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1)

$$e^{x}\cos(3x) = 1 + x - 4x^{2} - \frac{13}{3}x^{3} + \frac{7}{6}x^{4} + \frac{79}{30}x^{5} + o(x^{5})$$

$$\frac{\cos(x\sqrt{10})}{1-x} = 1 + x - 4x^{2} - 4x^{3} + \frac{1}{6}x^{4} + \frac{1}{6}x^{5} + o(x^{5})$$

$$\frac{1}{3}x^{3}e^{-3x} = \frac{1}{3}x^{3} - x^{4} + \frac{3}{2}x^{5} + o(x^{5})$$

Il numeratore $= \frac{119}{30}x^{5} + o(x^{5})$

il limite cercato $= \frac{7}{30}$

2) Serie NON a termini positivi, non a segni alterni

Convergenza assoluta: $|\cos n| \leq 1$, $e^{\operatorname{sen} n} \leq e$

confronto asintotico

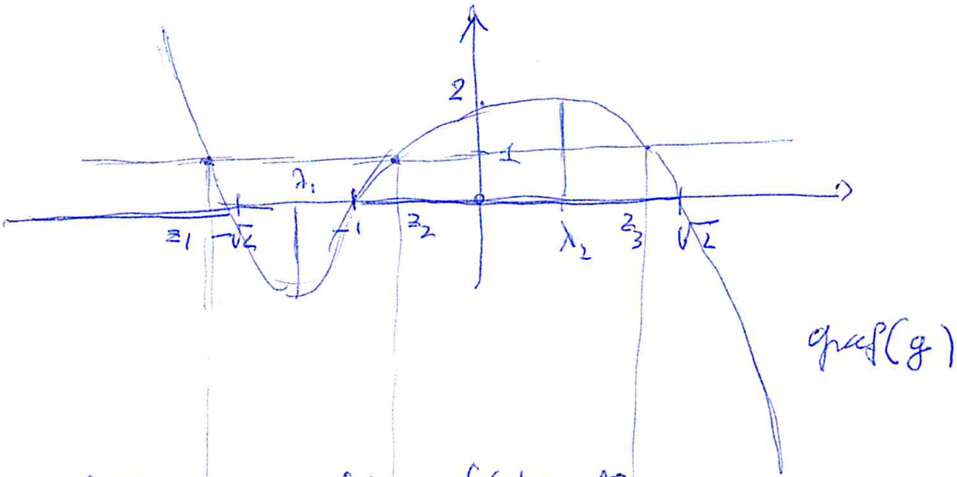
$$|a_n| \leq e \, \frac{\operatorname{sen}\frac{1}{n}}{\frac{1}{n}} \, \frac{1}{n} \, \frac{e^{\frac{1}{\sqrt{n}}} - 1}{\frac{1}{\sqrt{n}}} \, \frac{1}{\sqrt{n}} \sim \frac{1}{n^{3/2}}$$

Convergente ass. $(3/2 > 1)$ dunque convergente.

3) $f(x) = \log g(x), \qquad g(x) = (1+x)(2-x^2) = -x^3 - x^2 + 2x + 2$

$\mathrm{dom}(f) = \{x \in \mathbb{R} : g(x) > 0\} = (-\infty, -\sqrt{2}) \cup (-1, \sqrt{2})$

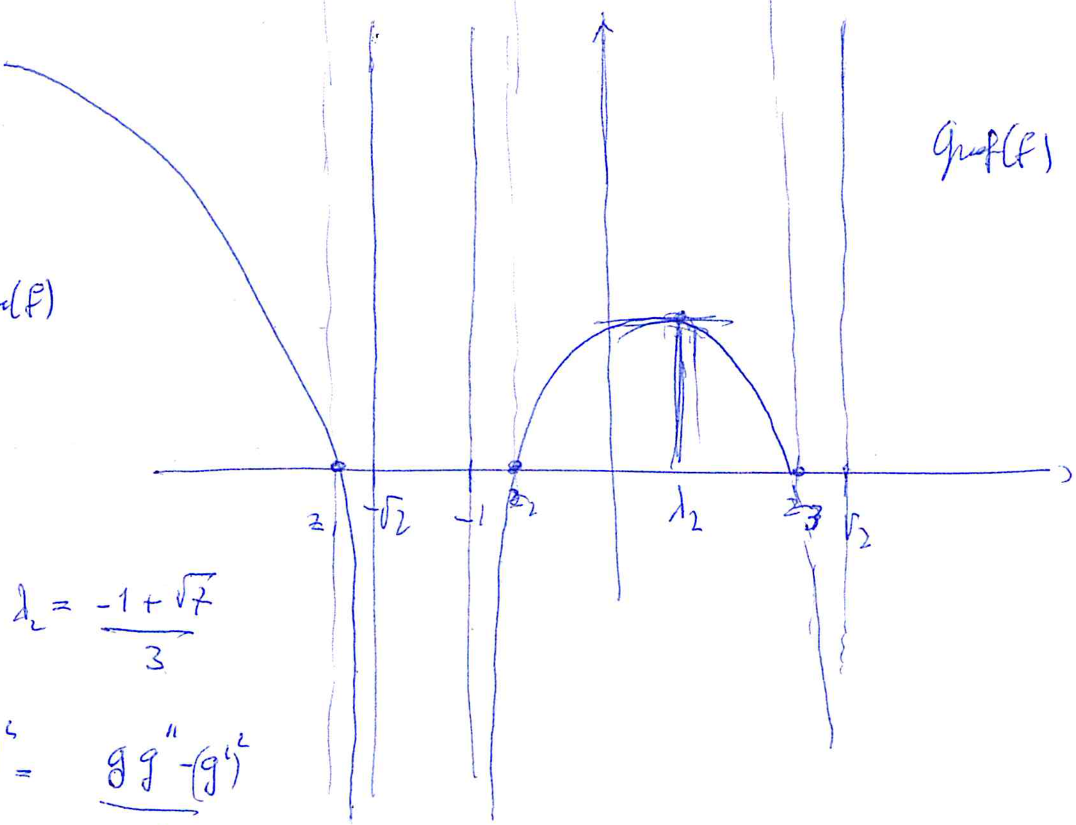
$f \in \mathcal{C}^\infty(\mathrm{dom}(f))$



$\lim_{x \to -\infty} f(x) = +\infty$

$\lim_{x \to (-\sqrt{2})^-} f(x) = -\infty, \quad \lim_{x \to (-1)^+} f(x) = -\infty, \quad \lim_{x \to (\sqrt{2})^-} f(x) = -\infty$

$\{f = 0\} = \{g = 1\} = \{z_1, z_2, z_3\}$



$f' = g'/g$ su $\mathrm{dom}(f)$

$= \dfrac{-3x^2 - 2x + 2}{g(x)}$

$\lambda_1 = -\dfrac{1-\sqrt{7}}{3}, \qquad \lambda_2 = \dfrac{-1+\sqrt{7}}{3}$

$f'' = \dfrac{g''}{g} - \dfrac{(g')^2}{g^2} = \dfrac{g g'' - (g')^2}{g^2}$

4)

$$\text{Int} = \int_{\sqrt{2}/2}^{\sqrt{3}/2} \frac{1}{\sqrt{1-t^2}}\, dt \qquad (\sqrt{x} = t)$$

$$= \left(\arcsin t \right)_{\sqrt{2}/2}^{\sqrt{3}/2} = \frac{\pi}{12}$$

5) $\sqrt{x} - 1 < 0 \Rightarrow$ l'integrale o converge o diverge negativamente.

Integrand illimitato in $(0, \delta)$ $\forall \delta > 0, \delta < 1$

Confronto con $g(x) = -\frac{1}{x^{1/4}}$

$$\lim_{x \to 0^+} \frac{-x^{\frac{1}{4}} \sqrt{x} (\sqrt{x} - 1)}{\log(1 + x^{3/4})(1 + x^3)} = \lim_{x \to 0^+} \frac{x^{1/4} \sqrt{x}}{\log(1 + x^{3/4})} = 1$$

Siccome $\int_0^1 \frac{1}{x^{1/4}}$ converge, per il criterio del confronto asintotico l'integrale converge