1)
$$\lim_n \frac{\log n}{n^2} = 0$$

$$\lim_n \frac{\log(n+1)}{\log n} = 1$$

Dunque il carattere della serie (che è a termini positivi) è lo stesso del carattere della serie

$$\sum_{n\geq 2}' \left(\frac{n^2}{n^\beta \log n} \right)^{1/2} = \sum_{n\geq 2} \frac{1}{n^{(\beta-2)/2} (\log n)^{1/2}}$$

Se $\frac{\beta-2}{2} > 1$ abbiamo

$$\sum_{n\geq 2}' \frac{1}{n^{\frac{\beta-2}{2}} (\log n)^{1/2}} \leq \sum_{n\geq 2}' \frac{1}{n^{\frac{\beta-2}{2}}} < +\infty$$

Dunque se $\frac{\beta-2}{2} > 1$, i.e. $\beta > 4$, la serie converge

Se $\frac{\beta-2}{2} \leq 1$ abbiamo

$$\sum_{n\geq 2}' \frac{1}{n^{\frac{\beta-2}{2}} (\log n)^{1/2}} \geq \sum_{n\geq 2}' \frac{1}{n (\log n)^{1/2}} = +\infty .$$

2) $x \to 0^- \Rightarrow (\operatorname{sen} x)^{-3} \longrightarrow -\infty$

$$\Rightarrow \lim_{x \to 0^-} \operatorname{arctg}\left(\frac{1}{(\operatorname{sen} x)^3}\right) = -\pi/2$$

Dunque il limite è " della forma $\frac{0}{0}$ "

Si ha $x^2 \operatorname{sen}(x^7) \sim x^9 \sim (\operatorname{sen} x)^9$

Il limite cercato è

$$= \frac{1}{7} \lim_{x \to 0^-} \frac{\operatorname{arctg}\left(\frac{1}{(\operatorname{sen} x)^3}\right) + \pi/2 + \operatorname{sen}((\operatorname{sen} x)^3)}{(\operatorname{sen} x)^9}$$

Cambiando variabile $-t = (\operatorname{sen} x)^3$

$$= -\frac{1}{7} \lim_{t \to 0^+} \frac{-\operatorname{arctg}\left(\frac{1}{t}\right) + \pi/2 - \operatorname{sen} t}{t^3}$$

De l'Hopital

$$= -\frac{1}{7} \lim_{t \to 0^+} \frac{\frac{1/t^2}{1+\frac{1}{t^2}} - \cos t}{3t^2}$$

$$= -\frac{1}{7} \lim_{t \to 0^+} \frac{\frac{1}{1+t^2} - \cos t}{3t^2}$$

$$= -\frac{1}{7} \lim_{t \to 0^+} \frac{1 - (1+t^2)\cos t}{3t^2}$$

Taylor

$$= -\frac{1}{t} \lim_{t \to 0^+} \frac{-\frac{1}{2}t^2 + o(t^2)\cos t}{3t^2} = \frac{1}{42}$$

3)

$$f(x) = \frac{|(x-1)^3|}{x^2-4}$$

$$\text{dom } f = \mathbb{R} \setminus \{-2, 2\}$$

$$f \in C(\text{dom } f)$$

$$\lim_{x \to \pm\infty} f(x) = +\infty$$

$$\lim_{x \to -2^-} f(x) = \lim_{x \to 2^+} f(x) = +\infty$$

$$\lim_{x \to -2^+} f(x) = \lim_{x \to 2^-} f(x) = -\infty$$

$f(x) = 0 \iff x = 1,$

$f(x) > 0$ in $x \in (-\infty, -2) \cup (2, +\infty)$

$f(x) \leq 0$ in $x \in (-2, 2)$

$f \in C^1(\text{dom } f),$

$$f'(x) = \frac{(x-1)|x-1|(x^2+2x-12)}{(x^2-4)^2}$$

$f \uparrow$ in $[-1-\sqrt{13}, -2)$, $(-2, 1]$, $[-1+\sqrt{13}, +\infty)$

$f \downarrow$ in $(-\infty, -1-\sqrt{13}]$, $[1, 2)$, $(2, -1+\sqrt{13}]$

$f'(x) = 0$ se $x = -1-\sqrt{13}$, $x = 1$, $x = -1+\sqrt{13}$

$f \in \mathcal{C}^2(\text{dom} f)$

$$f''(x) = \frac{2(x-1)\left(7x^2 - 32x + 52\right)}{(x^2-4)^3}$$

e $7x^2 - 32x + 52 > 0 \quad \forall x$

f convessa in $(-\infty, -2)$, $(2, +\infty)$

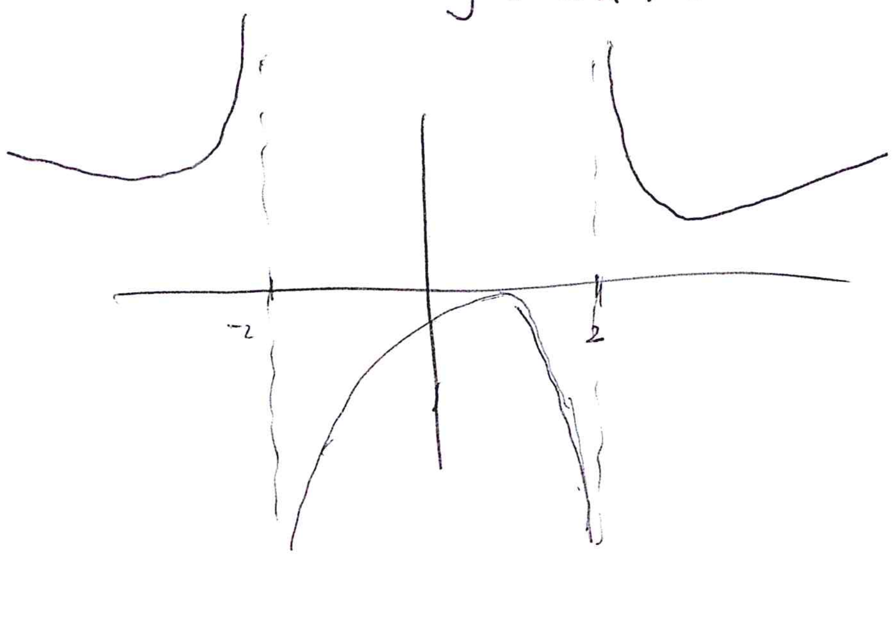
f concava in $(-2, 2)$

asintoti obliqui: $a = \lim_{x \to +\infty} \frac{f(x)}{x} = 1$, $\lim_{x \to +\infty} f(x) - ax = -3$

$y = x - 3$ a $+\infty$

$c = \lim_{x \to -\infty} \frac{f(x)}{x} = -1$, $d = \lim_{x \to -\infty} f(x) - cx = 3$

$y = -x + 3$ a $-\infty$



4)

$$\int \frac{x^2+1}{x^3-4x^2+5x-2}$$

$x=1$, $x=2$ sono radici del denominatore, e

$$x^3-4x^2+5x-2 = (x-1)^2(x-2)$$

$x=2$ radice semplice, $x=1$ radice doppia

$$\Longrightarrow \int = -4\int \frac{1}{x-1}\,dx + 5\int \frac{dx}{x-2} + \frac{2}{x-1}$$

$$= \log \frac{|x-2|^5}{|x-1|^4} + \frac{2}{x-1} + C$$