Exploiting weather forecasts for sizing photovoltaic energy bids

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***Abstract*—In this work, we study the problem of optimizing energy bids for a photovoltaic (PV) power producer taking part into a competitive electricity market characterized by financial penalties for generation shortfall and surplus. The optimal bidding strategy depends on the statistics of the PV power generation and on the monetary penalties applied. We show how to tune the bidding strategy on the basis of the weather forecasts. To this purpose, an optimization procedure is devised to mitigate the risk associated with the intermittent nature of PV generation and maximize the expected profit of the producer. We also investigate an approach to properly take into account the seasonal variation and non stationary nature of PV power generation statistics, by exploiting the knowledge of the amount of energy that the plant can generate under clear-sky conditions. The proposed bidding strategy is validated on a real data set from an Italian PV plant.**

***Index Terms*—Energy market, bidding strategy, photovoltaic power generation, weather forecasts.**

I. Introduction

Energy generation from renewable energy sources (RES) is one of the main targets for the development of the grid of the future. Due to their intrinsic intermittent nature, integration of RES in the grid causes serious problems to transmission and distribution system operators, asking for procurement of large quantities of reserve power. One possible way to mitigate the uncertainty of RES generation is to require that producers provide day-ahead generation profiles, and to apply penalties if the actual generation profile differs from the schedule. On the producers' side, this calls for the development of suitable bidding strategies to offer the right amount of energy and to avoid penalties.

In this paper, we address the optimal bidding problem for a photovoltaic (PV) power producer. Starting from a stochastic model for PV power generation and a model for the electricity market with financial penalties, we formulate and solve the problem of finding the optimal bids maximizing the expected profit of the producer. As for the case of wind power producers (see, e.g., [1] and references therein), the optimal bidding strategy turns out to depend on the PV power generation statistics and the relative weight of imbalance penalties. Specifically, the optimal offer at a certain time of the day is a suitable percentile of the PV power probability distribution at the same time.

A peculiarity of the solar source is that power generation is characterized by significant seasonal variations and its statistics exhibits a non stationary behaviour. For instance, the number of daylight hours, as well as the intensity of the solar radiation and the average air temperature, change remarkably over the year. As a consequence, PV power generation cannot be modelled as a stationary stochastic process, its probability distribution changing from day to day. Such a phenomenon has a prominent role in PV power generation and may negatively affect the optimal bidding strategy. A probability distribution of PV power generation should be available for each day (or some other coarser partition) of the year. Since such functions are usually estimated from past data, a huge amount of recordings, spanning several years of plant operation, should be collected for reliable probability distribution approximations. To overcome the problem, this paper provides a technique allowing one to modify the aforementioned bidding strategy to properly take into account fluctuations of PV power generation over the year, without having to resort to complex time-varying stochastic models of PV power generation. The proposed solution consists in normalizing the generated power with respect to the power that the plant can generate under clear-sky conditions.

Since the main source of uncertainty on PV generation is related to the weather conditions of the day the bids refer to (typically, the next day), the problem of obtaining accurate forecasts of solar radiation at a given site has been deeply studied. Widely adopted approaches involve neural networks [2] and classical linear models for time series forecasting [3]. A possible bidding strategy is to offer the power predicted from the forecasts of solar radiation and the power curve of the plant. Since this approach does not take into account the pricing scheme for penalizing schedule deviations, it does not handle properly the financial risk of incurring power imbalance. For this reason, we also investigate a different way to exploit the additional information contained in the meteorological forecasts to improve the bidding strategies. Similarly to what is done in [4] for a wind power plant, assuming that day-ahead forecasts of solar radiation and air temperature are available from a meteorological service, we propose a two-stage bidding strategy. Given the meteorological forecasts, each day is classified into one of several predefined classes on the basis of the expected normalized daily energy. Then, the bid profile for the next day is computed according to the class the day is assigned to. The proposed bidding strategies are tested and validated on historical data from a real PV plant.

The paper is structured as follows. In Section II, we formulate the bidding problem and derive the optimal bidding strategy. Section III describes how to include information about generation under clear-sky conditions in the bid process optimization. The use of weather forecasts and the related classification strategy are addressed in Section IV. Section V reports the experimental results obtained under different pricing scenarios using experimental data from a real Italian PV plant. Finally, conclusions are drawn in Section VI.

II. Optimal bidding strategy

In this section the problem of finding the optimal PV energy bids for an electricity market featuring financial penalties for energy imbalance is formulated. The optimal solution is then derived, in terms of the PV power statistics and the imbalance penalties.

Let w_m, $m = 1, \ldots, M$, be a set of random variables representing the average active power generated by a PV power plant over the m-th sampling interval of the day and let C_m denote the corresponding bid of active power for the same interval. Typically the sampling time is one hour, therefore $M = 24$. It is assumed that the PV producer is remunerated with unitary price $p > 0$ for the actual generated energy, whereas penalties are applied whenever the generated power deviates from the bid. In particular, $\bar{q} \geq 0$ and $\bar{\lambda} \geq 0$ are the unitary penalties applied for energy shortfall and surplus, respectively. Throughout the paper the prices p, $\bar{q}$ and $\bar{\lambda}$ are assumed to be constant and known beforehand. Hence, the net hourly profit for the PV producer amounts to

$$J(C_m, w_m) = p w_m - \bar{q} \max\{C_m - w_m, 0\} - \bar{\lambda} \max\{w_m - C_m, 0\}. \quad (1)$$

Since $J(C_m, w_m)$ in (1) is a stochastic quantity due to the uncertainty on the generated power w_m, the optimal bidding problem consists in determining the bid C_m^* maximizing the expected profit $\mathbf{E}[J(C_m, w_m)]$, i.e.

$$C_m^* = \arg\max_{C_m} \mathbf{E}[J(C_m, w_m)], \quad (2)$$

where $\mathbf{E}[\cdot]$ denotes expectation with respect to the generated power statistics.

Let $F_m(\omega)$ denote the cumulative distribution function (*cdf*) of the random variable w_m, i.e. $F_m(\omega) = \Pr(w_m \leq \omega)$. Moreover, let $F_m^{-1}(\nu) = \inf\{\omega : F_m(\omega) \geq \nu\}$, $\nu \in [0, 1]$, be the corresponding quantile function. It turns out that the optimal solution to (2) is as follows [1]:

$$C_m^* = F_m^{-1}\left(\frac{\bar{\lambda}}{\bar{\lambda} + \bar{q}}\right), \qquad m = 1, \ldots, M. \quad (3)$$

Computation of C_m^* is illustrated in Figure 1.

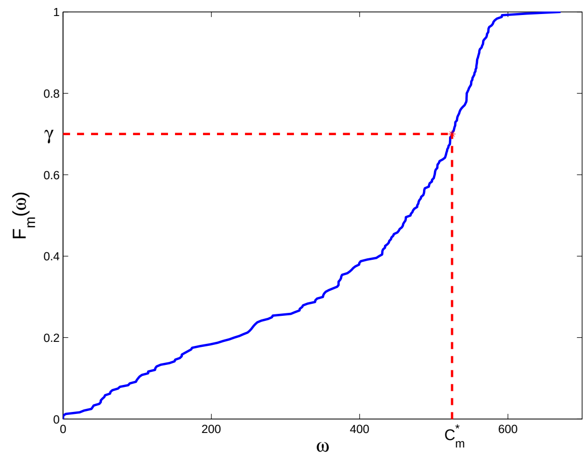

Fig. 1. Example of empirical *cdf* of w_m (solid line). The optimal hourly contract C_m^* is the value of ω such that the *cdf* is equal to the market parameter $\gamma = \bar{\lambda}/(\bar{\lambda} + \bar{q})$.

III. Exploiting clear-sky generation profiles

The effectiveness of the bidding strategy described above might be hindered in practice by the non stationary behaviour of the PV power statistics (see Figure 2). For this reason, we propose to normalize both the generated power and the bid at a given hour with respect to the power generated by the plant under clear-sky conditions.

For a cloudless sky, the solar radiation at ground level takes maximum values and is defined clear-sky solar radiation (I_{cs}). The generation profile of a PV plant hit by clear-sky solar radiation is called clear-sky generation profile (w_{cs}). It can be estimated by using clear-sky solar radiation and the power curve of PV modules. A general form of the power curve of PV modules is provided by the PVUSA method [5], which expresses the generated power as a function of solar radiation and air temperature according to the equation:

$$w = aI + bI^2 + cIT, \quad (4)$$

where P, I and T are respectively the generated power, solar radiation and ambient temperature, and a, b and c are the model parameters (typically $a > 0$, $b < 0$, $c < 0$). Model (4) is linear-in-the-parameters, so that parameter estimation can be carried out very efficiently via classical least-squares methods. Unfortunately, it is quite frequent that measurements of solar radiation and air temperature are not available at the plant site. A heuristic approach to estimate the parameters of the PVUSA model in the partial information case is presented in [6], which relies on historical data of generated power, air temperature forecasts and clear-sky solar radiation obtained through well-known analytical models [7]. Given model (4), the clear-sky generation profile can be computed by substituting in (4) the clear-sky solar radiation I_{cs} and air temperature T:

$$w_{cs} = aI_{cs} + bI_{cs}^2 + cI_{cs}T. \quad (5)$$

Denote $w_{cs,m}$ the average clear-sky PV power over the m-th hour of the day (to simplify notation, we omit the dependence of $w_{cs,m}$ on the day of the year), and let $w_m = \alpha_m w_{cs,m}$. It turns out that $\alpha_m \in [0, 1]$, since $w_{cs,m}$, being obtained under the best irradiation conditions, represents an upper bound on w_m (see Figure 3). Moreover, parameterize the bid C_m as $C_m = \beta_m w_{cs,m}$, $\beta_m \in [0, 1]$. By substituting

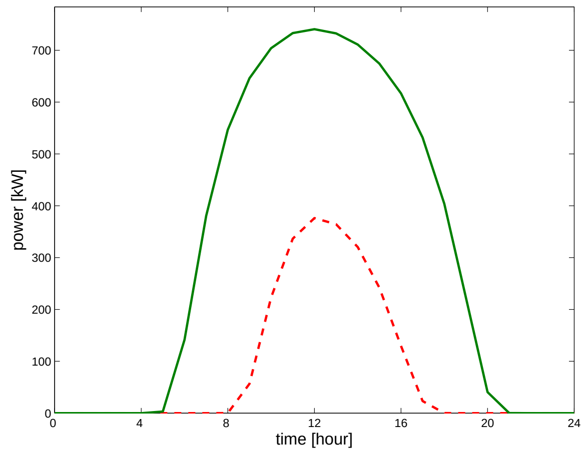

Fig. 2. Example of two different clear-sky generation profiles w_{cs}. The solid line refers to a generic summer day, while the dashed line refers to a generic winter day. Note the differences between the two curves both in the number of daylight hours and peak values.

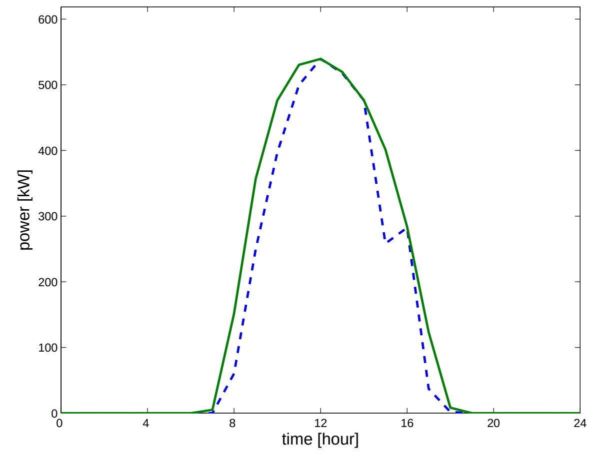

Fig. 3. Clear-sky generation profile w_{cs} (solid) and measured generation profile w (dashed) in a day of October. Note that w never exceeds w_{cs}.

the expressions of w_m and C_m into (1), we obtain that $J(C_m, w_m) = w_{cs,m} J(\beta_m, \alpha_m)$, where

$$J(\beta_m, \alpha_m) = p\alpha_m - \bar{q}\max\{\beta_m - \alpha_m, 0\} - \bar{\lambda}\max\{\alpha_m - \beta_m, 0\}. \quad (6)$$

The considered bidding problem can thus be reformulated as

$$\beta_m^* = \arg\max_{\beta_m} \mathbf{E}[J(\beta_m, \alpha_m)]. \quad (7)$$

Let $F_{cs,m}(\alpha)$ denote the *cdf* of the random variable α_m. Similarly to the previous case the optimal solution to (7) is as follows:

$$\beta_m^* = F_{cs,m}^{-1}\left(\frac{\bar{\lambda}}{\bar{\lambda} + \bar{q}}\right), \qquad m = 1, \ldots, M. \quad (8)$$

The optimal bid is finally computed as

$$C_m^* = \beta_m^* w_{cs,m}, \qquad m = 1, \ldots, M. \quad (9)$$

The main advantage of the proposed bidding strategy is that the seasonal variations of PV power generation are captured by the clear-sky PV power profile $w_{cs,m}$. As a consequence, the resulting normalized power α_m can be regarded as a stationary process, thus limiting the adverse effect of seasonality on the bidding strategy. Note that strategy (9) does not use weather forecasts to compute optimal bids.

IV. Exploiting weather forecasts

In the previous section, contracts were determined assuming to know only the prior PV power statistics, and clear-sky PV power. Since the generated power is highly dependent on the real solar radiation, meteorological forecasts for the day the bids refer to can help guess the generated power, and hence refine and improve the reliability of the amount of bidden energy. In this section, we assume that additional information is available, namely the forecasts of solar radiation $\hat{I}_m$ and air temperature $\hat{T}_m$, $m = 1, \ldots, M$, provided by a meteorological service, and investigate how to exploit these forecasts in the bidding strategy.

A. Bidding power forecasts

Assuming that the power curve of the PV plant is available, the most intuitive approach would be to offer the forecast PV power profile computed by substituting the solar radiation and air temperature forecasts into the equation of the power curve (4):

$$C_m = a\hat{I}_m + b\hat{I}_m^2 + c\hat{I}_m\hat{T}_m, \qquad m = 1, \ldots, M. \quad (10)$$

However, such a naive approach may lead to unsatisfactory performance for the PV producers. First, inaccurate weather forecasts may lead to unacceptable errors when predicting PV power. Second, and more importantly, forecasts of generated power do not take into account the price p and the penalties $\bar{q}$ and $\bar{\lambda}$. This implies that bidding these forecasts may be far from optimal. For instance, consider the limit case $\bar{q} = 0$, i.e. power shortfalls are not penalized. Clearly, under this assumption the optimal strategy is to offer $C_m = \bar{P}$, $m = 1, \ldots, M$, where $\bar{P}$ is an upper bound on the PV power, thus having all the generated power remunerated at price p. Similarly, if $\bar{\lambda} = 0$, i.e. power surplus is not penalized, then the optimal strategy is to offer $C_m = 0$, $m = 1, \ldots, M$.

B. Weather forecast based classification

Motivated by the above discussion, in this paper we propose a different approach to mitigate the effects of inaccurate weather forecasts, while accounting for imbalance penalties. The idea is to combine the optimal bidding strategy described in Section III with a suitable classification strategy based on power forecasts. Roughly speaking, the proposed approach consists in training a classifier which maps a day (represented by the corresponding power forecasts) to one of several possible classes associated to different levels of daily generated energy. Then, the bid made for that day is the optimal contract computed as in equation (9), but using the conditional normalized PV power *cdf* of the corresponding class.

Partition the interval $[0,1)$ into s contiguous, non overlapping intervals $\mathcal{E}_i = [\epsilon_{i-1}, \epsilon_i)$, $i = 1, \dots, s$, such that

$$0 = \epsilon_0 < \epsilon_1 < \cdots < \epsilon_{s-1} < \epsilon_s = 1. \tag{11}$$

The normalized PV energy generated over day d is computed as

$$E^{(d)} = \frac{\sum_{m=1}^{M} w_m^{(d)}}{\sum_{m=1}^{M} w_{cs,m}^{(d)}}, \tag{12}$$

where $w_m^{(d)}$ denotes the average PV power generated over the m-th sampling interval of day d, and $w_{cs,m}^{(d)}$ denotes the corresponding clear-sky PV power. Then, the classification rule is defined as:

$$d \in \mathcal{C}_i \quad \Leftrightarrow \quad E^{(d)} \in \mathcal{E}_i, \quad i = 1, \dots, s, \tag{13}$$

where $\mathcal{C}_i$ represents the i-th class. Clearly, the actual normalized daily energy $E^{(d)}$ can be computed only a posteriori. Hence, since the bids must be made in advance, day d is classified a priori on the basis of the corresponding PV power forecasts $\hat{w}_m^{(d)}$, $m = 1, \dots, M$. To this aim, we train an automatic classifier, which takes as inputs the PV power forecasts and returns the class the day will likely belong to. Training is performed by creating a training set from past data of generated power and forecast power. First, each day d of the training set is assigned to the corresponding true class $\mathcal{C}^{(d)} \in \mathcal{C} = \{\mathcal{C}_1, \dots, \mathcal{C}_s\}$ according to (13). Then, features represented by the predicted normalized PV energy $f^{(d)}$ for day d are computed as

$$f^{(d)} = \frac{\sum_{m=1}^{M} \hat{w}_m^{(d)}}{\sum_{m=1}^{M} w_{cs,m}^{(d)}}, \tag{14}$$

where $\hat{w}_m$ is computed as in equation (10) starting from forecasts of solar radiation and air temperature. The pairs $\left(\mathcal{C}^{(d)}, f^{(d)}\right)$, $d = 1, \dots, D_T$, where D_T is the cardinality of the training set, are used to train a classifier H which, given a feature f, returns a class $H(f) \in \mathcal{C}$. Several approaches can be adopted to estimate the function H [8]. In this paper, since the features $f^{(d)}$ are scalar, we adopt the approach based on pairwise separation and Robust Linear Programming (RLP) [9].

Having the classifier H available, the last step is to determine the optimal bidding strategy for each of the classes $\mathcal{C}_i \in \mathcal{C}$. This boils down to substituting the *cdf* $F_{cs,m}(\cdot)$ in (8) with the conditional *cdf* $F_{cs,m}(\alpha \,|\, \mathcal{C}_i) = \Pr(\alpha_m \leq \alpha \,|\, \mathcal{C}_i)$ for each class $\mathcal{C}_i$, where $\Pr(\cdot \,|\, \mathcal{C}_i)$ means that the statistics is restricted only to days belonging to the class $\mathcal{C}_i$.

V. Experimental results

In this section, the proposed bidding strategies are validated on experimental data taken from a real Italian PV plant. The bidding strategies introduced in Section II and III, without any information about weather forecasts, will be denoted by OB and OB+N respectively. The bidding strategy described in Section IV-A, which uses weather forecasts and PV power curve to compute (and offer) PV power forecasts will be denoted by WF+PC. The bidding strategy proposed in Section IV-B, which combines the use of weather forecasts for classification and normalization, will be denoted by WF+OB+N.

The following data about a real 825 KWp PV power plant are available:

- generated power $w_m^{(d)}$
- solar radiation forecasts $\hat{I}_m^{(d)}$
- air temperature forecasts $\hat{T}_m^{(d)}$,

where $m = 1, \dots, 24$, $d = 1, \dots, 366$. The number of days spanned by the data set corresponds to one year of recordings in 2012. The data set is split into a training set composed of the data of the first 240 days (about 8 months of recordings) and a validation set containing the data of the remaining 126 days (about 4 months of recordings).

For the bidding strategies OB and OB+N, the training set is used to compute the relevant empirical *cdf* $F_m(\cdot)$ or $F_{cs,m}(\cdot)$. Then, for fixed penalties $\bar{q}$ and $\bar{\lambda}$, the bids C_m are computed using (3) or (9), depending on the strategy adopted.

Concerning the bidding strategy WF+PC, data points $(w_m^{(d)}, I_{cs,m}^{(d)}, \hat{T}_m^{(d)})$ in the training set are used to estimate the power curve (4) of the PV plant by adopting the approach in [6]. The estimated power curve, the forecasts of solar radiation $\hat{I}_m^{(d)}$ and air temperature $\hat{T}_m^{(d)}$ are used to compute the bids C_m through (10) on the validation set.

With reference to the bidding strategy WF+OB+N, the normalized energy range $[0,1)$ is partitioned into two intervals by choosing $s = 2$, $\epsilon_1 = 0.6068$. In this way each day is classified as a "sunny" day, when the normalized generated energy exceeds ϵ_1 (about 60% of the maximum amount of producible energy), or a "cloudy" day, when the normalized generated energy is less than ϵ_1. For each day d in the training set, the feature $f^{(d)}$ is computed as in (14). Training data is then used to estimate a classifier H and to determine the bids C_m for each of the two classes $\mathcal{C}_1$ and $\mathcal{C}_2$, as described in Section IV-B. The resulting classifier has the form:

$$H(f) = \begin{cases} \mathcal{C}_1 & \text{if } f < f^* \\ \mathcal{C}_2 & \text{if} f \geq f^*, \end{cases} \tag{15}$$

where $f^* = 0.8009$ is a threshold computed by training the RLP classifier [9]. Note that f^* differs from ϵ_1 mainly due to the errors in the forecasts of solar radiation and air temperature.

The performance of the bidding strategies OB, OB+N, WF+PC and WF+OB+N has been evaluated using the data contained in the validation data set under two scenarios.

In Scenario I, the energy surplus is supposed to be not remunerated at all, i.e. $\bar{\lambda} = p$. The following market parameters are assumed: $p = 0.1027$ €/Kwh and $\bar{q} = 0.0150$ €/Kwh.

Differently, in Scenario II, we set $p = 0.1027$ €/KWh, and assume that the power exceeding the upper bound is penalized, but nevertheless remunerated at $0.5p$, i.e. $\bar{\lambda} = 0.5p$. Moreover, we set $\bar{q} = 0.0150$ €/KWh as in the first scenario.

The average daily profits achieved by the proposed bidding strategies in both scenarios are reported in Table I. By comparing the first with other rows, it is clear that, in both scenarios, WF+PC performs significantly worse than the other bidding strategies. As stated previously, this is due to the fact that available solar radiation forecasts are typically very coarse and inaccurate.

With reference to Scenario I, choosing WF+PC as a reference profit, we obtain an increase of the average net daily profit of 8.6% through OB, 10.1% through OB+N and 11.1% through WF+OB+N.

In Scenario II, the benefits introduced by normalization and classification are less evident than before (comparing the first column with the second one of Table I). This outcome is due to the fact that the power surplus is now remunerated to some extent and consequently, the PV producer incurs penalties less frequently.

For comparison purposes, we consider the ideal strategy, denoted by R, assuming that the generation profile of the next day is known in advance. Since $C_m = w_m$ in this case, the producer never incurs penalties, and therefore the profit is maximal. This makes it possible to evaluate the performance of the proposed bidding strategies with respect to the maximum achievable. With strategy R, the average daily profit would be 244.98 € (last row of Table I). This implies that applying WF+OB+N allows one to fill 47.62% of the gap between WF+PC and R in Scenario I, and 24.13% in Scenario II. In the first scenario, the improvement obtained by WF+OB+N is remarkable. In the second scenario, the improvement is smaller for the reasons described above, but still significant.

VI. Conclusions and future work

In this paper, the problem of optimizing energy bids for a PV power producer on a competitive electricity market has been addressed. Since electricity market regulations provide penalties for shortfall and surplus of the energy actually produced with respect to the contracted bids, an optimization approach has been proposed in which the producer profit is maximized as a function of the penalties and the generated power statistics.

A first improvement of the basic bidding strategy is obtained by normalizing the historical generated power data through the theoretical clear-sky daily power profile of the PV plant. This amendment allows one to get rid of non stationary components of the PV power historical data due to several variables, such as the number of daylight hours or the intensity of solar radiation in different days of the year.

If forecasts of air temperature and solar radiation are available at the plant site, the bidding strategy can still be improved by suitably exploiting this additional information. To this purpose a classification step has been added to the overall optimization procedure. Information in historical PV power data and weather forecasts is used to train a classifier which allows for using the PV power conditional probability distribution function in place of the unconditional distribution in the optimization problem. The approach improves consistently the quality of the bidding strategy, both with respect to the unconditional case and to the case in which the bidding plan is computed simply by offering the PV power corresponding to the solar radiation and air temperature forecasts.

The devised algorithms have been tested on historical real data of generated power, and solar radiation and air temperature forecasts gathered in a PV plant over a period covering one year. The obtained quantitative results confirm the effectiveness of the proposed procedure, showing the roles of normalization and classification in different scenarios.

Future research will be devoted to extend the proposed approach to the case where a storage device is available and the market price is a stochastic variable. An additional issue deserving investigation regards different cost functions to be optimized in scenarios where the PV plant is exploited by a transmission/distribution system operator as an additional degree of freedom to improve power dispatching and grid operation reliability.

TABLE I
Average daily profits (€) in Scenario I and II with different bidding strategies

	$\bar{\lambda} = p$	$\bar{\lambda} = 0.5p$
WF+PC	198.59	213.49
OB	215.68	218.57
OB+N	218.75	220.92
WF+OB+N	220.68	221.09
R	244.98	244.98

References

[1] E. Bitar, R. Rajagopal, P. Khargonekar, K. Poolla, and P. Varaiya, “Bringing wind energy to market,” *IEEE Trans. Power Systems*, vol. 27, no. 3, pp. 1225–1235, 2012.

[2] J. Wu and C. Chan, “Prediction of hourly solar radiation using a novel hybrid model,” *Solar Energy*, vol. 85, pp. 808–817, 2011.

[3] G. Reikard, “Predicting solar radiation at high resolutions: A comparison of time series forecasts,” *Solar Energy*, vol. 83, pp. 342–349, 2009.

[4] A. Giannitrapani, S. Paoletti, A. Vicino, and D. Zarrilli, “Optimal bidding strategies for wind power producers with meteorological forecasts,” in *Proc. SIAM Conf. on Control and Its Applications*, July 2013, to appear.

[5] R. Dows and E. Gough, “PVUSA procurement, acceptance, and rating practices for photovoltaic power plants,” Pacific Gas and Electric Company, San Ramon, CA, Tech. Rep., 1995.

[6] G. Bianchini, S. Paoletti, A. Vicino, F. Corti, and F. Nebiacolombo, “Model estimation of photovoltaic power generation using partial information.” *IEEE ISGT Europe*, 2013, submitted.

[7] L. Wong and W. Chow, “Solar Radiation Model,” *Applied Energy*, vol. 69, pp. 191–224, 2001.

[8] S. Boucheron, O. Bousquet, and G. Lugosi, “Theory of classification: A survey of some recent advances,” *ESAIM: Probability and Statistics*, vol. 9, pp. 323–375, 2005.

[9] K. Bennett and O. Mangasarian, “Robust linear programming discrimination of two linearly inseparable sets,” *Optimization Methods and Software*, vol. 1, pp. 23–34, 1992.