_2$	101	3	29	4.50
$\mathcal{R}3$	179	6	41	7.50
$\mathcal{R}_4$	325	6	30	2.00
$\mathcal{R}_5$	365	6	37	8.50

The application of the proposed method yields an objective value of 235.51 €, including a DR reward of 26.50 €. Fig. 9 displays the heater command and the temperature for one zone. The average time needed to compute heater commands for all zones at a given step is less than 20 seconds.[1] It is worth remarking that one of the novelty of this paper is the formulation of a relaxed optimization problem based on a decoupled linear regressive model of the building. In fact, thanks to decoupling, the computational burden of the proposed algorithm is proportional to the number of zones, thus allowing feasible computations even for large buildings. Moreover, such computations can be easily parallelized with a great reduction of computation time.

Like in the previously reported example, the decoupled identified model still shows good performance as depicted in Fig. 8. This is an essential prerequisite for the use of MPC techniques.

Regarding internal temperature reported in Fig. 9, one may notice similarities w.r.t. to the three-zone test case. In particular, preheating before a DR request as well as reduction of power consumption during on-peak times are again observed. In addition, one may notice a fast rising edge of zone temperature before 8 a.m. of each day (i.e., before hour 8, 32 and 56 in the simulation) due to the change in the lower comfort bound. By looking at the DR requests,

[1] Computations have been performed using *CPLEX* [31] to solve the LPs, on an Intel Core i5 M520 at 2.40 GHz with 4 GB of RAM.

it becomes apparent how the proposed technique tries to honour such requests by reducing the power consumption in the respective time intervals. However, it is worthwhile to note that in general not all DR requests will be satisfied, but only those that are economically convenient and feasible from the comfort point of view.

As opposed to the three-zone case, it is not possible to compare the suboptimal strategy with the optimal one, due to the numerical intractability of the optimal MILP problem. In order to arrange an experiment to evaluate the quality of the proposed method, the binary commands of the heaters have been relaxed to be continuous, i.e. it is assumed that heaters may change continuously their power from zero to their maximum. Although the considered scenario may not seem feasible from a realistic point of view, it turns out to be convenient for estimating the quality of performance of the proposed approach. In fact, the obtained results provide a lower bound to the optimal cost. The total costs and the DR rewards for both optimizations are reported in Table 10. Notice that the cost obtained by the proposed method turns out to be about 4% greater w.r.t. the relaxed optimal one. It is reasonable to expect that the devised heuristics will behave similarly in the original (non relaxed) problem, too. In Fig. 10, the behavior of the internal temperature of zone 20 for both strategies is reported. Notice that the temperature profiles are almost indistinguishable except in the proximity of the first and the fourth DR requests. As reported in Table 10, the proposed heuristics is able to fulfil 3 DR requests out of 5, contrary to the optimal algorithm which is able to honour all the DR requests. In any case, as previously stated, the gap of the proposed heuristics is just 4% greater w.r.t. the optimal one, showing good performance of the method.

Table 10: Large-scale case - Overall simulation results obtained by assuming continuous regulation of heaters power (relaxed model)

	Relaxed model - Simulated	
	Heuristic	Optimal
Overall cost [€]	200.39	192.47
Fulfilled DR reqs	3	5
DR reward [€]	20.50	28.50

5. Conclusion and future research

In this paper the problem of optimizing the operation of a building heating system under the hypothesis of participation in a demand response program has been addressed. The DR setup is based on price-volume signals sent by an aggregator to the building energy management system. The optimizer exploits a receding horizon control technique for minimizing the energy bill. Since the complexity of the overall optimization is intractable even for buildings of modest dimension, a heuristic search strategy based on problem decomposition has been devised to make the computational burden reasonable. Numerical results show that the heuristic strategy involves almost negligible loss of accuracy with respect to the exact optimal solution. The overall optimization procedure has been tested both on the simplified identified model used for design, and on a realistic building model computed using EnergyPlus. The results show excellent performance in terms of robustness of the control law to model uncertainties.

Future work will address two main problems. First, validate the proposed approach on the energy management system of a real large-scale building, by deeply evaluating benefits both from the economic and the environmental viewpoints. Second, investigate the applicability of the optimal predictive control approach to a more complex setup, where the building is considered as a microgrid, including the entire HVAC plant, different renewable generation sources, electric and thermal storage devices, electric appliances and other kinds of electric loads.

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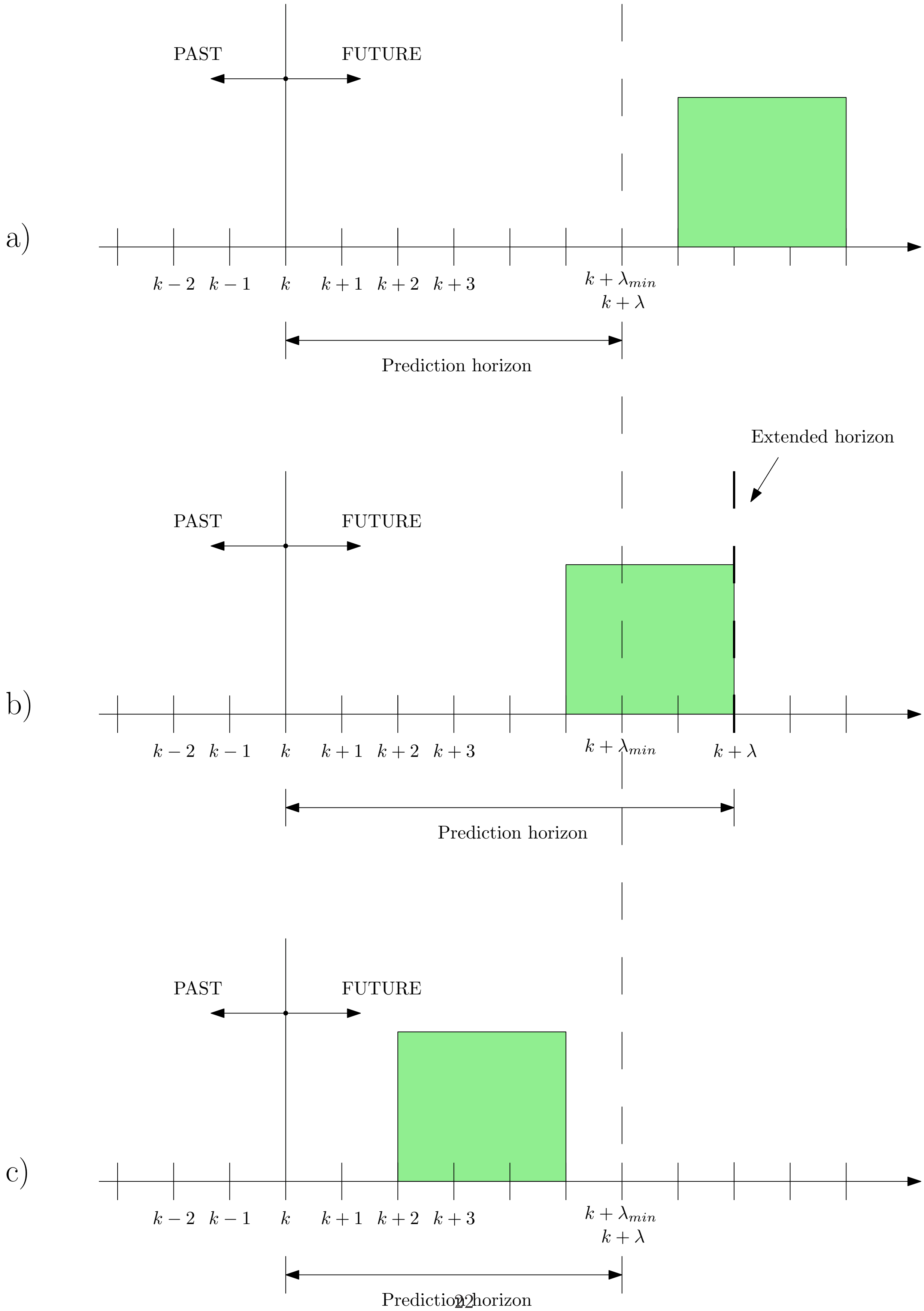

Figure 2: Effect of DR request on the horizon length of the MPC scheme. a) Standard condition: no DR request within the horizon ($\lambda = \lambda_{min}$). b) Prediction horizon adaptation: the prediction horizon λ is extended to fully cover the incoming DR request ($\lambda > \lambda_{min}$). c) Standard condition: DR fully contained in the prediction horizon.

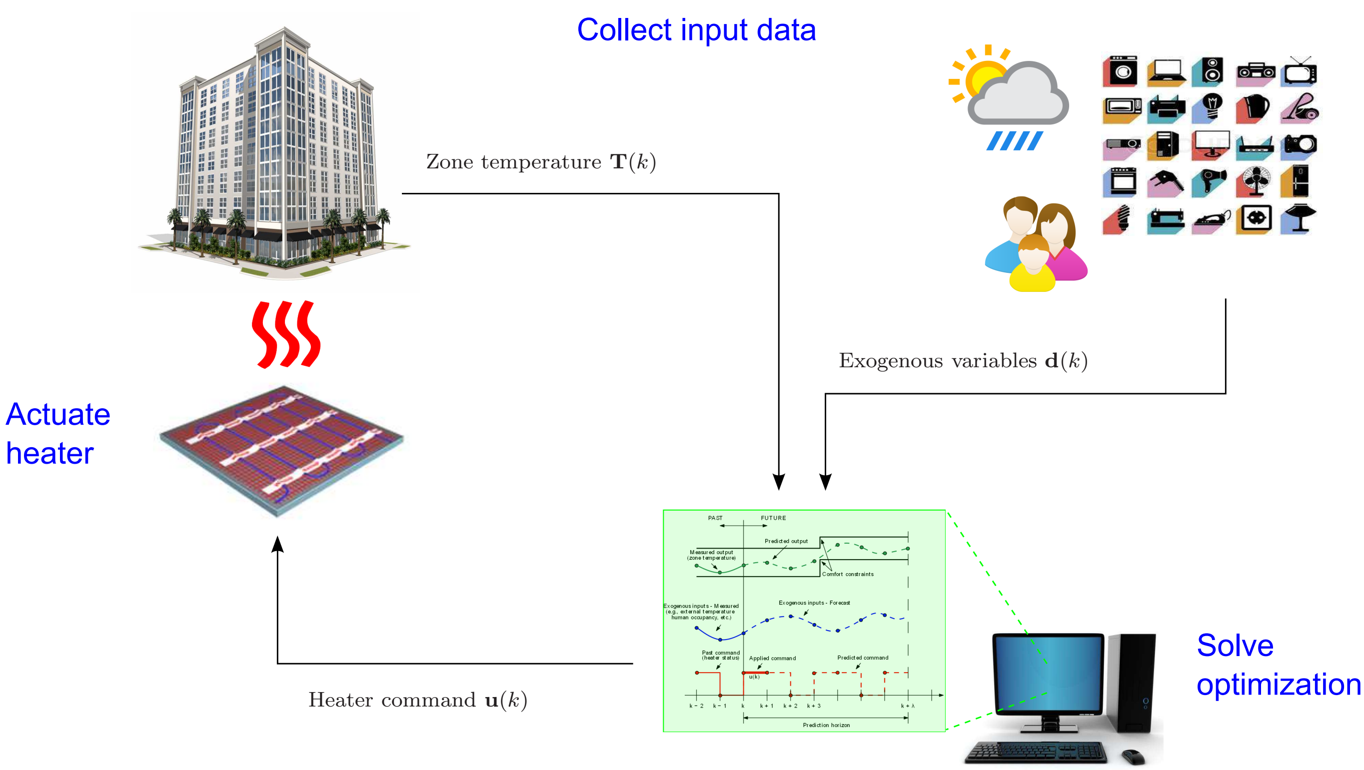

Figure 3: Control system architecture.

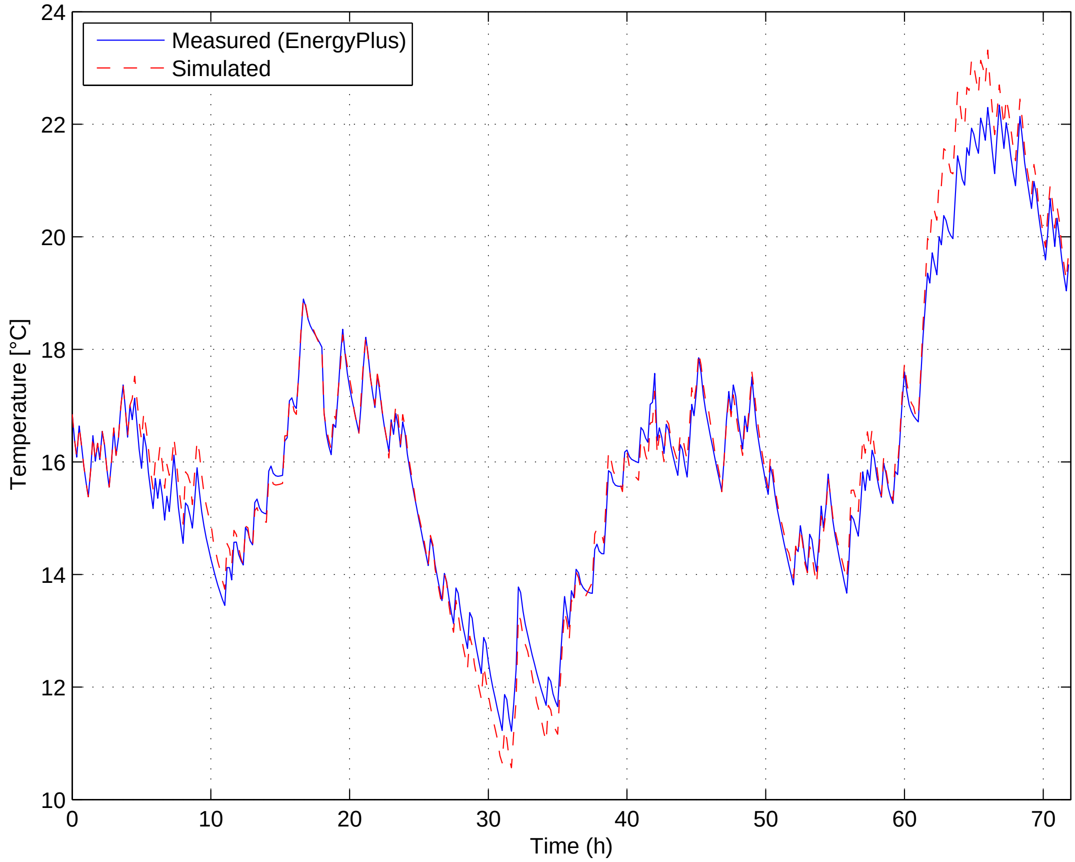

Figure 4: Three-zone case: model validation on 24-step ahead prediction. Comparison between real and predicted output for three validation days (January, 18-20).

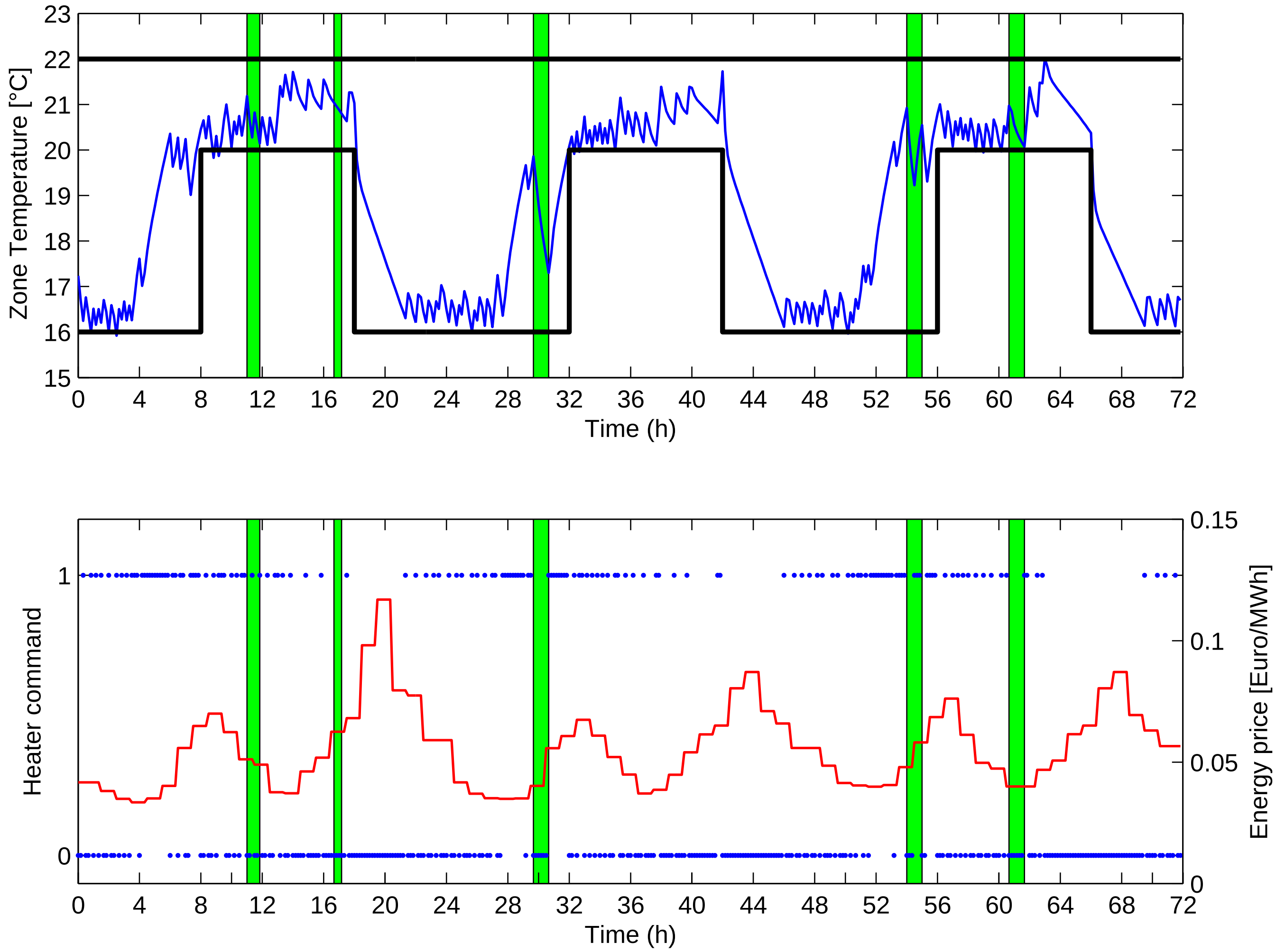

Figure 5: Three-zone case: results obtained through the proposed optimization heuristics on the real scenario (EnergyPlus model). Top: temperature of zone 1 (blue), comfort constraints (black), DR program (green). Bottom: Heater command (blue), energy price (red), DR program (green).

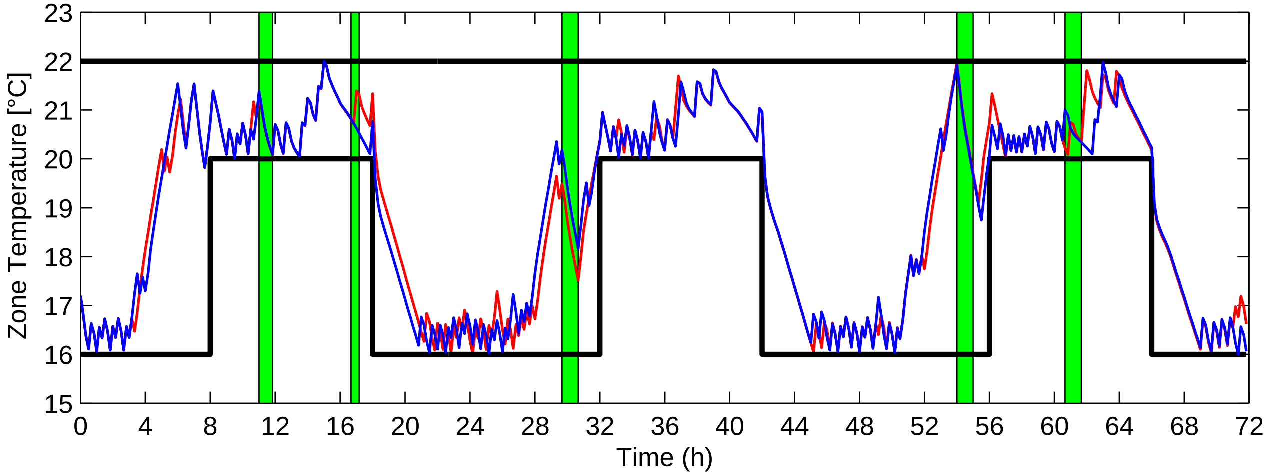

Figure 6: Three-zone case: simulation results obtained on the identified model. Temperature of zone 1 obtained by using the proposed heuristics (blue) and the optimal algorithm (red). Green bands denote DR requests.

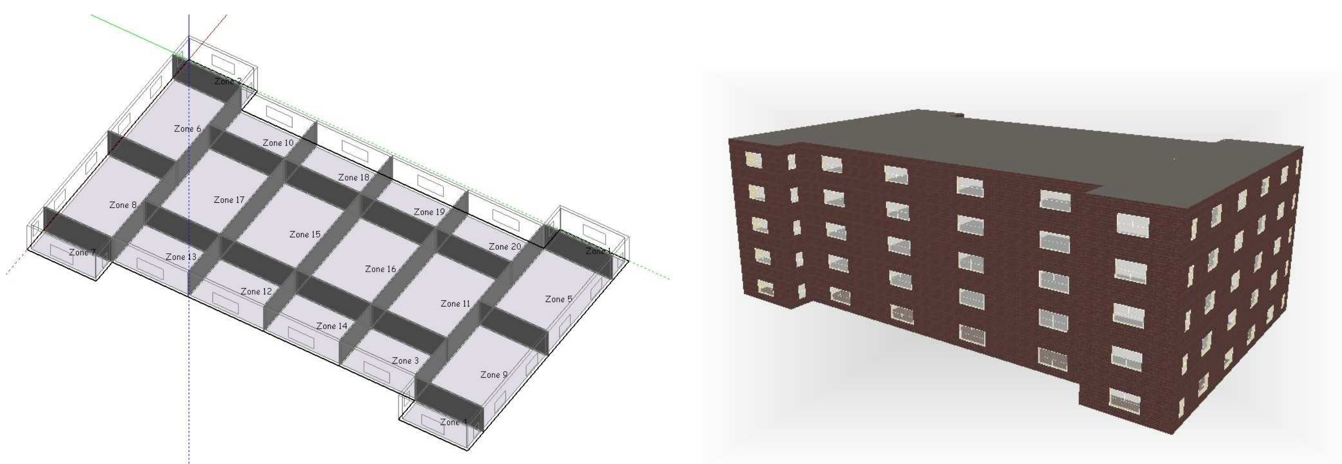

Figure 7: Large-scale case. Map of the first floor and building rendering.

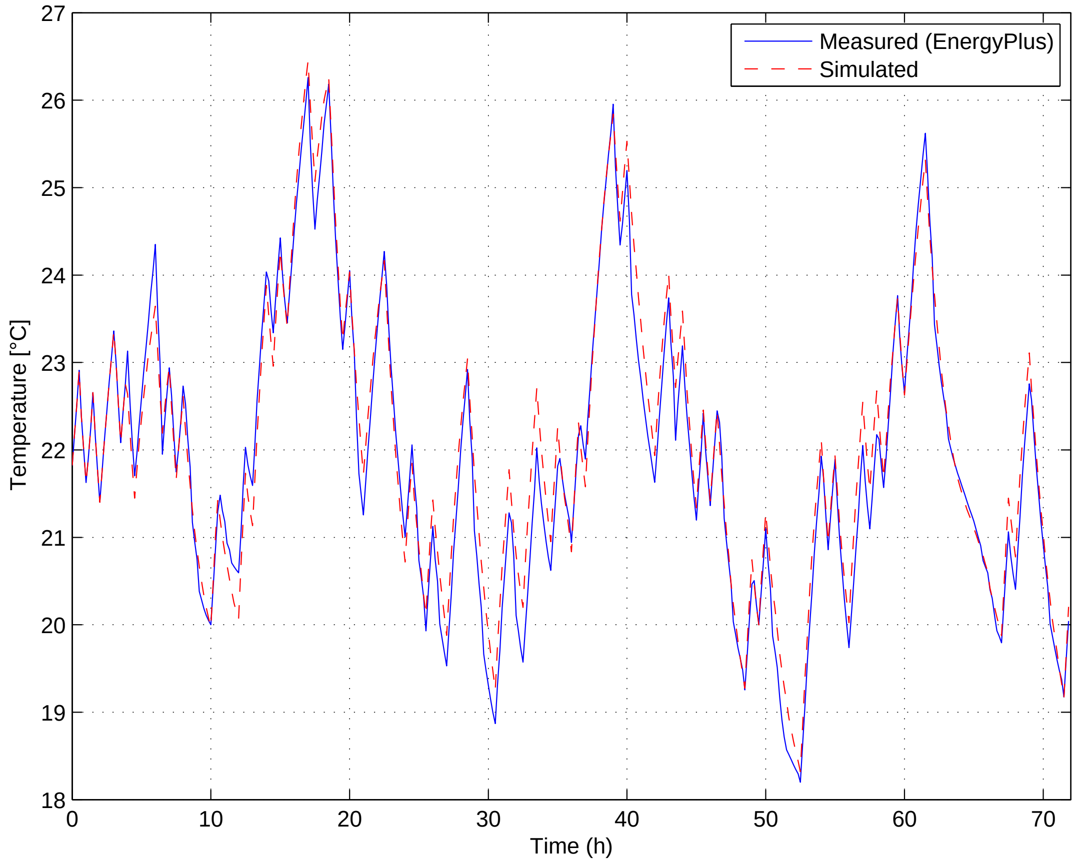

Figure 8: Large-scale case: model validation on 24-step ahead prediction. Comparison between real and predicted output for three validation days (January 25-27).

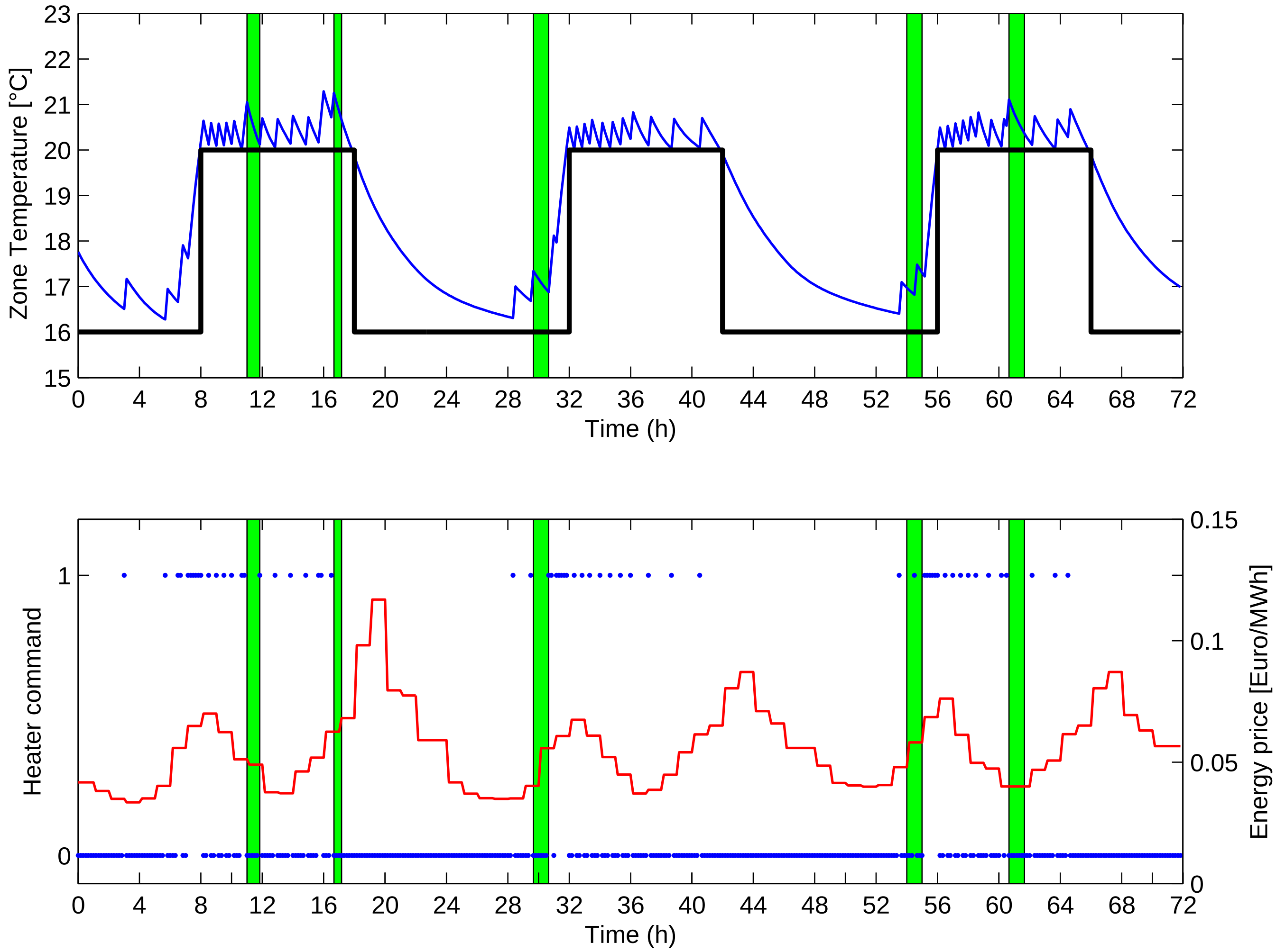

Figure 9: Large-scale case: simulation results obtained through the proposed optimization heuristics on the identified model. Top: temperature of zone 20 (blue), comfort constraints (black), DR program (green). Bottom: Heater command (blue), energy price (red), DR program (green).

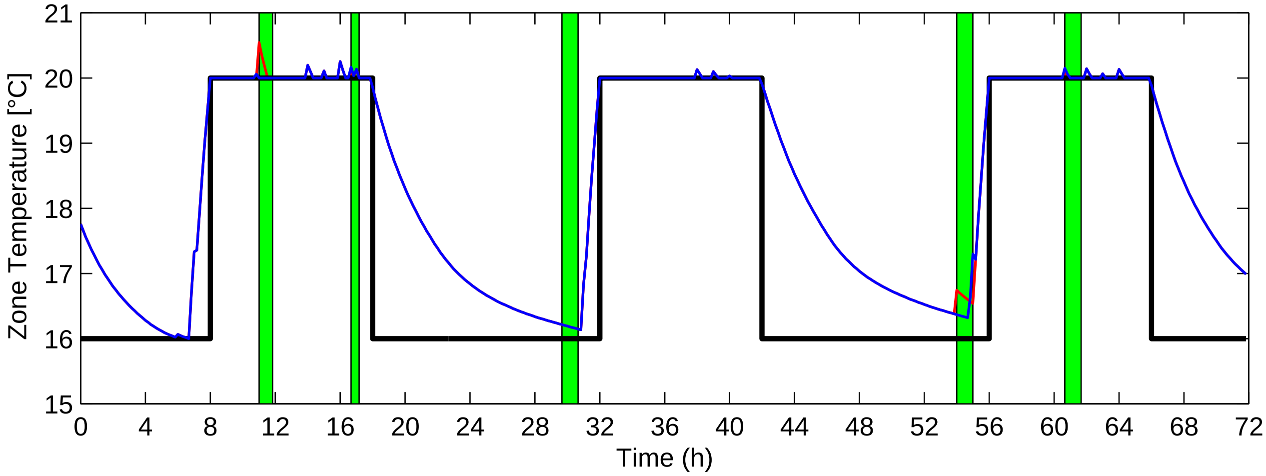

Figure 10: Large-scale case: simulation results obtained by assuming continuous regulation of heaters power (relaxed model). Temperature of zone 20 obtained by using the proposed heuristics (blue) and the optimal algorithm (red). Green bands denote DR requests.