Sizing of energy storage systems considering uncertainty on demand and generation

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Abstract: Over recent years there has been a general consensus about the necessary changes towards modernizing the existing power grid to meet environmental and socio-economic objectives. The adoption of low carbon technologies is a milestone in this process. On the other hand, the massive and uncoordinated connection of distributed generators (e.g. solar or wind) is making the operation of electrical distribution networks more challenging, e.g. causing energy balancing problems or voltage violations. Energy storage systems represent a possible means to cope with these issues. In this paper, we consider the problem of sizing the energy storage systems installed in a low voltage network with the aim of preventing voltage violations along the feeders. Since the problem is solved at the planning stage, when future realizations of demand and generation are unknown, we adopt a two-stage stochastic formulation where daily demand and generation profiles are modelled as random processes. The cost function to be minimized takes into account installation and operation costs related to storage use. By taking a scenario-based approach, the two-stage problem is approximated via a multi-scenario optimal power flow. To reduce the computational burden of the latter problem, a heuristic strategy consisting of solving separately a sizing problem for each scenario, and then combining the solutions of the single problems through a worst-case criterion, is proposed. The multi-scenario approach and the heuristic strategy are compared in terms of both computation time and quality of the solution using real data from an Italian low voltage network with photovoltaic generation.

Keywords: Energy storage sizing, voltage control, distribution network, two-stage stochastic programming, scenario-based approach.

1. INTRODUCTION

The ever growing penetration of non-dispatchable power generation is posing new challenges to distribution system operators (DSOs), called to guarantee power quality standards to consumers. In this respect, energy storage systems (ESSs) have been recently recognized as an instrumental tool for energy balancing and voltage support (Chu and Majumdar, 2012; U.S. DOE, 2013). The reason lies in the fact that ESSs represent the most flexible solution in this restructuring process, where new distributed energy resources such as wind and photovoltaic (PV) plants, electric vehicles and heat pumps, can be continuously connected to the grid. In view of such a dynamic scenario, ESSs can be easily relocated and operated both as loads or generators according to the grid requirements. Moreover, pros of the use of ESSs are that distributed supply and demand are balanced locally, thus reducing recourse to curtailment of renewable energy sources. The interested reader is referred to (IEC, 2011; U.S. DOE, 2013) for surveys on the general benefits that ESSs bring to the whole electricity system.

Many recent papers deal with optimal ESS allocation in power networks, which consists of deciding the number of ESSs to be deployed, their locations (siting) and sizes (sizing). A recent review of methods for ESS allocation in distribution networks can be found in (Zidar et al., 2016). Different methods are classified according both to the methodology used to find a solution and to the application for which ESSs are utilized. When dealing with voltage support in distribution networks, the problem is often formulated in a full AC optimal power flow (OPF) framework, where storage locations and sizes are optimally determined subject to power balance equations and storage dynamics. To cope with the computational burden of the derived OPF problems, a second-order cone programming OPF approach is considered in (Nick et al., 2014), while convex relaxations based on semidefinite programming (SDP) are used in (Gayme and Topcu, 2013). The alternating direction method of multipliers is exploited in (Nick et al., 2015) to break down the original problem into a parallel convex optimization. Giannitrapani et al. (2016) propose a heuristic approach based on voltage sensitivity analysis to circumvent the combinatorial nature of the siting problem.

A challenging issue for DSOs is to cope with a significant level of uncertainty both in planning and operation of distribution networks. For this reason there is a growing interest of researchers in including uncertainty explicitly in the formulated problems. As far as ESS sizing is concerned, in some cases a deterministic formulation of the problem is combined with an approach based on scenarios, see, e.g., (Nick et al., 2014; Giannitrapani et al., 2016). Baker et al. (2014) optimize ESS energy capacity using two-stage stochastic optimization in order to account for the intrinsic intermittency of load and wind supply. A formulation of the ESS sizing problem including intermittencies as chance constraints is provided in (Kargarian et al., 2016). Different stochastic techniques, such as Monte Carlo simulation, simulated annealing and robust multi-period OPF, have been adopted in (Baziar and Kavousi-Fard, 2013; Giannakoudis et al., 2010; Jabr et al., 2015), when considering several optimal energy management problems for active grids including ESSs.

In this paper we consider the problem of ESS sizing for voltage support in low voltage (LV) distribution networks. The problem is addressed by taking explicitly into account uncertainty on demand and generation through a two-stage stochastic programming formulation with AC power flow constraints and storage dynamics. Considering a finite number of scenarios, the two-stage problem is approximated by a multi-scenario OPF problem, for which a heuristic solution strategy based on a worst-case criterion is devised. The two-stage approach and its heuristic are evaluated and compared using real data from an Italian LV network featuring overvoltages in the absence of ESSs. Differences of this paper compared to (Baker et al., 2014) consist mainly in the adopted network model (a bus injection model enabling access to bus voltages) and the formulated optimization problem (a full AC OPF). Moreover, the two-stage stochastic programming formulation of the sizing problem was not present in our previous work (Giannitrapani et al., 2016).

The paper is organized as follows. The bus injection model of a LV network with ESSs is introduced in Section 2. Section 3 presents the formulation of the ESS sizing problem via two-stage stochastic optimization, while the proposed heuristic solution strategy is described in Section 4. Section 5 reports numerical results. Finally, conclusions are drawn in Section 6.

2. NETWORK MODEL

In this section we introduce the bus injection model of a LV network equipped with ESSs. For a fixed sampling time ΔT, the value of a variable x at time $t\Delta T$ is denoted by $x(t)$, where $t = 1, 2, \ldots$ is the discrete time index. Moreover, the real part, imaginary part, modulus and complex conjugate of $z \in \mathbb{C}$ are denoted by $\mathrm{Re}(z)$, $\mathrm{Im}(z)$, $|z|$ and z^*, respectively.

Consider a LV network described by a graph $(\mathcal{N}, \mathcal{E})$, where $\mathcal{N} = \{1, 2, \ldots, n\}$ is the set of nodes (*buses*) and $\mathcal{E}$ is the set of edges (*lines*). Adopting the classical π-model for the n-bus system, the admittance-to-ground at bus i is denoted by y_{ii}, while $y_{ij} = y_{ji}$ is the line admittance between buses i and j. If $(i, j) \notin \mathcal{E}$, $y_{ij} = 0$. Let $V_k(t) \in \mathbb{C}$, $P_k(t) \in \mathbb{R}$ and $Q_k(t) \in \mathbb{R}$ denote the complex voltage, active power injection and reactive power injection at bus k and time t, respectively. These quantities are linked by the power balance equations

$$P_k(t) = \mathrm{Re}\Big(V_k(t) \sum_{j\in\mathcal{N}} V_j^*(t) Y_{kj}^*\Big) \tag{1a}$$

$$Q_k(t) = \mathrm{Im}\Big(V_k(t) \sum_{j\in\mathcal{N}} V_j^*(t) Y_{kj}^*\Big), \tag{1b}$$

where

$$Y_{kj} = \begin{cases} y_{kk} + \sum_{h\neq k} y_{kh} & \text{if } j = k \\ -y_{kj} & \text{otherwise.} \end{cases}$$

Bus 1 is assumed to be a slack bus, characterized by fixed voltage magnitude and phase, i.e. $V_1(t)$ is known for all t. Conversely, all other buses in the set $\mathcal{K} = \{2, \ldots, n\}$ are treated as load buses. This means that, given $P_k(t)$ and $Q_k(t)$ for all $k \in \mathcal{K}$, bus voltages $V_j(t)$, $j \in \mathcal{K}$, are determined by solving the power flow equations (1).

Voltage quality requirements set by current regulation take the following form:

$$\underline{v}_k^2 \leq |V_k(t)|^2 \leq \overline{v}_k^2, \tag{2}$$

where $\underline{v}_k \leq \overline{v}_k$ are given positive bounds. The real power flow from bus i to bus j is also bounded to reflect the physical limits of the lines:

$$\mathrm{Re}\left(V_i(t)\big[V_i(t) - V_j(t)\big]^* y_{ij}^*\right) \leq \overline{P}_{ij}. \tag{3}$$

The left-hand side of (3) is the real power transferred from bus i to bus j at time t, while $\overline{P}_{ij} = \overline{P}_{ji}$ is a given upper bound.

Let $\mathcal{S} \subseteq \mathcal{K}$ be the set of buses equipped with ESSs. For $s \in \mathcal{S}$, the storage level at bus s and time t is denoted by $e_s(t)$, while $r_s(t)$ and $b_s(t)$ are the average active and reactive power exchanged between $t\Delta T$ and $(t+1)\Delta T$. The quantities $r_s(t)$ and $b_s(t)$ are bounded as follows:

$$\underline{R}_s \leq r_s(t) \leq \overline{R}_s \tag{4a}$$

$$\underline{B}_s \leq b_s(t) \leq \overline{B}_s, \tag{4b}$$

where $\underline{R}_s < 0$, $\overline{R}_s > 0$ and $\underline{B}_s < \overline{B}_s$ are given bounds depending on the ESS technology adopted. For the sake of simplicity, charging and discharging efficiencies are assumed equal to 1 and the dynamics of $e_s(t)$ are modelled through the difference equation

$$e_s(t+1) = e_s(t) + r_s(t)\Delta T, \tag{5}$$

with known initial condition $e_s(1)$. For all t, the storage level $e_s(t)$ is bounded as follows:

$$0 \leq e_s(t) \leq E_s, \tag{6}$$

with E_s being the ESS capacity installed at bus s. Since in this paper we consider time horizons of one day, the additional constraint stating that the storage level at the beginning and at the end of the day are equal, is imposed to decouple ESS operation in different days:

$$\textstyle\sum_{t\in\mathcal{T}} r_s(t) = 0, \tag{7}$$

where $\mathcal{T} = \{1, \ldots, T\}$ and T is the number of time samples per day.

For a generic bus $k \in \mathcal{K}$ having loads, generators and ESSs connected to it, $P_k(t)$ and $Q_k(t)$ in (1) can be written as

$$P_k(t) = P_k^G(t) - P_k^D(t) - r_k(t) \tag{8a}$$

$$Q_k(t) = Q_k^G(t) - Q_k^D(t) - b_k(t), \tag{8b}$$

where the superscript ^{G} refers to generation and the superscript ^{D} refers to demand. The following additional constraints apply to buses not equipped with ESSs:

$$r_h(t) = b_h(t) = 0, \quad h \in \mathcal{K} \backslash \mathcal{S}. \tag{9}$$

Moreover, in case no load/generator is connected to bus k, demand/generation at bus k is assumed to be zero.

3. PROBLEM FORMULATION VIA TWO-STAGE STOCHASTIC OPTIMIZATION

At the planning stage, the fundamental decision problem related to the adoption of ESSs in a power network concerns ESS allocation, i.e. one has to decide the number and the locations of the ESSs (siting problem), as well as their sizes (sizing problem). For voltage support applications in LV networks, an iterative technique for ESS allocation is proposed in (Giannitrapani et al., 2016). In particular, ESS siting is accomplished by exploiting voltage sensitivity analysis to identify the most effective connections between nodes with the aim of maximizing the impact of power injection/consumption on voltage variations. In this paper we address the companion ESS sizing problem in a setting with uncertainty. Since the sizing problem is solved at the planning stage, when future realizations of demand and generation are unknown, we adopt a stochastic formulation where daily demand and generation profiles are considered as stochastic quantities. The cost function to be minimized takes into account both installation and operation costs related to ESS use. In particular, installation costs are assumed to be proportional to the total installed ESS capacity, while for the sake of simplicity only average line losses are considered as operation costs. More composite choices are possible without changing the nature of the problem, see, e.g., (Zarrilli et al., 2017).

Following (Shapiro et al., 2014), the considered problem can be cast in a two-stage stochastic optimization framework. Let ξ be the random vector containing the values $P_k^D(t)$, $Q_k^D(t)$, $P_k^G(t)$ and $Q_k^G(t)$ for all $k \in \mathcal{K}$ and $t \in \mathcal{T}$ (recall that the time horizon $\mathcal{T}$ is assumed to span one day), and denote by Ξ the support of the probability distribution of ξ. Moreover, let x denote the vector of ESS capacities E_s for all $s \in \mathcal{S}$. The total installed ESS capacity corresponding to x is

$$\mathcal{C}(x) = \textstyle\sum_{s\in\mathcal{S}} E_s. \tag{10}$$

Then, the first-stage problem aiming at finding the ESS sizes which minimize a linear combination of installation and operation costs, reads as

$$\min_{x\in X} \mathbb{E}_\xi[\mathcal{L}(x,\xi)] + \gamma\mathcal{C}(x), \tag{11}$$

where $X = \{x : x \geq 0\}$ (the inequality is to be intended component-wise) and $\gamma > 0$ is a weighting parameter which trades off installation and operation costs. The latter are represented by the term $\mathbb{E}_\xi[\mathcal{L}(x,\xi)]$, namely the expected value, taken with respect to the probability distribution of ξ, of the line losses under an optimal ESS operation policy which minimizes them. In other words, $\mathcal{L}(x,\xi)$ is the optimal value of the second-stage problem:

$$\mathcal{L}(x,\xi) = \min_{y\in\mathcal{G}(x,\xi)} \ell(x,y,\xi), \tag{12}$$

where the vector y contains the values $V_k(t)$, $r_s(t)$ and $b_s(t)$ for all $k \in \mathcal{K}$, $s \in \mathcal{S}$ and $t \in \mathcal{T}$; the objective function $\ell(x,y,\xi)$ denotes the average total line losses per sampling time:

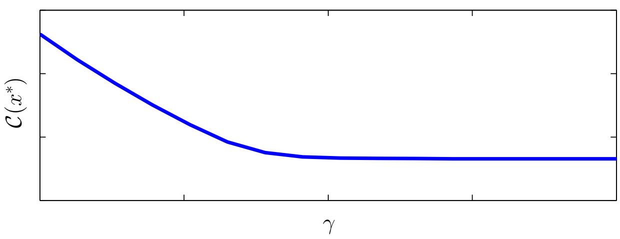

Fig. 1. Typical plot of the installation cost $\mathcal{C}(x^*)$ versus the weighting parameter γ.

$$\ell(x,y,\xi) = \textstyle\sum_{t\in\mathcal{T}} \sum_{k\in\mathcal{N}} P_k(t)\Delta T/T, \tag{13}$$

with $P_k(t)$ given by (1a), and the feasible set $\mathcal{G}(x,\xi)$ is defined by power balance equations, ESS dynamics and constraints on voltage, real power flow and ESS power exchanges:

$$\mathcal{G}(x,\xi) = \{y : (1) - (9), k \in \mathcal{K}, s \in \mathcal{S}, (i,j) \in \mathcal{E}, t \in \mathcal{T}\}. \tag{14}$$

In practical applications, only a finite set of scenarios of demand and generation is available. Each scenario is represented by a realization ξ_d of the random variable ξ, either extracted from historical data sets, or generated via simulation of suitable identified models. In these cases, problem (11)-(12) can be approximated in a discrete fashion. Let D be the number of scenarios and define D distinct vectors of unknowns with the same meaning as y in (12), denoted by y_d, $d = 1,\ldots,D$. Then, problem (11)-(12) is approximated by

$$\min_{x,y_1,\ldots,y_D} \sum_{d=1}^{D} p_d\,\ell(x,y_d,\xi_d) + \gamma\mathcal{C}(x) \tag{15}$$
$$\text{s.t.} \quad x \in X,\ y_d \in \mathcal{G}(x,\xi_d),\ d = 1,\ldots,D,$$

where p_d represents the probability associated to scenario ξ_d (see Remark 2)[1]. Approximation of (11)-(12) through (15) is justified by the fact that, if ξ were discrete-valued with possible values $\xi_1,\ldots,\xi_D$, then the two problems would be equivalent thanks to the interchangeability principle, see (Shapiro et al., 2014). In the following, x^* is used to denote the value of x at the optimum of problem (15), while E_s^* denotes the optimal size of the ESS at bus s corresponding to the solution x^*.

Problem (15) is a multi-scenario, multi-period OPF, which is very hard to solve due to non-convexity (typical of OPF problems), time-coupling constraints (determined by ESS dynamics) and scenario-coupling constraints (represented by ESS sizes). A possible approach to cope with this computational burden is by resorting to SDP convex relaxations (Lavaei and Low, 2012; Low, 2014). In the next section, a heuristic approach based on solving separately a sizing problem for each scenario, thus allowing for parallel computation, is proposed.

Remark 1. The weighting parameter γ is used to trade off installation and operation costs. The installation cost $\mathcal{C}(x)$ at the optimum of problem (15), i.e. for $x = x^*$, is plotted in Fig. 1 as a function of the weighting parameter γ. As expected, when greater weight is given to the installation costs, the total installed ESS capacity decreases. However,

[1] With a slight abuse of terminology, from now on we will denote a scenario by the corresponding realization of the random vector ξ.

the term $\mathcal{C}(x^*)$ cannot decrease below a lower bound mainly imposed by the satisfaction of voltage quality constraints under all scenarios.

Remark 2. The probability p_d of scenario ξ_d is a measure of its representativeness with respect to the probability distribution of ξ. When information on this distribution is missing, a uniform distribution $p_d = 1/D$ can be adopted.

4. HEURISTIC METHOD BASED ON SCENARIOS

The proposed heuristic method to tackle problem (15) consists of solving a sizing problem for each scenario, and then combining the solutions found, e.g. through a worst-case criterion. Let x_d be the vector of ESS sizes in scenario ξ_d. The sizing problem for scenario ξ_d reads as

$$\min_{x_d, y_d} \ell(x_d, y_d, \xi_d) + \gamma_d \mathcal{C}(x_d) \quad (16)$$
$$\text{s.t.} \quad x_d \in X, \ y_d \in \mathcal{G}(x_d, \xi_d),$$

where $\gamma_d > 0$ is the weighting parameter. This problem is a multi-period OPF, for which solution strategies based on SDP convex relaxations can be adopted as for problem (15). Indeed, (16) is obtained from (15) by considering scenario ξ_d only, hence with $D = 1$ and $p_d = 1$. The values of x_d and y_d at the optimum of problem (16) are denoted by $\widetilde{x}_d$ and $\widetilde{y}_d$, respectively. Moreover, $\widetilde{E}_{s,d}$ is the size of the ESS at bus s corresponding to the solution $\widetilde{x}_d$. The weighting parameter γ_d of problem (16) plays the same role as γ in problem (15). Therefore, the plot of the cost $\mathcal{C}(\widetilde{x}_d)$ versus γ_d will be like the one of Fig. 1, see also (Giannitrapani et al., 2016).

The final decision on the size of the ESS at bus s is made according to the rule

$$\widetilde{E}_s = \max_{d=1,\ldots,D} \widetilde{E}_{s,d}. \quad (17)$$

This rule is justified by the fact that ESS sizes E_s enter the definition of the feasible set of problem (15) only as upper bounds in constraints of the type (6), one for each scenario. Hence, (17) ensures that the vector $\widetilde{x}$ constructed with values $\widetilde{E}_s$, together with the vectors $\widetilde{y}_1, \ldots, \widetilde{y}_D$, form a feasible solution for problem (15).

5. NUMERICAL RESULTS

The proposed methods for ESS sizing are tested on a real LV network whose topology was provided by the main Italian DSO. The test network is shown in Fig. 2. It consists of 17 buses and 16 lines. A total of 26 loads and 4 PV generators are connected to the network. Installed power of the PV generators is 9 kWp at buses 6 and 7, 6 kWp at bus 11 and 4 kWp at bus 15. For all loads and PV generators, historical data are available with time step $\Delta T = 60$ min over a period of 90 days. In the network model, 10% tolerance around the nominal voltage magnitude is allowed at all buses $k \in \mathcal{K}$, i.e. $\underline{v}_k = 0.9$ pu and $\overline{v}_k = 1.1$ pu in (2), in accordance with the European Norm 50160. With these bounds, the available demand and generation profiles are such that the network often experiences overvoltages in the absence of ESSs. For this reason, an empty initial storage level is assumed for all ESSs, i.e. $e_s(1) = 0$ kWh, in order to compensate overvoltages with as small as possible storage capacity.

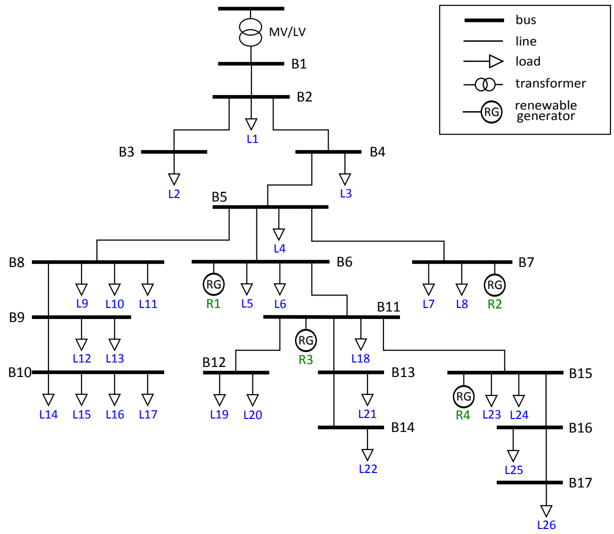

Fig. 2. Test LV network with 17 buses.

The bounds $\overline{P}_{ij}$ in (3) are all set to 35 kW. Bounds in (4) are chosen such that $\overline{R}_s = -\underline{R}_s = 25$ kW and $\overline{B}_s = -\underline{B}_s = 25$ kVar.

The siting algorithm presented in (Giannitrapani et al., 2016) is run in order to locate two ESSs in the network. In this case when only overvoltages show up, locations selected by the algorithm are buses 11 and 15, i.e. two buses with PV generators. In the following, all SDP relaxations of OPF problems for ESS sizing are solved using the CVX modelling package for convex optimization (Grant and Boyd, 2014) and the SeDuMi solver (Sturm, 1999) on an Intel Xeon 2.4 GHz CPU with 32 GB RAM.

5.1 Sizing via two-stage programming

In order to evaluate how the computation time of problem (15) scales with the number D of scenarios, the SDP relaxation of (15) is solved for increasing values of D. Ten scenarios are initially selected by picking ten days randomly from the available data set. Then, larger groups of scenarios are iteratively constructed by adding ten days (picked randomly among those remaining) to the ones previously selected. Table 1 shows the approximate computation time of the SDP problems solved for the different values of D considered. The choice $\gamma = 0.001$ is made for all problems, while $p_d = 1/D$ is the probability associated to each scenario when D scenarios are considered. The computation time turns out to be approximately quadratic in D up to $D = 70$. Most importantly, for $D = 80$ it is not

Table 1. Computation time for solving the SDP-relaxed version of problem (15).

D	time [min]
1	0.3
10	8
20	25
30	51
40	91
50	139
60	205
70	302

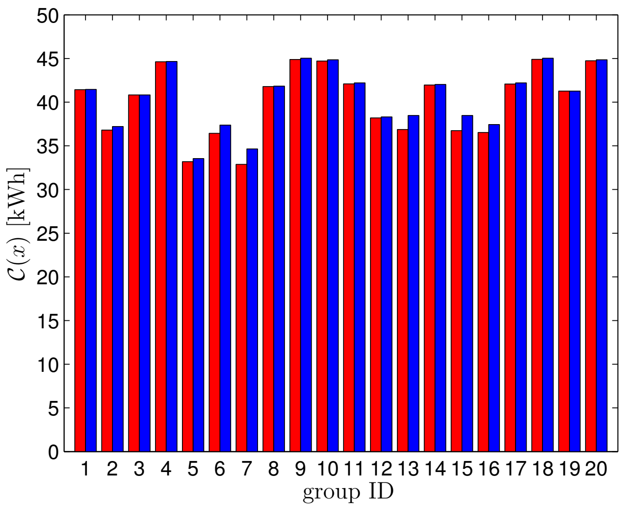

Fig. 3. Comparison of the total ESS capacities $\mathcal{C}(x^*)$ (red bars) and $\mathcal{C}(\widetilde{x})$ (blue bars) for $N = 20$ groups of $D = 20$ scenarios.

possible to solve the SDP problem due to out-of-memory issues caused by the prohibitive number of variables involved. Similar computational issues are reported in the literature for applications with a large number of scenarios, see, e.g., (Nick et al., 2015). This highlights that the real drawback of the formulation (15) is not the computation time (whose weight at the planning stage may be relative), rather the fact to become rapidly intractable as D grows.

5.2 *Sizing via heuristic method*

A point in favor of the heuristic method of Section 4 is that the computation time of the solution $\mathcal{C}(\widetilde{x})$ scales linearly with the number D of scenarios (assuming sequential computation). The first row of Table 1 shows the approximate computation time for solving the SDP-relaxed version of problem (16). This implies that obtaining $\mathcal{C}(\widetilde{x})$ requires about 24 min for $D = 80$, when a solution of the SDP-relaxed version of (15) cannot be either computed. It is also stressed that computation times can be further reduced by parallelizing the computation.

To assess the performance of the heuristic method, we evaluate the gap between the total ESS capacity $\mathcal{C}(x^*)$ obtained by solving the multi-scenario problem (15) and the one $\mathcal{C}(\widetilde{x})$ obtained by applying the heuristic method. To evaluate this gap under different data realizations, $N = 20$ groups of $D = 20$ scenarios are formed by picking days randomly from the available data set. For each group of scenarios, the SDP-relaxed version of problem (15) is solved with probability $p_d = 1/D$ associated to each scenario. Moreover, SDP-relaxed versions of problem (16), one for each scenario, are solved, and then the solutions of the single-scenario problems are combined according to (17). In all the multi- and single-scenario problems solved, the weighting parameters $\gamma, \gamma_1, \ldots, \gamma_D$ are selected in order to get as small as possible total ESS sizes $\mathcal{C}(x^*), \mathcal{C}(\widetilde{x}_1), \ldots, \mathcal{C}(\widetilde{x}_D)$ (see Fig. 1). With this choice and by defining $\bar{x} = \arg\max_{d=1,\ldots,D} \mathcal{C}(\widetilde{x}_d)$, it holds that $\mathcal{C}(\bar{x}) \leq \mathcal{C}(x^*) \leq \mathcal{C}(\widetilde{x})$, which represents a computable, guaranteed interval for $\mathcal{C}(x^*)$. Since in this case $\mathcal{C}(x^*)$ can be computed, the bar plot in Fig. 3 compares the total ESS sizes $\mathcal{C}(x^*)$ (red bars) and $\mathcal{C}(\widetilde{x})$ (blue bars) obtained. For all groups of scenarios, it can be observed that $\mathcal{C}(x^*) \leq \mathcal{C}(\widetilde{x})$ as expected, and the two solutions are typically very close. For a quantitative assessment, let the relative gap (in percent) be defined as

$$\nu(\widetilde{x}, x^*) = \frac{\mathcal{C}(\widetilde{x}) - \mathcal{C}(x^*)}{\mathcal{C}(x^*)} \times 100\%. \tag{18}$$

It turns out that the maximum of $\nu(\widetilde{x}, x^*)$ over the considered groups of scenarios is 5.4%, with an average value of 1.2%. For 13 groups out of 20, the relative gap is less than 0.4%. Far from being an exhaustive test, these results show the good performance of the proposed heuristic on the available data set also in terms of quality of the solution found.

Remark 3. The solution of the SDP problem may be infeasible for the original problem (15) or (16), leading in particular to violations of the voltage magnitude and real power flow constraints. For this reason, feasibility of the solution of the SDP problem for the corresponding original problem should be always checked a posteriori. For each scenario considered, this amounts to solve a series of T power flow problems (1), one for each time period $t = 1, \ldots, T$, where the complex voltages $V_k(t)$ are the unknowns and the active and reactive power $P_k(t)$ and $Q_k(t)$ are given by (8), with $r_s(t)$ and $b_s(t)$ provided by the solution of the relaxed problem for all $s \in \mathcal{S}$. Then, satisfaction of constraints (2) and (3) is verified with the complex voltages $V_k(t)$ returned by the power flow problems. This check was always successful for the solutions of all relaxed problems considered in this section. An example is presented in Fig. 4, showing the daily voltage profiles at bus 6 in one of the considered scenarios both in the absence and in the presence of ESSs. When ESSs are not installed, overvoltages due to high PV generation occur between 11 AM and 5 PM (blue curve). The problem is prevented with the use of the ESSs (red curve). Since ESSs are initially empty, they are not used until overvoltages show up (the blue and red curve overlap until 10 AM). Then, voltage is pulled back below the upper bound by charging the ESSs. Notice that the bus voltage is higher after 6 PM in the case with ESSs. This is due to constraint (7), causing the ESSs to discharge for restoring the initial energy level at the end of the day.

6. CONCLUSIONS AND FUTURE WORK

This paper dealt with the deployment of ESSs for voltage support applications in LV networks. Sizing of these ESSs was addressed by taking explicitly into account uncertainty on demand and generation through a two-stage stochastic programming formulation. Considering a finite number of scenarios, the two-stage problem was then approximated by a multi-scenario sizing problem. This made it possible to devise a heuristic solution strategy based on solving a sizing problem for each scenario, and then combining the solutions of the single problems through a worst-case criterion. The presented numerical results, obtained for a test LV network with 17 buses, showed that, even when one resorts to SDP convex relaxations, the multi-scenario sizing problem suffers from computational issues as the number of scenarios grows. On the other

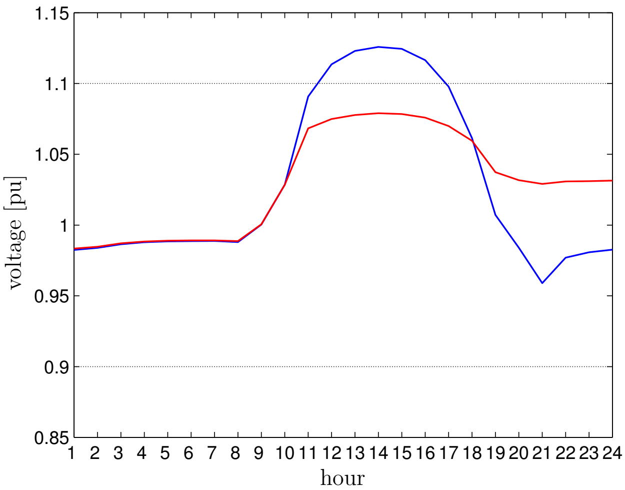

Fig. 4. Voltage magnitude at bus 6 in one of the considered scenarios: (blue) in the absence of ESSs, (red) in the presence of ESSs.

hand, the proposed heuristic method based on solving multi-period OPF problems of fixed size, is computationally less demanding. Moreover, the gap between the solution of the heuristic method and that of the multi-scenario problem was acceptable, and typically very small, in all presented instances.

Future work will provide an extensive validation of the heuristic method in terms of solution gap using different network topologies and scenarios of demand and generation. Moreover, in order to limit the computation time of the multi-scenario sizing problem, suitable scenario reduction techniques will be investigated. A scenario reduction technique makes it possible to downsize a given scenario set to a scenario subset of prescribed cardinality while preserving as much as possible the information useful to the solution of the underlying optimization problem. To the best of the authors' knowledge, scenario reduction techniques appropriate to solve ESS sizing problems are missing. Standard clustering techniques such as k-means are used in (Nick et al., 2014). On the other hand, scenario reduction techniques based on the notion of probability distance (Dupačová et al., 2003) are shown in (Bucciarelli et al., 2016) to be unsuitable to the purposes of ESS sizing.

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