An Exact Solution to the Market Clearing Problem with Uniform Purchase Price

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***Abstract*—The electricity market clearing process can be affected to varying degrees by norms set by regulators. One possible rule is the Uniform Purchase Price, which is implemented, for example, in the Italian day-ahead market with the name of *Prezzo Unico Nazionale*, which literally means unique national price. This rule requires that all the consumers pay the same price in all the market zones. On the contrary, producers receive the zonal prices, which may differ from one zone to another. As a consequence, traditional market clearing techniques cannot be employed because of this difference in the paid and received price, and current state-of-the-art methods still rely on heuristic search procedures. Starting from a non-linear mixed integer bilevel formulation of the clearing process in presence of uniform purchase price, this paper shows how to obtain a mixed integer linear programming model, which is computationally tractable and able to solve exactly the uniform purchase price problem. Numerical results are reported by testing the algorithm using real data from the Italian day-ahead market.**

***Index Terms*—Bilevel programming, Italian electricity market, market clearing, marginal pricing, uniform purchase price.**

NOMENCLATURE

A. Sets and Indices

$\mathcal{Z}^\pi$	Set of UPP zones.
$\mathcal{Z}^\zeta$	Set of non-UPP zones.
$\mathcal{Z}$	Set of all zones, $\mathcal{Z} = \mathcal{Z}^\pi \cup \mathcal{Z}^\zeta$.
i	Market zone index, $i \in \mathcal{Z}$.
$\mathcal{K}_i^\pi$	Set of UPP consumers in zone $i \in \mathcal{Z}^\pi$, $\mathcal{K}^\pi = \cup_i \mathcal{K}_i^\pi$.
$\mathcal{K}_i^p$	Set of non-UPP consumers in zone $i \in \mathcal{Z}^\pi$, $\mathcal{K}^p = \cup_i \mathcal{K}_i^p$.
$\mathcal{K}_i^\zeta$	Set of consumers in zone $i \in \mathcal{Z}^\zeta$, $\mathcal{K}^\zeta = \cup_i \mathcal{K}_i^\zeta$.
$\mathcal{K}_i$	Set of all consumers in zone $i \in \mathcal{Z}$, $\mathcal{K} = \cup_i \mathcal{K}_i$.
$\mathcal{P}_i$	Set of all producers in zone $i \in \mathcal{Z}$, $\mathcal{P} = \cup_i \mathcal{P}_i$.

B. Constants

p_k^d	Price submitted by consumer k in €/MWh, which represents the maximum price he/she is willing to pay in order to buy the quantity d_k.
p_p^s	Price submitted by producer p in €/MWh, which represents the minimum price he/she is willing to receive in order to sell the quantity s_p.
d_k^{max}	Maximum quantity demanded by consumer k, in MWh.
m_k	Economic merit order for consumer k, lower values mean higher priority. If $p_h^d > p_k^d$ then $m_h < m_k$, with $k, h \in \mathcal{Z}^\pi$. If $p_h^d = p_k^d$ the merit order is assigned by the market operator.
B	Number of bits used in the binary conversion.
c	Number of significant digits in the binary conversion.
ε	Arbitrarily small positive parameter.

C. Variables

D_i^π	Total demand quantity executed in zone $i \in \mathcal{Z}^\pi$ and pertaining to consumers $k \in \mathcal{K}_i^\pi$, in MWh.
d_k	Allocated/executed demand quantity for consumer k, in MWh.
s_p	Allocated/executed supply quantity for producer p, in MWh.
π	Uniform purchase price, in €/MWh.
ζ_i	Zonal price in zone i, in €/MWh.
u_k^g	Binary variable, if $u_k^g = 1$ then $p_k^d > \pi$.
u_k^e	Binary variable, if $u_k^e = 1$ then $p_k^d = \pi$.
b_{ji}	Binary variable, used to convert an integer in binary form.
F_{ij}	Flow from zone i to j, in MWh.

I. INTRODUCTION

The *Uniform Purchase Price* (UPP) is a market rule that affects deeply the market clearing process. In a multi-zone market, this rule requires that all the consumers (with few exeptions) pay the same price in all the market zones. On the contrary, the producers receive the zonal prices, which may differ from one zone to another. The Italian day-ahead market [1] is based on the UPP pricing method and the unique price paid is termed *Prezzo Unico Nazionale* (PUN), i.e. unique national price.

The UPP π is defined implicitly by the following condition:

$$\pi \sum_{i \in \mathcal{Z}^\pi} D_i^\pi = \sum_{i \in \mathcal{Z}^\pi} D_i^\pi \zeta_i \,, \tag{1}$$

where D_i^π is the total demand quantity executed in zone $i \in \mathcal{Z}^\pi$ and pertaining to consumers $k \in \mathcal{K}_i^\pi$, and ζ_i is the zonal price in zone i. Therefore, the UPP π is the common value paid by all the consumers such that the *total monetary amount* collected by the market operator (MO) is the same as if the consumers paid the zonal prices ζ_i. This allows to collect exactly the amount needed to pay both the transmission system operator and the producers [2]. Notice that, however, all consumers are forced to pay a common price, hence, the market clearing *solution* is not the same as if the consumers had actually to pay the zonal prices.

In this work, a *producer* is any unit who submits a sell or *offer order*. By contrast, a *consumer* is any operator who submits a buy or *bid order*, and the consumers $k \in \mathcal{K}^\pi$ paying the UPP are termed UPP consumers. Any bid order belonging to the UPP zones $\mathcal{Z}^\pi$, i.e. the zones where the UPP pricing method is implemented, is classified as follows:

1) *in-the-money*, if the submitted price p_k^d is *strictly greater* than the UPP;
2) *at-the-money*, if the submitted price p_k^d is exactly *equal* to the UPP;
3) *out-of-the-money*, if the submitted price p_k^d is *strictly lower* than the UPP.

Given the above definitions, we can state the *UPP rule* as follows:

1) any in-the-money bid order must be fully accepted, i.e. $d_k = d_k^{max}$;
2) any at-the-money bid order can be partially executed, i.e. $0 \leq d_k \leq d_k^{max}$;
3) any out-of-the-money bid order must be fully rejected, i.e. $d_k = 0$.

The UPP pricing scheme allows exceptions to the UPP rule, as for example the pumping units of hydroelectric production plants in the Italian day-ahead market. These operators belong to the UPP zones but pay the zonal prices and not the UPP. In our model, these consumers are represented by the set $\mathcal{K}^p$.

Another peculiarity of the UPP pricing method is that the market orders are ranked according to a value assigned by the MO and called *merit order*. The MO uses these values to rank the orders by price and for those at the same price, the ranking is established by using a set of given conditions, as for example the time-stamp of submission. The priority established by the merit order has to be respected, unless it conflicts with the problem constraints.

Traditionally, the clearing problem for day-ahead markets is solved by using a *social welfare* maximization [3], [4], defined as follows:

$$\max_{d_k, s_p} \sum_{k \in \mathcal{K}} p_k^d d_k - \sum_{p \in \mathcal{P}} p_p^s s_p \,, \tag{2}$$

which is desirable because the solution is both Pareto-optimal [5] and economic efficient, i.e. it maximizes both the consumer and producer surplus [6] (Fig. 1). Despite this, the approach in (2) cannot be used with the UPP pricing scheme, because it implies that both the consumers and the producers have to pay and receive the same price, i.e. the zonal price [3]. By contrast, the UPP problem allows for different prices within the same zone. This means that it is necessary to develop a specific method to solve the UPP market clearing problem.

Indeed, the current state-of-the-art methods for solving the UPP problem (as the Italian PUN) are based mostly on

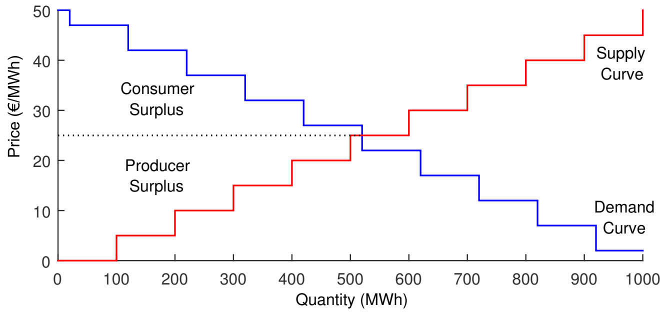

Fig. 1. In blue the demand curve. In red the supply curve. The intersection gives the market price.

heuristic methods. In [2] the UPP is fixed iteratively according to any possible price of the demand curve. Then, for each fixed UPP the bid orders are accepted or rejected by applying the UPP rule, and a social welfare problem is employed to compute the remaining variables. Finally, the optimum is chosen as the one which attains the greatest social welfare.

In Europe, the algorithm used for market clearing, named EUPHEMIA, has to face a UPP problem since February 2015, when the Italian day-ahead market was coupled with the European market. The method adopted is still based on a heuristic approach [4]. In this case the whole aggregate market demand curve is iteratively analyzed until a solution is found that satisfies (1). The European market clearing problem has to face all the possible types of European orders (e.g. block, complex, flexible orders [4]), and the iterative technique used for handling the Italian PUN stressed even further the whole process.

To summarize, the fundamental issues when facing the UPP clearing problem can be outlined as follows:

1) the traditional social welfare maximization method cannot be used to solve the UPP problem, because it implies a common price for both bid and offer orders;
2) the method employed should be heuristic-free and tractable, in order to avoid computational issues.

The main contribution of the approach proposed in this paper is to formulate a mixed integer linear program (MILP) able to address the above two issues. That is, the UPP solution obtained from our MILP model is heuristic-free, tractable and exact (at least up to the level of resolution of the current market data). The proposed model is based on the marginal pricing framework [3], [7], i.e. the zonal price is defined as the dual variable of the relevant power balance constraint, and our approach ensures that these variables correctly represent the zonal prices, overcoming the specific problem posed by the possible difference in the price paid and received within the same zone. The MILP model is developed starting from a non-linear bilevel problem with both continuous and binary variables. Then, the bilevel model is recast as an equivalent single level problem exploiting the linear form of its lower level part and the strong duality property. This allows us to gain access to the dual variables of the power balance con-

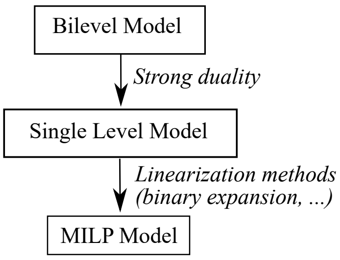

Fig. 2. Derivation of the optimization model for UPP market clearing. Step 1: definition of the initial bilevel model. Step 2: reformulation as an equivalent single level problem, exploiting the linear form of its lower level part and the strong duality property. Step 3: linearization to obtain an equivalent MILP model by using a binary expansion and other techniques.

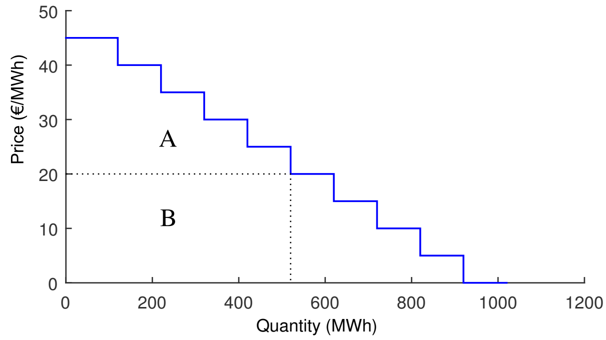

Fig. 3. Aggregate demand curve, $\pi = 20$ €/MWh. The part A is the (net) consumer surplus. The amount actually paid by consumers is the rectangular part B. The area A+B is the gross consumer surplus.

straints, i.e. the zonal prices. Finally, appropriate linearizations are introduced, leading to an equivalent MILP model, Fig. 2.

This paper is organized as follows. Section II describes how the MILP is derived. In Section III we compare both the real PUN and the Italian zonal prices with the results obtained by using our model. Section IV reports conclusions and ongoing work.

II. The Model

The proposed model is built starting from a bilevel optimization problem. In general, a bilevel optimization problem is structured as follows [8]:

$$\max_{x \in \mathcal{X}} \; F(x, y^*) \tag{3}$$

$$\text{s.t.} \quad y^* = \max_{y \in \mathcal{Y}} \; f(x, y)\,, \tag{4}$$

where (3) is the upper optimization problem and (4) is the lower optimization problem, F, f are their respective objective functions, and $\mathcal{X}$, $\mathcal{Y}$ are constraint sets. A fundamental characteristic of a bilevel model is that the decision variables x of the upper level problem are fixed parameters inside the lower level problem. In the proposed method, the upper level is used to compute the UPP and to manage all the integer variables. By contrast, the lower level is used to compute the zonal prices.

The upper optimization problem is specified as follows:

$$\max_{u_k^g, u_k^e, \pi} \sum_{k \in \mathcal{K}^\pi} u_k^g p_k^d d_k^{max} + \sum_{k \in \mathcal{K}^\pi} u_k^e p_k^d {d_k}^* \tag{5}$$

$$\text{s.t.} \quad \pi \sum_{i \in \mathcal{Z}^\pi} \Big(\sum_{k \in \mathcal{K}_i^\pi} u_k^g d_k^{max} + \sum_{k \in \mathcal{K}_i^\pi} u_k^e {d_k}^* \Big) =$$

$$\sum_{i \in \mathcal{Z}^\pi} \zeta_i^* \Big(\sum_{k \in \mathcal{K}_i^\pi} u_k^g d_k^{max} + \sum_{k \in \mathcal{K}_i^\pi} u_k^e {d_k}^* \Big) \tag{6}$$

$$u_k^g \left(p_k^d - \pi - \varepsilon \right) \geq 0 \qquad \forall k \in \mathcal{K}^\pi \tag{7}$$

$$u_k^e \left(p_k^d - \pi \right) = 0 \qquad \forall k \in \mathcal{K}^\pi \tag{8}$$

$$u_h^g \geq u_k^g \qquad \forall h, k \in \mathcal{K}^\pi : m_h < m_k \tag{9}$$

$$u_h^g \geq u_k^e \qquad \forall h, k \in \mathcal{K}^\pi : p_h^d > p_k^d \tag{10}$$

$$u_k^g \in \{0, 1\}\,,\; u_k^e \in \{0, 1\} \qquad \forall k \in \mathcal{K}^\pi, \tag{11}$$

where ${d_k}^*$ and ${\zeta_i}^*$ are solutions of the lower level problem, and ε is an arbitrarily small positive parameter.

Assuming piecewise-constant market curves, the aggregate demand curve can be graphically represented as in Fig. 3. The key property is that any demand curve is always downward-sloping (excluding the case of degenerate goods). This feature is actively exploited by the objective function (5). Indeed, (5) maximizes the area under the aggregate demand curve, labeled by both A and B in Fig. 3, where A is the aggregate consumer surplus. Relying on the downward-shape of the market demand curve, the maximization of the area under the curve implies the maximization of the consumer surplus, which is the objective of any rational consumers. The condition (7) ensures that the variable u_k^g can be equal to one only if the price p_k^d is *strictly greater* than the UPP. Similarly, (8) requires that u_k^e can be equal to one only if the price p_k^d is exactly *equal* to the UPP. This means that u_k^g and u_k^e cannot be both equal to one. Equation (6) is the UPP definition (1) reformulated by using both the UPP rule and the binary variables u_k^g and u_k^e, where we use the following identity:

$$D_i^\pi = \sum_{k \in \mathcal{K}_i^\pi} u_k^g d_k^{max} + \sum_{k \in \mathcal{K}_i^\pi} u_k^e d_k \,. \tag{12}$$

The conditions (9)-(10) ensure the consecutive execution of the bid orders, with a significant reduction in the search space of the binary variables and a substantial improvement in the computation time.

The objective function of the lower level problem is defined as follows :

$$\left(d_k^*, s_p^*, F_{ij}^*, [\zeta_i^*]\right) = \arg \max_{d_k, s_p, F_{ij}} \sum_{k \in \mathcal{K}^\varsigma} p_k^d d_k + \sum_{k \in \mathcal{K}^p} p_k^d d_k$$

$$+ \sum_{k \in \mathcal{K}^\pi} u_k^e p_k^d d_k - \sum_{p \in \mathcal{P}} p_p^s s_p \;. \tag{13}$$

The objective function (13) acts as a social welfare maximization for all the operators present in the market with the exception of both the in-the-money and the out-of-the-money bid orders for the UPP consumers $\mathcal{K}^\pi$ within the UPP zones $\mathcal{Z}^\pi$. In the first case, the demand quantities have to be fully executed, i.e. $d_k = d_k^{max}$, and dispatched. In the second case, the orders must be fully rejected, i.e. $d_k = 0$. By contrast, if $u_k^e = 1$, with $k \in \mathcal{K}^\pi$, then the bid orders can be partially executed, i.e. $0 \leq d_k \leq d_k^{max}$, and the volumes are determined by the lower level problem.

The power balance constraints in the lower level problem are defined as follows:

$$\sum_{k\in\mathcal{K}_i^p} d_k + \sum_{k\in\mathcal{K}_i^\pi} u_k^e d_k - \sum_{p\in\mathcal{P}_i} s_p + \sum_{j\in\mathcal{Z}} F_{ij} = - \sum_{k\in\mathcal{K}_i^\pi} u_k^g d_k^{max} \qquad [\zeta_i \in \mathbb{R}] \; \forall i \in \mathcal{Z}^\pi \tag{14}$$

$$\sum_{k\in\mathcal{K}_i^\zeta} d_k - \sum_{p\in\mathcal{P}_i} s_p + \sum_{j\in\mathcal{Z}} F_{ij} = 0 \qquad [\zeta_i \in \mathbb{R}] \; \forall i \in \mathcal{Z}^\zeta \,, \tag{15}$$

where (14) is the power balance for a UPP zone, and (15) is the power balance for a non-UPP zone. Notice that the dual variables, i.e. the zonal prices ζ_i, are represented in square brackets. The fundamental difference in these two constraints is that (14) has to manage separately the in-the-money and the at-the-money bid orders. That is, all the in-the-money bid orders have to be fully dispatched, i.e. $d_k = d_k^{max}$, while the at-the-money bid orders can be partially executed.

Notice that the bilevel approach precisely reflects the consequence of the UPP rule, which requires to fully dispatch all the in-the-money bid orders, and implies the full knowledge of the zonal prices to compute properly the UPP.

Notice also that the lower level is a linear model because the binary variables u_k^g and u_k^e are fixed parameters inside the lower level. Additional linear constraints can be added to describe the network topology and to enforce bounds on the lower level decision variables.

To compute the UPP π in (6) it is necessary to gain access to the dual variables ζ_i. For this reason, the bilevel model has to be recast as a single level problem. In the single level problem, the objective function is that of the upper level problem, and the constraints are the same of the upper level with the addition of a set of necessary and sufficient conditions to embed the lower level into the single level problem, represented by:

1) the constraints of the lower level (the primal problem);
2) the constraints of its dual problem;
3) the strong duality property, i.e. the requirement that the values of the objective functions of both the primal and the dual problem have to be equal.

Notice that this approach relies on the linearity of the lower level, the interested reader is referred to [9, Sec. III], [10, Ch. 6]. In the single level problem there is no difference between upper and lower level variables, and all these variables become decision variables. Notice also that in (13), the variables d_k with $k \in \mathcal{K}^\pi$ are meaningful, i.e. they represent the executed volumes for the UPP consumers, only if $u_k^e = 1$ (at-the-money bid orders). When $u_k^e = 0$, the executed quantity is d_k^{max} if $u_k^g = 1$ (in-the-money bid orders), and zero otherwise, as exploited in (12).

The single level model is a non-linear mixed integer problem. In particular, there are two types of non-linearities:

1) the product between a binary and a continuous variable, as in (5)-(8), (13)-(14);
2) the product of two continuous variables with a binary variable in (6), i.e., after rearranging:

$$\sum_{i\in\mathcal{Z}^\pi} \left((\pi - \zeta_i) \sum_{k\in\mathcal{K}_i^\pi} u_k^e d_k \right) . \tag{16}$$

The non-linearity of the first type, i.e. the product of a binary with a continuous variable, can be removed by introducing an auxiliary continuous variable, as is standard practice (see [11, Ch. 2.8]).

The critical part is to linearize (16) without introducing any approximation. Firstly, notice that the term $\sum_{k\in\mathcal{K}_i^\pi} u_k^e d_k$ is most of the times zero. Indeed, it can differ from zero only when $p_k^d = \pi$ (we exploit strongly this property). Then, the crucial step is to implement a binary expansion as in [12]. That is, to convert a positive integer number in binary form using binary variables. Usually, this method is computationally expensive because it requires a large number of additional binary variables. However, we rely on the key feature that most of the times the term under conversion is simply zero.

The binary conversion is implemented as follows:

$$10^c \sum_{k\in\mathcal{K}_i^\pi} u_k^e d_k = \sum_{j=0}^{B-1} 2^j b_{ji} \quad \forall i \in \mathcal{Z}^\pi \tag{17}$$

$$b_{ji} \in \{0,1\} \qquad \forall j \in \mathcal{H}, \forall i \in \mathcal{Z}^\pi \,, \tag{18}$$

where c is the number of significant digits in d_k, $B = \lfloor \log_2(U) \rfloor + 1$, U is the upper bound of the left-hand side (LHS) in (17), $\lfloor \cdot \rfloor$ is the floor operator, and $\mathcal{H} = \{0, \dots, B-1\}$. Notice that the upper bound U can be computed in advance from the market data. In (17), the term on the LHS is multiplied by 10^c in order to obtain an integer number. This is a fundamental step because it leads to an exact discretization. Hence, (16) is replaced by:

$$\sum_{i\in\mathcal{Z}^\pi} \sum_{j=0}^{B-1} \frac{2^j (\pi - \zeta_i) b_{ji}}{10^c} . \tag{19}$$

After removing the remaining non-linearities, i.e. the products between binary and continuous variables, the final model results in a MILP.

III. Numerical Results

In this section, we compare actual Italian market data [1] with the results obtained from our model. We recall that the Italian day-ahead market implements the UPP pricing method, and the unique price paid by all the consumers is termed PUN. The market data refer to May 5th, 2010. Notice that the PUN is an hourly price, this means that there are up to 24 different PUN each day. Fig. 4 shows the actual zonal prices and the Italian PUN on the 8th hour. On average, for the considered day, each hourly problem involves, 483 bid orders, 1268 offer orders and 578 binary variables. The average time employed to find the hourly solution is 3.036 seconds. In Fig. 5 are reported the Cplex time needed to find the hourly solution and the number of binary variables actually used in the problem. The MILP model was run for each of the 24 hours and the

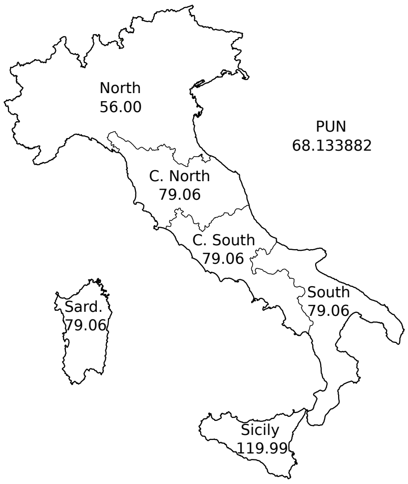

Fig. 4. Real PUN and zonal prices in the Italian day-ahead market on the 8th hour of May 5th, 2010. The Italian UPP zones are: North, Centre-North, Centre-South, South, Sicily and Sardinia.

TABLE I
MAY 5TH, 2010

Hour	PUN (Real)	PUN (Model)	# Bid[a]	# Offer[b]
1	49.200000	49.200000	483	1194
2	44.333458	44.333458	492	1176
3	47.696949	47.696949	491	1174
4	44.391946	44.391946	490	1173
5	44.233295	44.233295	485	1178
6	36.529486	36.529486	489	1185
7	47.692413	47.692413	488	1200
8	68.133882	68.133882	481	1258
9	87.487872	87.487872	482	1297
10	88.087935	88.087935	482	1318
11	95.113566	95.113566	481	1327
12	94.086209	94.086209	480	1312
13	76.133796	76.133796	481	1307
14	72.139688	72.139688	481	1308
15	76.122751	76.122751	481	1311
16	82.873750	82.873750	480	1306
17	77.270914	77.270914	481	1315
18	74.044218	74.044218	485	1312
19	59.569805	59.569805	481	1301
20	59.789623	59.789623	481	1307
21	82.141510	82.141510	482	1322
22	77.172780	77.172780	483	1322
23	61.561795	61.561795	480	1293
24	55.877169	55.877169	475	1251

[a] Number of bid orders involved.
[b] Number of offer orders involved.

results compared with the actual data. The only assumption made is that $\pi < 3000$ €/MWh. Notice that 3000 €/MWh is the maximum price allowed in the Italian day-ahead market. This assumption is used to set both $u_k^e = 0$ and $u_k^g = 1$ if $p_k^d = 3000$. The model was implemented in GAMS 24.7.3 [13] and solved with Cplex 12.6.3.0 [14] on the NEOS-7 server [15]. In Cplex, the relative gap was set to zero, i.e. the results are proven optimal solutions. The values of the parameters in

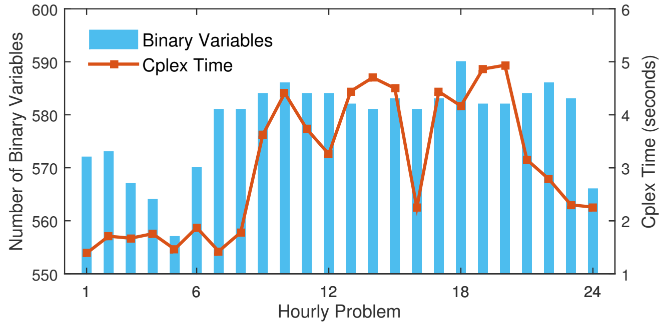

Fig. 5. Number of binary variables actually used in each hourly problem (bar chart, left scale), and Cplex time needed to find the solution in seconds (line chart, right scale)

(7) and (17) are $\varepsilon{=}10^{-6}$, $c{=}3$, $B{=}24$.

For each of the 24 hours involved, the PUN, the real zonal prices and the volumes executed at each submitted price, exactly match the results of our model. Table I reports for each hour the numerical values for the Italian PUN, the PUN obtained from our model, the number of bid orders and the number of offer orders involved.

The hours from the 1st to the 7th, and the 24th include bid orders belonging to pumping units. We recall that these orders are excluded from the UPP rule in the Italian market. The proposed model correctly processes these orders, yielding the correct PUN value (see the corresponding rows in Table I).

IV. CONCLUSION

The Italian PUN regulation of the internal electricity market has generated a considerable increase of the computational complexity of the European market clearing process. In fact, the European algorithm for market coupling (EUPHEMIA) exploits a heuristic iterative technique to solve the Italian market subproblem. The proposed approach shows how to obtain a MILP model computationally tractable and able to solve the UPP problem without introducing any approximation, at least as long as the model parameters are properly tailored to fit the market data. The proposed MILP model can be solved by using standard solvers. Ongoing research activities focus on introducing block and complex orders in our framework in order to solve the European market clearing problem as a whole.

REFERENCES

[1] *Gestore dei Mercati Energetici S.p.A.* Accessed: 2017-05-03. [Online]. Available: www.mercatoelettrico.org/en/Esiti/MGP/EsitiMGP.aspx

[2] Tabors Caramanis and Associates Inc., *UPPO Auction Module User Manual*, 2002. [Online]. Available: www.mercatoelettrico.org/en/MenuBiblioteca/Documenti/20041206UniformPurchase.pdf

[3] F. Schweppe, M. Caraminis, R. Tabors, and R. Bohn, *Spot pricing of electricity*. Kluwer Academic Publishers, Norwell, MA, 1988.

[4] PCR PXs, *EUPHEMIA Public Description PCR Market Coupling Algorithm*, Version 1.3. [Online]. Available: www.mercatoelettrico.org/en/MenuBiblioteca/Documenti/20160127EuphemiaPublicDescription.pdf

[5] A. Mas-Colell, M. D. Whinston, J. R. Green *et al.*, *Microeconomic theory*. Oxford University Press New York, 1995, vol. 1.

[6] D. Rubinfeld and R. Pindyck, *Microeconomics*. Pearson, 2013.
[7] D. S. Kirschen and G. Strbac, *Fundamentals of power system economics*. John Wiley & Sons, 2004.
[8] J. Bard, *Practical bilevel optimization: applications and algorithms*. Kluwer Academic Press Dordrecht, Netherlands, 1998.
[9] R. Fernández-Blanco, J. M. Arroyo, and N. Alguacil, "A unified bilevel programming framework for price-based market clearing under marginal pricing," *IEEE Trans. Power Syst.*, vol. 27, no. 1, pp. 517–525, 2012.
[10] S. A. Gabriel, A. J. Conejo, J. D. Fuller, B. F. Hobbs, and C. Ruiz, *Complementarity modeling in energy markets*. Springer Science & Business Media, 2012, vol. 180.
[11] FICO™ Xpress Optimization Suite, *MIP formulations and linearizations - Quick reference*, 2009.
[12] M. V. Pereira, S. Granville, M. H. Fampa, R. Dix, and L. A. Barroso, "Strategic bidding under uncertainty: a binary expansion approach," *IEEE Trans. Power Syst.*, vol. 20, no. 1, pp. 180–188, 2005.
[13] B. A. McCarl, A. Meeraus, P. van der Eijk, M. Bussieck, S. Dirkse, P. Steacy, and F. Nelissen, *McCarl GAMS user guide*, 2016.
[14] IBM-ILOG, *CPLEX User's Manual - Version 12 Release 7*, 2016.
[15] J. Czyzyk, M. P. Mesnier, and J. J. Moré, "The NEOS server," *IEEE Comput. Sci. Eng.*, vol. 5, no. 3, pp. 68–75, 1998.