An Optimization Model for the Electricity Market Clearing Problem with Uniform Purchase Price and Zonal Selling Prices

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Abstract

Electricity markets can be designed in different ways. One rule that is sometimes enforced is the uniform purchase price. Under this pricing method, all the consumers pay the same price regardless of the zone they belong to. By contrast, each producer receives its zonal price. This asymmetry in the price paid and received makes the clearing process not easily treatable through standard optimization techniques. Within the framework of marginal pricing, this paper shows how it is possible to formulate the market clearing problem with uniform purchase price and zonal selling prices as a computationally tractable mixed integer linear programming problem. The proposed approach is tested using real data from the Italian day-ahead market, which is actually based on the aforementioned rule.

Index Terms

Uniform purchase price, market clearing, marginal pricing, bilevel programming, binary expansion, power system economics.

NOMENCLATURE

A. Sets and Indices

$\mathcal{Z}^\pi$ Set of UPP zones.

$\mathcal{Z}^\zeta$ Set of non-UPP zones.

$\mathcal{Z}$ Set of all zones, $\mathcal{Z} = \mathcal{Z}^\pi \cup \mathcal{Z}^\zeta$.

i Market zone index, $i \in \mathcal{Z}$.

$\mathcal{K}_i^\pi$ Set of UPP consumers in zone $i \in \mathcal{Z}^\pi$, $\mathcal{K}^\pi = \cup_i \mathcal{K}_i^\pi$.

$\mathcal{K}_i^p$ Set of non-UPP consumers in zone $i \in \mathcal{Z}^\pi$, $\mathcal{K}^p = \cup_i \mathcal{K}_i^p$.

$\mathcal{K}_i^\zeta$ Set of consumers in zone $i \in \mathcal{Z}^\zeta$, $\mathcal{K}^\zeta = \cup_i \mathcal{K}_i^\zeta$.

$\mathcal{K}_i$ Set of all consumers in zone $i \in \mathcal{Z}$, $\mathcal{K} = \cup_i \mathcal{K}_i$.

$\mathcal{P}_i$ Set of all producers in zone $i \in \mathcal{Z}$, $\mathcal{P} = \cup_i \mathcal{P}_i$.

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B. Constants

p_k^d	Price submitted by consumer k in €/MWh, which represents the maximum price he/she is willing to pay in order to buy the quantity d_k.
p_p^s	Price submitted by producer p in €/MWh, which represents the minimum price he/she is willing to receive in order to sell the quantity s_p.
d_k^{max}	Maximum quantity demanded by consumer k, in MWh.
s_p^{max}	Maximum quantity offered by producer p, in MWh.
m_k	Economic merit order for consumer k, lower values mean higher priority. If $p_h^d > p_k^d$ then $m_h < m_k$, with $k, h \in \mathcal{Z}^\pi$. If $p_h^d = p_k^d$ the merit order is assigned by the market operator.
F_{ij}^{max}	Maximum flow capacity from zone i to j, in MWh.
B	Number of bits used in the binary conversion.
c	Number of significant digits in the binary conversion.

C. Variables

d_k	Allocated/executed demand quantity for consumer k, in MWh.
s_p	Allocated/executed supply quantity for producer p, in MWh.
π	Uniform purchase price, in €/MWh.
ζ_i	Zonal price in zone i, in €/MWh.
u_k^g	Binary variable, if $u_k^g = 1$ then $p_k^d > \pi$.
u_k^e	Binary variable, if $u_k^e = 1$ then $p_k^d = \pi$.
F_{ij}	Flow from zone i to j, in MWh.
φ_k^d	Dual variable of constraint $d_k \leq d_k^{max}$.
φ_p^s	Dual variable of constraint $s_p \leq s_p^{max}$.
δ_{ij}^{max}	Dual variable of constraint $F_{ij} \leq F_{ij}^{max}$.
η_{ij}	Dual variable of constraint $F_{ij} + F_{ji} = 0$.

D. Auxiliary Variables

$y_k^{g\pi}$	Auxiliary variable, it replaces the product $u_k^g \pi$.
$y_{ki}^{g\zeta}$	Auxiliary variable, it replaces the product $u_k^g \zeta_i$.
y_k^{ed}	Auxiliary variable, it replaces the product $u_k^e d_k$.
$y_k^{e\pi}$	Auxiliary variable, it replaces the product $u_k^e \pi$.
$y_k^{e\varphi}$	Auxiliary variable, it replaces the product $u_k^e \varphi_k^d$.
$y_{ji}^{\Delta b}$	Auxiliary variable, it replaces the product $(\pi - \zeta_i) b_{ji}$.
b_{ji}	Binary variable, used to convert an integer in binary form.

I. Introduction

Since electricity is a strategic commodity, policy makers may establish rules that force energy markets to deviate from economic principles in favour of specific social targets. One of these rules is the *uniform purchase price* (UPP).

Assuming a multi-zone market, its main characteristic is that each zone can potentially have its own zonal price at which producers are rewarded, but the price paid by all the consumers has to be unique. This rule is implemented for example in the Italian day-ahead market [1].

The UPP π is defined as a weighted average of the zonal prices ζ_i, with weights given by the quantities allocated to the consumers within each UPP zone, i.e., the zones where the UPP pricing method is enforced. Formally,

$$\pi = \frac{\sum_{i \in \mathcal{Z}^\pi} \sum_{k \in \mathcal{K}_i^\pi} d_k \zeta_i}{\sum_{k \in \mathcal{K}^\pi} d_k}. \tag{1}$$

Notice that (1) can be generalized in different ways [2], [3]. However, the form (1) is usually adopted because it ensures that the amount paid by consumers covers exactly the revenues for both the producers and the transmission system operator [2].

In the present paper, the term *producer* refers generically to an operator who submits a sell order, also termed supply or *offer order*. An offer order is a pair $(s_p^{max}; p_p^s)$, where s_p^{max} is the maximum quantity that the pth producer is willing to sell and p_p^s is the minimum price requested. Similarly, the term *consumer* refers generically to an operator who submits a buy order, also termed demand or *bid order*. A bid order is a pair $(d_k^{max}; p_k^d)$, where d_k^{max} is the maximum quantity that the kth consumer is willing to buy, paying a price not more than p_k^d. Specifically, the consumers $k \in \mathcal{K}^\pi$ that pay the UPP are termed *UPP consumers*. Notice that we assume piecewise-constant market curves (Fig. 1).

Under the UPP definition (1), we can state the *UPP rule* as follows. All bid orders with submitted price p_k^d strictly greater than the UPP, i.e., *in-the-money* orders, have to be fully accepted. The bid orders at the same price as the UPP, i.e., *at-the-money* orders, can be partially executed. Those with a submitted price strictly lower than the UPP, i.e., *out-of-the-money* orders, have to be fully rejected.

In order to generalize even further the UPP pricing scheme, it is possible to consider the presence of particular non-UPP consumers $k \in \mathcal{K}^p$, belonging to an UPP zone, but excluded from the UPP rule. That is, we assume the existence of consumers that belong to UPP zones, but pay zonal prices instead of the UPP. For example, bid orders submitted by pumping units belonging to hydroelectric production plants are usually excluded from the UPP rule.

Market orders belonging to UPP zones are ranked according to a parameter termed *economic merit order*. This ranking coincides with the price ranking for orders with different prices. For orders at the same price, the economic merit order is assigned by the market operator according to a set of hierarchical rules. In the following, lower values imply higher priority. As long as this ranking is compatible with the problem constraints, the priority in the execution has to be fulfilled.

Market clearing methods for non-UPP markets as in [4], [5], are usually based on the *social welfare* maximization [6], [7], [8], which is defined as follows:

$$\max_{d_k, s_p} \sum_{k \in \mathcal{K}} p_k^d d_k - \sum_{p \in \mathcal{P}} p_p^s s_p \,. \tag{2}$$

This approach is adopted because, in a competitive market, the equilibrium obtained in (2) is Pareto-optimal [9] and economically efficient, i.e., it maximizes the aggregate consumer and producer surplus [10]. However, it also

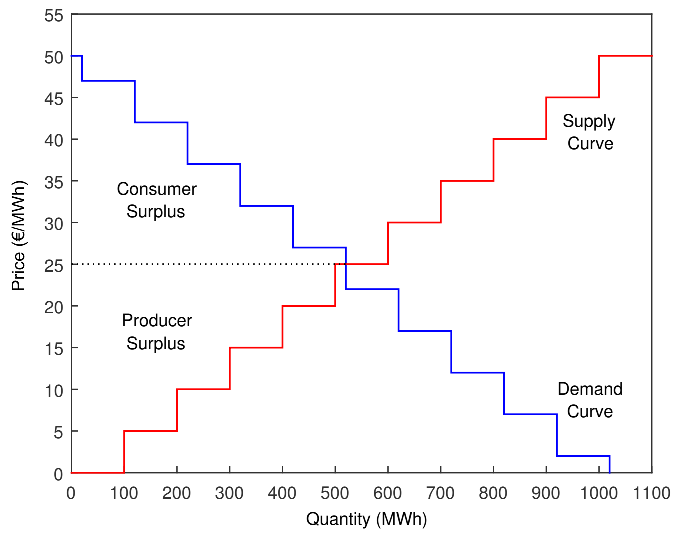

Fig. 1. Consumer and producer surplus. In red the supply curve. In blue the demand curve. The zonal price is given by the intersection of the two curves.

implies that both the demand and supply orders, belonging to the same zone, have to pay and receive the same price, i.e., the zonal price (Fig. 1) [7]. For this reason, the UPP market clearing problem cannot be solved by using the traditional social welfare maximization (2).

To the best of our knowledge, UPP-like problems have been addressed only in a limited number of references with different techniques. In [11] a UPP problem is handled within the context of the consumer payment minimization problem by using a multi-period single-bus model with inter-temporal constraints, assuming an inelastic demand and no network flows. In that work the market price is defined as the highest accepted supply order.

In [3] the problem of having mixed pricing rules in multi-zone markets is addressed by formulating a complementarity program [12], which is further extended to reserve capacity in [13]. Block and complex orders are also considered in [14]. The strength of the complementarity programming approach is that it allows to satisfy a set of conditions involving simultaneously both consumer and producer objectives. However, as acknowledged by the authors in [3], it is not always possible to revert this group of conditions back to an originating optimization problem. This means that it might be impossible to recast a complementarity program as an optimization problem with a well specified objective function [15].

A heuristic method for solving the UPP problem is based on the UPP optimization searching procedure [2]. In this approach, the UPP is iteratively selected among the prices belonging to the submitted bid orders, starting from the value of zero. Then, for each selected and fixed UPP price, the demand orders are accepted or rejected according to the UPP rule. Afterward, a social welfare maximization problem is run to obtain the values of the remaining variables. This process is iterated until all the submitted demand prices are explored and then the optimum solution is selected among the feasible candidates which yield the maximum value of the objective function. One problem of this approach is that it is based on a heuristic search procedure which forces the algorithm to explore all the bid orders, making the process computationally intensive.

The European algorithm for market coupling (EUPHEMIA) currently must handle a UPP problem when it clears the Italian market [8]. This market is characterized by the PUN, i.e., the unique national price paid by all the consumers in the Italian UPP zones, and EUPHEMIA still relies on a heuristic search procedure to solve this problem. At the beginning, the algorithm computes an early solution considering all the possible types of European orders but without enforcing the UPP rule for the Italian market. This step is used to fix all the binary variables. Then, a specific search procedure is used to compute the PUN. In this case, the bid orders are explored heuristically until a solution is found that falls within an accepted tolerance range. If the solution is incompatible with the European constraints, the algorithm starts a new cycle from the beginning. The problem of this method is that it has to go through many steps in order to accommodate all the possible European orders, which makes the overall procedure very cumbersome. In particular, the introduction of the Italian PUN in February 2015 led to a significant increase in the computation time needed to reach an initial feasible candidate solution [16].

Examples of electricity markets with asymmetries between paid and received prices exist also in the US. For instance, in the Californian market producers receive the nodal prices, whereas consumers pay the zonal prices [17]. The nodal prices are computed through a security constrained unit commitment involving the minimization of procurement costs. Then, the zonal prices are obtained with an ex-post process as a load-weighted average of the nodal prices within each zone. This differs from the UPP framework of this paper, where purchase price and selling prices have to be computed simultaneously, and considering elastic market curves for both consumers and producers.

As described above, the existing techniques to deal with the UPP pricing scheme are limited and mainly iterative and/or heuristic. In this context, the contribution of this paper is to propose a heuristics-free and exact mathematical formulation for the UPP market clearing problem, with a coherent interpretation in economic terms. More precisely, the proposed model is developed in three steps. First, a specific non-linear bilevel program is formalized to address the UPP problem. Second, to gain access to market prices, the bilevel model is translated into an equivalent single level problem by exploiting the linear form of its lower level part and the strong duality property. Then, all the non-linearities are removed by using a binary expansion and appropriate auxiliary variables. The final formulation results in a single, computationally tractable mixed integer linear program (MILP), which can be efficiently solved using state-of-the-art solvers. We stress that the solution obtained from this final model is exact, at least at the level of resolution of current market prices and volumes.

The remaining sections of this paper are organized as follows. Section II shows the MILP model and how it is developed starting from a non-linear mixed integer bilevel program. In Section III, the results obtained with our model are compared with real data from the Italian day-ahead market, highlighting the most relevant findings. Finally, Section IV outlines the conclusions and ongoing work.

II. The Model

The UPP problem cannot be solved by using the standard social welfare maximization. Still, it would be desirable to find a formalization within a marginal pricing framework, in which the dual variables of the relevant power balance constraints represent the zonal prices. However, great care has to be adopted when using the marginal pricing framework with the UPP, because in this case, within the same zone, the bid and offer orders may be valued at two different prices. The bilevel model proposed hereafter makes it possible to overcome this problem and provides a formulation consistent with well developed and accepted theory on spot market pricing [6], [7]. This section provides a detailed step-by-step description of how the final model is obtained.

A. Step 1 - The Bilevel Model

The proposed MILP model for market clearing with UPP is built starting from a non-linear bilevel problem with both binary and continuous variables. In general, a bilevel model is composed by two nested optimization problems [18]:

$$\max_{x\in\mathcal{X}} \; F(x, y^*) \tag{3}$$

$$\text{s.t.} \quad y^* = \max_{y\in\mathcal{Y}} \; f(x, y)\,, \tag{4}$$

where (3) is the upper level problem with objective function F, (4) is the lower level problem with objective function f, and $\mathcal{X}$, $\mathcal{Y}$ represent generic constraint sets. Indeed, a bilevel model can be regarded as if it were a two-step optimization with two different, but intrinsically related, optimization problems. Historically, bilevel models trace back to the field of game theory, and in particular, to the class of non-cooperative *Stackelberg* games [15], [18]. These games are also called leader-follower, because the upper level problem can be regarded as a leader who acts in its own interest and before the follower, that, in turn, is represented by the lower level problem. The key feature of the bilevel approach is that the decision variables x of the upper level enter the lower level problem as fixed parameters. This means that the resulting optimum values y^* of the lower level are parametrized by the decision variables of the upper level, i.e., $y^* = y^*(x)$.

In our model, the upper level handles the UPP rule and all the binary variables, which therefore enter the lower level as parameters. In turn, the lower level solves a specific optimization problem to clear the market and to obtain the correct zonal prices, which are used to compute the UPP within the upper level problem. A schematic representation of the proposed bilevel model is shown in Fig. 2.

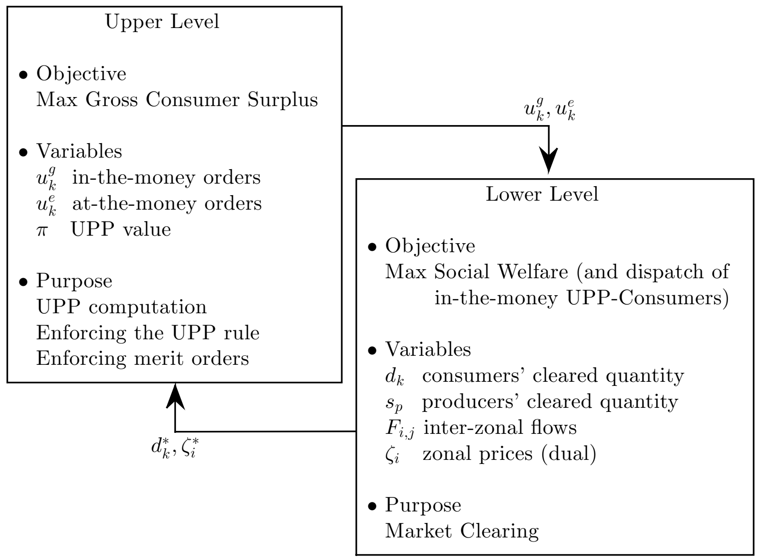

Fig. 2. Scheme of the bilevel optimization model. The upper level variables u_k^g and u_k^e enter the lower level as parameters. In turn, the optimum values d_k^* and ζ_i^* of the lower level variables d_k and ζ_i are inputs to the upper level.

The upper level problem is defined as follows:

$$\max_{u_k^g, u_k^e, \pi} \sum_{k \in \mathcal{K}^\pi} u_k^g p_k^d d_k^{max} + \sum_{k \in \mathcal{K}^\pi} u_k^e p_k^d {d_k}^* \tag{5}$$

$$\text{s.t.} \quad \pi\Big(\sum_{k \in \mathcal{K}^\pi} u_k^g d_k^{max} + \sum_{k \in \mathcal{K}^\pi} u_k^e {d_k}^*\Big) = \sum_{i \in \mathcal{Z}^\pi} \zeta_i^* \Big(\sum_{k \in \mathcal{K}_i^\pi} u_k^g d_k^{max} + \sum_{k \in \mathcal{K}_i^\pi} u_k^e {d_k}^*\Big) \tag{6}$$

$$u_k^g \left(p_k^d - \pi - \varepsilon\right) \geq 0 \qquad \forall k \in \mathcal{K}^\pi \tag{7}$$

$$u_k^e \left(p_k^d - \pi\right) = 0 \qquad \forall k \in \mathcal{K}^\pi \tag{8}$$

$$u_h^g \geq u_k^g \qquad \forall h, k \in \mathcal{K}^\pi : m_h < m_k \tag{9}$$

$$u_h^g \geq u_k^e \qquad \forall h, k \in \mathcal{K}^\pi : p_h^d > p_k^d \tag{10}$$

$$u_k^g \in \{0, 1\}, \, u_k^e \in \{0, 1\} \quad \forall k \in \mathcal{K}^\pi, \tag{11}$$

where ε is an arbitrarily small positive constant. Notice that in (5) and (6) d_k^* and ζ_i^* are solutions of the lower level problem.

The aggregate market demand curve of the UPP consumers, i.e., the collection of all the individual demand orders $(d_k^{max}; p_k^d)$ with $k \in \mathcal{K}^\pi$, can be graphically represented by a non-increasing piecewise-constant function (Fig. 3). The upper level objective function (5) relies strongly on this characteristic. Indeed, it maximizes the *gross* consumer surplus [19], which is the "trapezoidal" area beneath the aggregate demand curve made by the sum of two components: an upper part denoted by "A" in Fig. 3, which is the *net* consumer surplus (commonly referred to

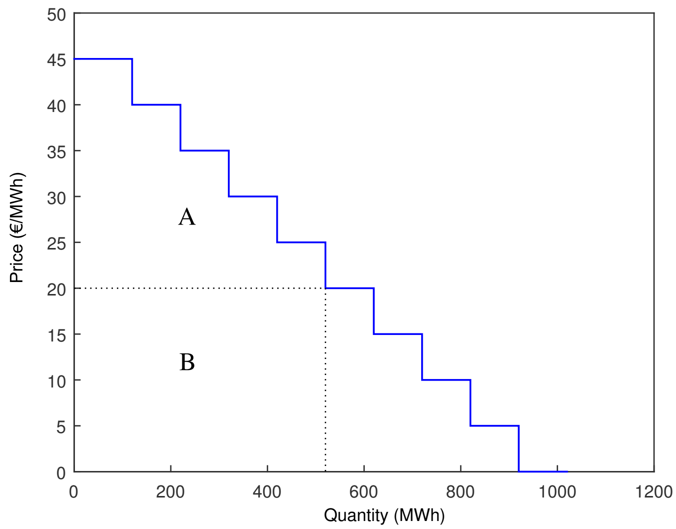

Fig. 3. Aggregate demand curve, $\pi = 20$ €/MWh. The part A is the (net) consumer surplus. The amount actually paid by consumers is the rectangular part B. The "trapezoidal" area A+B is the gross consumer surplus.

as consumer surplus), and a lower rectangular part, which is the amount actually paid by consumers. Exploiting the downward shape of the aggregate demand curve, the maximization of this trapezoidal area implies the maximization of the upper part "A", which coincides with the maximization of the consumer surplus. Together with (6)-(11), this means that all the bid orders are executed as long as they cause an increase in consumer surplus, which is the goal of each rational consumer and a requirement for a true social welfare maximization. Note that this method maximizes also the executed quantities of the at-the-money bid orders ($p_k^d = \pi$). Constraint (7) implies that the variable u_k^g can be equal to one only if the price p_k^d is *strictly greater* than the UPP. Constraint (8) implies that the variable u_k^e can be equal to one only if the price p_k^d is exactly *equal* to the UPP. Note that u_k^g and u_k^e cannot be both equal to one, and this is a key element. Constraint (9) is used to enforce the consecutiveness of the economic merit order for the bid orders strictly greater than the UPP. It also imposes, together with (10), the sequential execution of bid orders, which leads to a significant reduction in the search space of the binary variables, leading to a considerable reduction in the computation time. Constraint (6) is the UPP definition.

Remark 1: In electricity markets there exist the concepts of paradoxically rejected orders (PROs) and paradoxically accepted orders (PAOs). PROs are in-the-money orders that are not executed, while PAOs are out-of-the-money orders that are accepted. In particular, when considering a UPP pricing scheme, PROs are orders with submitted price $p_k^d > \pi$ which are not executed. This happens when the execution of these orders would lead to an increase in the UPP so large to make them actually out-of-the-money, as a joint effect of the UPP rule and the shape of the aggregate offer curve, which is a non-decreasing piecewise-constant function (see Fig. 4). Notice that, if PROs are not allowed, the market clearing problem under the UPP rule might be infeasible. In the proposed bilevel formulation, a PRO occurs when $u_k^g = 0$ and $p_k^d > \pi$, which is allowed due to the specific formulation of (7). By contrast, the conditions (7)-(8) always exclude the occurrence of PAOs. Indeed, if $u_k^g = 1$ then $p_k^d > \pi$, i.e., the executed order is in-the-money, whereas if $u_k^e = 1$ then $p_k^d = \pi$, i.e., the executed order is at-the-money. Notice that a solution with a PRO is acceptable. On the contrary, a PAO would exhibit a design fallacy because it implies that a consumer should pay a price greater than the maximum price he/she is willing to pay.

Now, we examine the lower level problem, which is defined as follows :

$$\left(d_k^*, s_p^*, F_{ij}^*, [\zeta_i^*]\right) = \arg \max_{d_k, s_p, F_{ij}} \sum_{k \in \mathcal{K}^\zeta} p_k^d d_k + \sum_{k \in \mathcal{K}^p} p_k^d d_k + \sum_{k \in \mathcal{K}^\pi} u_k^e p_k^d d_k - \sum_{p \in \mathcal{P}} p_p^s s_p \tag{12}$$

s.t.

$$d_k \le d_k^{max} \qquad [\varphi_k^d \ge 0] \quad \forall k \in \mathcal{K} \tag{13}$$

$$s_p \le s_p^{max} \qquad [\varphi_p^s \ge 0] \quad \forall p \in \mathcal{P} \tag{14}$$

$$F_{ij} \le F_{ij}^{max} \qquad [\delta_{ij}^{max} \ge 0] \, \forall i, j \in \mathcal{Z} \tag{15}$$

$$F_{ij} + F_{ji} = 0 \qquad [\eta_{ij} \in \mathbb{R}] \quad \forall i, j \in \mathcal{Z} \tag{16}$$

$$\sum_{k \in \mathcal{K}_i^p} d_k + \sum_{k \in \mathcal{K}_i^\pi} u_k^e d_k - \sum_{p \in \mathcal{P}_i} s_p + \sum_{j \in \mathcal{Z}} F_{ij} = - \sum_{k \in \mathcal{K}_i^\pi} u_k^g d_k^{max} \qquad [\zeta_i \in \mathbb{R}] \quad \forall i \in \mathcal{Z}^\pi \tag{17}$$

$$\sum_{k \in \mathcal{K}_i^\zeta} d_k - \sum_{p \in \mathcal{P}_i} s_p + \sum_{j \in \mathcal{Z}} F_{ij} = 0 \quad [\zeta_i \in \mathbb{R}] \quad \forall i \in \mathcal{Z}^\zeta \, . \tag{18}$$

Notice that dual variables are represented in square brackets in (12)-(18). The objective function (12) is a social welfare maximization for: 1) all the orders in the non-UPP zones; 2) all the orders of the non-UPP consumers within the UPP zones; 3) all the *at* the money orders of the UPP consumers within the UPP zones (which can be partially executed). For all the *in* the money orders of the UPP consumers within the UPP zones it behaves as a simple optimal constrained economic dispatch [19]. In fact, the quantities of these orders must be fully executed ($d_k = d_k^{max}$) and dispatched. In this sense, the logic of the lower level problem is similar to [2]. Notice that, if there are only in-the-money bid orders, then the market price coincides with the marginal cost of the marginal producer, i.e., the minimum price in the supply curve requested to dispatch all the allocated bid quantities. The constraints (13)-(14) enforce bounds on the desired quantities for consumers and producers. Constraints (15)-(16) set the inter-zonal flow capacities, and notice that (16) implies $F_{i,i} = 0$. The equation (17) represents the power balance within each UPP zone $i \in \mathcal{Z}^\pi$. In those zones, with the exception of the non-UPP consumers, the quantities associated with the bid orders whose price is strictly greater than the UPP, i.e., $u_k^g = 1$, have to be fully executed and dispatched. The quantities associated with the bid orders whose price is exactly equal to the UPP, i.e., $u_k^e = 1$, can be partially executed. Constraint (18) represents the power balance for non-UPP zones. Notice that the lower level problem is a linear program (LP), because the binary variables u_k^g and u_k^e are fixed parameters inside the lower level. The proposed approach relies deeply on this characteristic.

The variables $d_k^*, s_p^*, F_{ij}^*, \zeta_i^*$ represent the solution of the lower level problem for fixed u_k^g and u_k^e. Notice that ζ_i^* are dual variables and it is fundamental to gain access to those variables to compute the UPP in (6). For doing this, in the next subsection we recast the above bilevel model as a single level problem.

Remark 2: Bilevel models are usually *non-cooperative* as in the case of Stackelberg games [15], [18], i.e., the objective function of the upper level differs substantially from the one of the lower level. However, in power system economics these models are sometimes implemented as *cooperative* games, i.e., the objective functions of the upper and lower level coincide (usually being either a social welfare maximization or a cost minimization). In these cases, the bilevel model is not used to build a game, but only to get access to the dual variables of the power balance constraints, which represent the market prices in the marginal pricing framework [15], [20], [21]. In this paper, we adopt a hybrid approach, where the upper level (the leader problem) maximizes the gross consumer surplus, while enforcing the UPP rule and the merit orders. In turn, the lower level (the follower problem) clears the market through a social welfare maximization, while enforcing the dispatch of in-the-money UPP orders determined by the upper level.

B. Step 2 - The Single Level Equivalent Model

In terms of complexity, a bilevel problem belongs to the class of strongly NP-hard problems [18]. For this reason, in this section we show how the bilevel model is transformed into an equivalent, computationally tractable single level problem.

The constraints (6)-(11) and the objective function (5) remain unchanged in the single level problem, with the only difference that, since both the primal and the dual variables of the lower level become decision variables of the single level, (5) is recast as:

$$\max_{\substack{u_k^g, u_k^e, \pi, d_k, s_p, F_{ij}, \\ \zeta_i, \varphi_k^d, \varphi_p^s, \delta_{ij}^{max}, \eta_{ij}.}} \sum_{k \in \mathcal{K}^\pi} u_k^g p_k^d d_k^{max} + \sum_{k \in \mathcal{K}^\pi} u_k^e p_k^d d_k \, . \tag{19}$$

Notice that the starred notation used in (5)-(11) has been removed because in the single level model there is no distinction between upper and lower level variables.

The part that significantly changes is the lower level. We recall that the lower level is an LP problem because the binary variables u_k^g and u_k^e enter the lower level as parameters. Therefore, it can be directly inserted into the single level by using its first order Karush-Kuhn-Tucker (KKT) conditions, where, equivalently, the complementary slackness is replaced by the *strong duality* property [15], [22], [23]. It follows that the lower level is substituted with the optimality conditions:

$$\sum_{k\in\mathcal{K}^{\zeta}} p_k^d d_k + \sum_{k\in\mathcal{K}^{p}} p_k^d d_k + \sum_{k\in\mathcal{K}^{\pi}} u_k^e p_k^d d_k - \sum_{p\in\mathcal{P}} p_p^s s_p =$$
$$\sum_{k\in\mathcal{K}^{\pi}} u_k^e \varphi_k^d d_k^{max} + \sum_{k\in\mathcal{K}^{p}\cup\mathcal{K}^{\zeta}} \varphi_k^d d_k^{max} + \sum_{p\in\mathcal{P}} \varphi_p^s s_p^{max}$$
$$+\sum_{i\in\mathcal{Z}}\sum_{j\in\mathcal{Z}} \delta_{ij}^{max} F_{ij}^{max} - \sum_{i\in\mathcal{Z}^{\pi}}\sum_{k\in\mathcal{K}_i^{\pi}} \zeta_i u_k^g d_k^{max} \tag{20}$$

$$\varphi_k^d + \zeta_i \geq p_k^d \qquad \forall\, k\in\mathcal{K}_i, \forall i\in\mathcal{Z} \tag{21a}$$

$$\varphi_p^s - \zeta_i \geq -p_p^s \qquad \forall\, p\in\mathcal{P}_i, \forall i\in\mathcal{Z} \tag{21b}$$

$$\delta_{ij}^{max} + \eta_{ij} + \eta_{ji} + \zeta_i = 0 \qquad \forall\, i,j\in\mathcal{Z} \tag{21c}$$

$$d_k \leq d_k^{max} \qquad \forall\, k\in\mathcal{K} \tag{22a}$$

$$s_p \leq s_p^{max} \qquad \forall\, p\in\mathcal{P} \tag{22b}$$

$$F_{ij} \leq F_{ij}^{max} \qquad \forall\, i,j\in\mathcal{Z} \tag{22c}$$

$$F_{ij} + F_{ji} = 0 \qquad \forall\, i,j\in\mathcal{Z} \tag{22d}$$

$$\sum_{k\in\mathcal{K}_i^{p}} d_k + \sum_{k\in\mathcal{K}_i^{\pi}} u_k^e d_k - \sum_{p\in\mathcal{P}_i} s_p + \sum_{j\in\mathcal{Z}} F_{ij} =$$
$$-\sum_{k\in\mathcal{K}_i^{\pi}} u_k^g d_k^{max} \qquad \forall i\in\mathcal{Z}^{\pi} \tag{22e}$$

$$\sum_{k\in\mathcal{K}_i^{\zeta}} d_k - \sum_{p\in\mathcal{P}_i} s_p + \sum_{j\in\mathcal{Z}} F_{ij} = 0 \quad \forall i\in\mathcal{Z}^{\zeta}\,. \tag{22f}$$

The constraint (20) represents the strong duality property, i.e., at the optimum the value of the objective function of the primal problem (12) must be equal to the value of the objective function of its dual problem. A fundamental characteristic is that for an LP this property implies the complementary slackness (see [15, Ch. 6], [22, Sec. III], [23, Ch. 4.5]), which allows us to avoid the use of the KKT conditions. Relations (22a)-(22f) are the primal feasibility constraints which are exactly the same as (13)-(18), whereas (21a)-(21c) are the feasibility constraints of the dual problem. Notice that in (12) the terms d_k with $k \in \mathcal{K}^{\pi}$ are meaningful (i.e., they represent the actually executed volumes) only if $u_k^e = 1$. For this reason, in the right-hand side (RHS) of (20) the terms φ_k^d are multiplied by u_k^e when $k \in \mathcal{K}^{\pi}$. We recall that, when $u_k^e = 0$, the executed quantity is d_k^{max} if $u_k^g = 1$, and zero otherwise.

C. Step 3 - The exact MILP Equivalent Model

The single level problem (19) subject to (6)-(11), (20)-(22f) is a non-linear mixed integer model. The purpose of this section is to show how it can be recast as a MILP without introducing any approximation, at least up to the level of detail of current market data. There are two kinds of non-linearities in the single level model: 1) the product of a binary variable with a continuous bounded variable; 2) the product of two continuous variables with a binary variable, which is present only in the UPP definition (6).

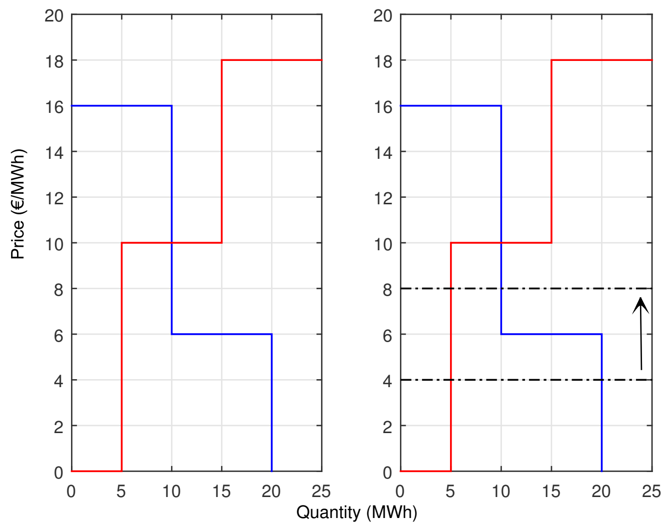

Fig. 4. In blue the demand, in red the supply. The left figure shows a non-UPP zone. The zonal price, i.e., the intersection point of the curves, is 10 €/MWh and only the bid order with price of 16 €/MWh is in-the-money and is executed. The right figure shows the same curves in a UPP zone, and suppose $\pi = 4$ €/MWh. Then, also the bid order with price 6 €/MWh has to be fully executed. As a consequence 20 MWh have to be dispatched and the zonal price rises to 18 €/MWh (as if the demand was inelastic, i.e., a vertical line). The increase in the zonal price determines an increase in the UPP, say it becomes 8 €/MWh. Now, the last accepted bid becomes out-of-the-money and it must be rejected. This bid order is termed Paradoxically Rejected Order.

To linearize the product of a binary and a continuous bounded variable we use a standard approach [24]. Suppose that we want to eliminate the product ux, where u is a binary variable and x is a continuous bounded variable. This can be done by introducing another continuous bounded auxiliary variable y, subject to the following two additional conditions:

$$M_1 u \le y \le M_2 u \tag{23}$$

$$M_1(1-u) \le x - y \le M_2(1-u)\,, \tag{24}$$

where M_1 and M_2 are the lower and upper bounds for x. Then, at each occurrence of the product ux, it is replaced with the auxiliary variable y, and the two additional constraints (23)-(24) are embedded into the single level problem. Notice that the auxiliary variables become additional decision variables.

To linearize (6) without introducing any approximation, we recast the UPP definition as follows:

$$\sum_{k\in\mathcal{K}^\pi} \pi u_k^g d_k^{max} + \sum_{i\in\mathcal{Z}^\pi} \left((\pi - \zeta_i) \sum_{k\in\mathcal{K}_i^\pi} u_k^e d_k \right) = \sum_{i\in\mathcal{Z}^\pi} \sum_{k\in\mathcal{K}_i^\pi} \zeta_i u_k^g d_k^{max}\,. \tag{25}$$

The first term in the left-hand side (LHS) and the one in the RHS in (25) can be linearized as shown in (23)-(24). The challenging part is to find an exact and efficient linear-integer representation for the second term in the LHS, namely:

$$\sum_{i \in \mathcal{Z}^\pi} \Big((\pi - \zeta_i) \sum_{k \in \mathcal{K}_i^\pi} u_k^e d_k \Big). \tag{26}$$

Firstly, notice that the term $\sum_{k \in \mathcal{K}_i^\pi} u_k^e d_k$ is very often zero, because u_k^e can be equal to one only if $p_k^d = \pi$. Moreover, regardless of the UPP value, if $u_k^e = 1$ this term sums some demand quantities belonging to the same zone and having the same price. Then, its upper bound can be computed in advance from the demand curves. Therefore, to linearize (26) we propose a binary expansion as in [25]. The basic idea is to rewrite a positive bounded integer number in binary form using binary variables. In normal circumstances this process is inefficient because it increases significantly the number of binary variables involved. However, we exploit the fact that most of the times the term to be converted is actually zero. The binary form of a positive integer number x requires B bits to be represented, where $B = \lfloor \log_2(x) \rfloor + 1$, and $\lfloor y \rfloor$ is the largest integer smaller than or equal to y. This number represents the amount of auxiliary binary variables needed in our conversion. If the demand quantities d_k are expressed with c decimal digits, the term to be converted has to be multiplied by 10^c in order to obtain an integer value. This allows us to obtain an exact discretization, which leads to an exact solution.

The binary expansion, for given parameters B and c, is made as follows:

$$0 \le y_k^{ed} \le d_k^{max} u_k^e \qquad \forall k \in \mathcal{K}^\pi \tag{27}$$

$$0 \le d_k - y_k^{ed} \le d_k^{max} (1 - u_k^e) \qquad \forall k \in \mathcal{K}^\pi \tag{28}$$

$$10^c \sum_{k \in \mathcal{K}_i^\pi} y_k^{ed} = \sum_{j=0}^{B-1} 2^j b_{ji} \qquad \forall i \in \mathcal{Z}^\pi \tag{29}$$

$$b_{ji} \in \{0, 1\} \qquad \forall j \in \mathcal{H}, \forall i \in \mathcal{Z}^\pi \tag{30}$$

$$M_1 b_{ji} \le y_{ji}^{\Delta b} \le M_2 b_{ji} \qquad \forall j \in \mathcal{H}, \forall i \in \mathcal{Z}^\pi \tag{31}$$

$$M_1 (1 - b_{ji}) \le (\pi - \zeta_i) - y_{ji}^{\Delta b} \le M_2 (1 - b_{ji}) \qquad \forall j \in \mathcal{H}, \forall i \in \mathcal{Z}^\pi, \tag{32}$$

where $\mathcal{H} = \{0, \dots, B-1\}$ and M_1, M_2 are appropriate lower and upper bounds. The constraints (27)-(28) define the auxiliary variables y_k^{ed}, which are used to replace the non-linear product $u_k^e d_k$. In (29), the term in the LHS uses the auxiliary variables y_k^{ed} to sum the quantities $u_k^e d_k$, with $k \in \mathcal{K}_i^\pi$, and expresses the result as an integer, while the RHS enforces the actual conversion into the binary form. Finally, (31)-(32) define the auxiliary variables $y_{ji}^{\Delta b}$, used to replace the products between $(\pi - \zeta_i)$ and the binary variables b_{ji}.

After substituting the terms $u_k^g \pi$ and $u_k^g \zeta_i$ with their auxiliary variables $y_k^{g\pi}$ and $y_{ki}^{g\zeta}$ by using (23)-(24), the final UPP definition equivalent to (6) and (25) is:

$$\sum_{k \in \mathcal{K}^\pi} y_k^{g\pi} d_k^{max} + \sum_{i \in \mathcal{Z}^\pi} \sum_{j=0}^{B-1} \frac{2^j y_{ji}^{\Delta b}}{10^c} = \sum_{i \in \mathcal{Z}^\pi} \sum_{k \in \mathcal{K}_i^\pi} y_{ki}^{g\zeta} d_k^{max}, \tag{33}$$

which is a linear equation in the transformed variables. We stress that, given appropriate values for the parameters

B and c in (29), the linear equation (33) yields the exact UPP. The complete MILP for the UPP market clearing problem is reported in Appendix A.

Remark 3: In (29), the term in the LHS is almost always zero, because u_k^e can be equal to one only if $p_k^d = \pi$, and the binary expansion in the RHS exploits deeply this characteristic. Indeed, when the LHS is zero, the only possible value for all the binary variables b_{ji} is zero, with a considerable gain in computational speed. This property is exploited by assigning to the variables u_k^e the highest priority in the Branch-and-Bound used to solve the MILP. Notice also that another computational improvement is achieved by using the constraints (9)-(10) which reduce greatly the search space of the binary variables by imposing the consecutive execution of the bid orders.

III. RESULTS

In this section we compare the results of our model with real market data downloaded from the website of the Italian market operator [1]. We recall that the UPP rule is adopted in the Italian day-ahead market and the Italian UPP is termed PUN. Hereafter, the two terms are used interchangeably. In this market, there are pumping units which are used to refill the reservoir of hydroelectric production plants, and their demand orders are excluded from the PUN rule. In our model, these pumping units are represented by the set $\mathcal{K}^p$. The computations were performed by using GAMS 24.8.5 [26] and Cplex 12.7.1.0 [27] on the NEOS-7 server [28]. The gap, defined as the relative difference between the best integer and the best remaining node relaxation, was imposed to be zero, i.e., the obtained results are guaranteed to be optimal solutions. The initialization value for PUN was set to 50 €/MWh. In the Branch-and-Bound process, the highest priority was imposed to the binary variables u_k^e, then to u_k^g and b_{ij} with equal priority. The values of the parameters in (7) and (29)-(32) are ε=10^{-6}, c=3, B=24, M_1=-3000 and M_2=3000. The only assumption made in our computations is that $\pi < 3000$ €/MWh, which is a very plausible assumption. Actually, having a PUN equal to such a value, would mean that in all the PUN zones, simultaneously, the zonal prices would be 3000 €/MWh, i.e., the maximum value allowed on the Italian market, which is extremely unlikely. This assumption is used to fix both $u_k^e = 0$ and $u_k^g = 1$ if $p_k^d = 3000$ €/MWh. The Italian market accepts also bid orders without a specified price. In these cases, the actual price has to be considered equal to 3000 €/MWh. If inside a PUN zone there were only bid orders without a specified price and no offer orders at all, one dummy

TABLE I
SUMMARY OF THE RESULTS OF THE MILP MODEL

	Avg. Time[a]	Avg. # Binary	Avg. # Bid	Avg. # Offer
May 1st	1.46	363	472	1107
May 2nd	2.05	365	480	1094
May 3rd	1.69	384	480	1226
May 4th	2.18	382	481	1277

[a] Average Cplex time, as reported by "resource usage" parameter in GAMS, in seconds.

offer was added with quantity equal to 10^{-9} MWh and price zero in order to avoid degeneracy issues.

We test our model using data from May 1st to May 4th, 2010. In all the 96 instances of the MILP problem (one for each hour of the four days considered), both the zonal prices and the PUN coincide with the true values. Also the volumes are perfectly matched, with the exception of a possible different allocation in case of partial execution for those orders with the same price as the one partially executed. This happens because we did not enforce the economic merit order in that case. However, this requirement can be checked ex-post and the quantities involved reallocated accordingly. As a summary of the results, Table I reports the computation time and the number of binary variables, bid orders and offer orders averaged over the 24 hourly instances of the MILP problem solved for each day.

In Table II we show the detailed results for May 4th. For each hour the following quantities are reported: presence of market split, the true Italian PUN, the PUN obtained from our model, the computation time, and the number of binary variables actually used by the model. Notice that *market split* is the state when the transmission lines are congested and the zonal prices can differ from one zone to another.

The 4th hour of May 4th represents a case where $u_k^e = 1$. When this happens, the binary expansion (29) effectively takes place. The bid order involved belongs to the Sicilian zone. It had $p_k^d = 33.36$ €/MWh and $d_k^{max} = 1$ MWh, and was partially executed for 407 kWh, which is the value to be converted in binary form. Notice that in this case the Cplex time required to obtain the final solution is 1.667 seconds (see Table II, row 4), showing that the binary expansion method is computationally sustainable.

The 24th hour of May 4th exhibits another interesting result. In this case, a bid order is present with $d_k = 140$ MWh, belonging to the pumping unit of the hydroelectric plant of Edolo. During this hour there is market split, and the zonal prices and the Italian PUN are different. We recall that the pumping units are excluded from the PUN rule. That order was submitted without price specification so its effect on the clearing problem can be appreciated only with respect to its quantity. Indeed, our model correctly processes this order and excludes that quantity from the computation in (33), yielding the correct value for the PUN which is $\pi = 56.011383$ €/MWh. If that quantity was not excluded, the PUN computation would lead to an incorrect value, i.e., 55.998580 €/MWh, with a difference of 0.012803 €/MWh. Notice that this effect is revealed by the presence of market split, otherwise it would remain hidden. Indeed, in practical terms, when there is no market split, the PUN and the zonal prices are coincident and everything works as if the PUN rule were not present.

IV. Conclusion

We showed how the UPP problem can be solved starting from a non-linear bilevel model with both continuous and binary variables and obtaining a single equivalent MILP which is computationally tractable, exact, and solvable using standard software. The market prices were obtained within the marginal pricing framework, overcoming the non trivial problem posed by the duality in the paid and received prices.

In the context of the European market clearing, where the introduction of the Italian PUN determined a significant computational overhead, our model may help solve efficiently at least the Italian part of the problem. We stress that the European electricity market is characterized also by other classes of orders, which pose additional burden on the European market clearing process, as the block orders [4], [5], or the minimum income condition orders [29], [30]. A block order is a single order that spans over multiple hours, and it must be either fully cleared or fully rejected [31]. An order with the minimum income condition is a single order that usually spans over the whole day, and can be cleared only if the producer collects sufficient revenues able to cover at least its fixed and variable costs [20]. Notice that, in both cases, the indivisible nature of these orders, that span over multiple hours, can lead to PROs or PAOs. Ongoing work aims at introducing these orders in our framework in order to mimic closer the European scenario.

TABLE II

RESULTS OF THE MILP MODEL, MAY 4TH

Hour	Split[a]	PUN (True)	PUN (Model)	Time[b]	# Binary
1	N	38.520000	38.520000	1.487	361
2	N	32.650000	32.650000	1.905	340
3	N	32.480000	32.480000	0.898	321
4	N	33.360000	33.360000	1.319	324
5	N	38.510000	38.510000	1.829	316
6	N	36.000000	36.000000	1.572	346
7	N	47.500000	47.500000	1.708	358
8	Y	60.463068	60.463068	1.480	395
9	Y	68.498829	68.498829	1.723	379
10	Y	73.130026	73.130026	2.391	415
11	Y	74.534459	74.534459	2.435	414
12	Y	71.013683	71.013683	1.866	420
13	Y	66.716025	66.716025	1.669	373
14	Y	64.584130	64.584130	2.374	372
15	Y	66.355690	66.355690	3.508	396
16	Y	68.543706	68.543706	2.452	404
17	Y	67.958972	67.958972	2.582	389
18	Y	66.299237	66.299237	2.844	375
19	Y	62.250964	62.250964	4.391	385
20	Y	61.753370	61.753370	1.388	387
21	Y	73.973516	73.973516	1.265	452
22	Y	71.045257	71.045257	3.414	450
23	Y	64.348253	64.348253	3.477	411
24	Y	56.011383	56.011383	2.410	380

[a] N: No market split; Y: market split.

[b] Cplex time, as reported by “resource usage” parameter in GAMS, in seconds.

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Appendix A
The Final UPP Model

$$\max_{\substack{u_k^g, u_k^e, \pi, d_k, s_p, F_{ij}, \\ \zeta_i, \varphi_k^d, \varphi_p^s, \delta_{ij}^{max}, \eta_{ij}, \\ y_k^{g\pi}, y_{ki}^{g\zeta}, y_k^{ed}, y_k^{e\pi}, y_k^{e\varphi}, y_{ji}^{\Delta b}, b_{ji}.}} \quad \sum_{k \in \mathcal{K}^\pi} u_k^g p_k^d d_k^{max} + \sum_{k \in \mathcal{K}^\pi} y_k^{ed} p_k^d \tag{34}$$

s.t.

$$\sum_{k \in \mathcal{K}^\pi} y_k^{g\pi} d_k^{max} + \sum_{i \in \mathcal{Z}^\pi} \sum_{j=0}^{B-1} \frac{2^j y_{ji}^{\Delta b}}{10^c} = \sum_{i \in \mathcal{Z}^\pi} \sum_{k \in \mathcal{K}_i^\pi} y_{ki}^{g\zeta} d_k^{max} \tag{35}$$

$$u_k^g \left(p_k^d - \varepsilon\right) - y_k^{g\pi} \geq 0 \qquad \forall k \in \mathcal{K}^\pi \tag{36}$$

$$u_k^e p_k^d - y_k^{e\pi} = 0 \qquad \forall k \in \mathcal{K}^\pi \tag{37}$$

$$u_h^g \geq u_k^g \qquad \forall h, k \in \mathcal{K}^\pi : m_h < m_k \tag{38}$$

$$u_h^g \geq u_k^e \qquad \forall h, k \in \mathcal{K}^\pi : p_h^d > p_k^d \tag{39}$$

$$u_k^g \in \{0, 1\}, \; u_k^e \in \{0, 1\} \qquad \forall k \in \mathcal{K}^\pi \tag{40}$$

$$\begin{aligned} &\sum_{k \in \mathcal{K}^\zeta} p_k^d d_k + \sum_{k \in \mathcal{K}^p} p_k^d d_k + \sum_{k \in \mathcal{K}^\pi} y_k^{ed} p_k^d - \sum_{p \in \mathcal{P}} p_p^s s_p = \\ &\sum_{k \in \mathcal{K}^\pi} y_k^{e\varphi} d_k^{max} + \sum_{k \in \mathcal{K}^p \cup \mathcal{K}^\zeta} \varphi_k^d d_k^{max} + \sum_{p \in \mathcal{P}} \varphi_p^s s_p^{max} \\ &+ \sum_{i \in \mathcal{Z}} \sum_{j \in \mathcal{Z}} \delta_{ij}^{max} F_{ij}^{max} - \sum_{i \in \mathcal{Z}^\pi} \sum_{k \in \mathcal{K}_i^\pi} y_{ki}^{g\zeta} d_k^{max} \end{aligned} \tag{41}$$

$$\varphi_k^d + \zeta_i \geq p_k^d \qquad \forall\, k \in \mathcal{K}_i, \forall i \in \mathcal{Z} \tag{42}$$

$$\varphi_p^s - \zeta_i \geq -p_p^s \qquad \forall\, p \in \mathcal{P}_i, \forall i \in \mathcal{Z} \tag{43}$$

$$\delta_{ij}^{max} + \eta_{ij} + \eta_{ji} + \zeta_i = 0 \qquad \forall\, i, j \in \mathcal{Z} \tag{44}$$

$$d_k \leq d_k^{max} \qquad \forall\, k \in \mathcal{K} \tag{45}$$

$$s_p \leq s_p^{max} \qquad \forall\, p \in \mathcal{P} \tag{46}$$

$$F_{ij} \leq F_{ij}^{max} \qquad \forall\, i, j \in \mathcal{Z} \tag{47}$$

$$F_{ij} + F_{ji} = 0 \qquad \forall\, i, j \in \mathcal{Z} \tag{48}$$

$$\begin{aligned} &\sum_{k \in \mathcal{K}_i^p} d_k + \sum_{k \in \mathcal{K}_i^\pi} y_k^{ed} - \sum_{p \in \mathcal{P}_i} s_p + \sum_{j \in \mathcal{Z}} F_{ij} = \\ &- \sum_{k \in \mathcal{K}_i^\pi} u_k^g d_k^{max} \qquad \forall i \in \mathcal{Z}^\pi \end{aligned} \tag{49}$$

$$\sum_{k\in\mathcal{K}_i^\zeta} d_k - \sum_{p\in\mathcal{P}_i} s_p + \sum_{j\in\mathcal{Z}} F_{ij} = 0 \qquad \forall i \in \mathcal{Z}^\zeta \tag{50}$$

$$0 \le y_k^{ed} \le d_k^{max} u_k^e \qquad \forall k \in \mathcal{K}^\pi \tag{51}$$

$$0 \le d_k - y_k^{ed} \le d_k^{max}(1 - u_k^e) \qquad \forall k \in \mathcal{K}^\pi \tag{52}$$

$$10^c \sum_{k\in\mathcal{K}_i^\pi} y_k^{ed} = \sum_{j=0}^{B-1} 2^j b_{ji} \qquad \forall i \in \mathcal{Z}^\pi \tag{53}$$

$$b_{ji} \in \{0, 1\} \qquad \forall j \in \mathcal{H}, \forall i \in \mathcal{Z}^\pi \tag{54}$$

$$M_1 b_{ji} \le y_{ji}^{\Delta b} \le M_2 b_{ji} \qquad \forall j \in \mathcal{H}, \forall i \in \mathcal{Z}^\pi \tag{55}$$

$$M_1(1 - b_{ji}) \le (\pi - \zeta_i) - y_{ji}^{\Delta b} \qquad \forall j \in \mathcal{H}, \forall i \in \mathcal{Z}^\pi \tag{56}$$

$$(\pi - \zeta_i) - y_{ji}^{\Delta b} \le M_2(1 - b_{ji}) \qquad \forall j \in \mathcal{H}, \forall i \in \mathcal{Z}^\pi \tag{57}$$

$$M_1 u_k^g \le y_k^{g\pi} \le M_2 u_k^g \qquad \forall k \in \mathcal{K}^\pi \tag{58}$$

$$M_1(1 - u_k^g) \le \pi - y_k^{g\pi} \le M_2(1 - u_k^g) \quad \forall k \in \mathcal{K}^\pi \tag{59}$$

$$M_1 u_k^g \le y_{ki}^{g\zeta} \le M_2 u_k^g \qquad \forall k \in \mathcal{K}_i^\pi \, \forall i \in \mathcal{Z}^\pi \tag{60}$$

$$M_1(1 - u_k^g) \le \zeta_i - y_{ki}^{g\zeta} \le M_2(1 - u_k^g) \quad \forall k \in \mathcal{K}_i^\pi \, \forall i \in \mathcal{Z}^\pi \tag{61}$$

$$M_1 u_k^e \le y_k^{e\pi} \le M_2 u_k^e \qquad \forall k \in \mathcal{K}^\pi \tag{62}$$

$$M_1(1 - u_k^e) \le \pi - y_k^{e\pi} \le M_2(1 - u_k^e) \quad \forall k \in \mathcal{K}^\pi \tag{63}$$

$$M_1 u_k^e \le y_k^{e\varphi} \le M_2 u_k^e \qquad \forall k \in \mathcal{K}^\pi \tag{64}$$

$$M_1(1 - u_k^e) \le \varphi_k^d - y_k^{e\varphi} \le M_2(1 - u_k^e) \; \forall k \in \mathcal{K}^\pi \tag{65}$$

$$d_k = u_k^g d_k^{max} + y_k^{ed} \qquad \forall k \in \mathcal{K}^\pi, \tag{66}$$

where: $\mathcal{H} = \{0, \ldots, B-1\}$, ε is an arbitrarily small positive constant and M_1, M_2 appropriate bound values. For ease of reading, the constraint (66) is added to recap the executed quantities into single variables.

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