# Bidding strategies for renewable energy generation with non stationary statistics

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**Abstract:** The intrinsic variability in non-dispatchable power generation raises important challenges to the integration of renewable energy sources into the electricity grid. This paper studies the problem of optimizing energy bids for a photovoltaic (PV) power producer taking part into a competitive electricity market characterized by financial penalties for generation shortfall and surplus. To this purpose, an optimization procedure is devised to cope with the intermittent nature of PV generation and maximize the expected profit of the producer. Since the optimal offer turns out to be a suitable percentile of the PV power cumulative distribution function (*cdf*), we investigate two approaches to properly take into account the effects of seasonal variation and non stationary nature of PV power generation in the estimation of PV power statistics. The first one normalizes the generated power with the power obtainable under clear-sky conditions. The second approach estimates a time-varying PV power *cdf* using only power data in a moving window of suitable width. A numerical comparison of the different bidding strategies is performed on a real data set from an Italian PV plant.

## 1. INTRODUCTION

With the increasing penetration level of renewable energy sources (RES), grid system operators have to face more and more challenging technical issues. While RES bring in obvious advantages in terms of production costs and environmental impact, their intrinsic intermittent and non-dispatchable nature causes several difficulties for a correct grid operation. In order to mitigate such problems, several countries are promoting regulatory frameworks forcing the producers to actively participate in the technical and economical integration of renewables [Klessmann et al., 2008]. In an attempt to reduce the uncertainty affecting generation from RES, producers are required to provide day-ahead schedules of their generation. Energy is then remunerated according to the conformity of the actual generation profile to the schedule, by applying financial penalties to shortfall or surplus of energy generation. From a producer perspective, this calls for the development of suitable bidding strategies to offer the maximum amount of energy while avoiding imbalance costs.

Optimal bidding strategies for a wind power producer have been studied in [Bathurst et al., 2002, Matevosyan and Soder, 2006, Pinson et al., 2007, Morales et al., 2010, Dent et al., 2011], and recently in [Bitar et al., 2012]. In a market where penalties are applied whenever the delivered power deviates from the schedule, the optimal bid for a certain hour of the day turns out to be a suitable percentile of the cumulative distribution function (*cdf*) of the power generation at the same hour. Under the assumption of time-invariant statistics of power generation, the *cdf*s can be estimated from all past data of the power generated by the plant. In principle, one could apply the same bidding strategy to other RES. However, some RES like photovoltaic (PV) and hydro are characterized by remark-

able seasonal variations of power generation and exhibit a significant non stationary behavior. Such a phenomenon may negatively affect the optimal bidding strategy, if not properly considered.

The main contribution of the paper is to present two approaches to account for the fluctuations of the generated power over the year and thus tune the optimal bidding strategies originally developed for a wind source to the case of a PV producer. The first solution consists in normalizing the generated power with respect to the power that could be obtained from the plant under clear-sky conditions. In the second approach, a moving window on the most recent power generation data is adopted to estimate a time-varying *cdf* of generated power. Both techniques are experimentally compared, in terms of average daily profit, to the straightforward application of the optimal bidding strategy for wind power producers.

The paper is organized as follows. Section 2 presents the mathematical formulation of the bidding problem for a generic non-dispatchable RES and recalls the optimal solution. Section 3 describes the proposed approaches to deal with the non stationarity of the PV power generation statistics. Section 4 reports experimental results obtained under different pricing scenarios using data from a real Italian PV plant. Finally, some conclusions are drawn in Section 5.

## 2. OPTIMAL BIDDING STRATEGY

In this section we consider a power producer from non-dispatchable RES (e.g. wind, solar), and formulate the problem of finding the optimal energy bids in an electricity market featuring financial penalties for energy imbalance. We also recall the optimal solution to this problem, which

is derived in [Bitar et al., 2012] in terms of power statistics and imbalance penalties.

Let $w_m$ be a random variable describing the energy generated by the power plant over the $m$-th hour of the day, $m = 1, \ldots, 24$, and let $C_m$ denote the corresponding energy bid for the same interval. It is assumed that the power producer is remunerated with unitary price $p > 0$ for the actual generated energy. Moreover, the power producer is penalized whenever the generated energy differs from the bid. In particular, $\bar{q} \geq 0$ and $\bar{\lambda} \geq 0$ are the unitary penalties applied for energy shortfall ($w_m < C_m$) and surplus ($w_m > C_m$), respectively. It follows that the net hourly profit for the power producer amounts to

$$
\begin{aligned}
J(C_m, w_m) = & p w_m - \bar{q} \max\{C_m - w_m, 0\} \\
& - \bar{\lambda} \max\{w_m - C_m, 0\}.
\end{aligned} \tag{1}
$$

Since $J(C_m, w_m)$ in (1) is a stochastic quantity due to the uncertainty on the generated energy $w_m$, the optimal bidding problem consists in finding the bid $C_m^*$ which maximizes the expected profit $\mathbf{E}[J(C_m, w_m)]$, i.e.

$$
C_m^* = \arg\max_{C_m} \mathbf{E}[J(C_m, w_m)], \tag{2}
$$

where $\mathbf{E}[\cdot]$ denotes expectation with respect to the statistics of $w_m$. We define $F_m(\cdot)$ the *cdf* of the random variable $w_m$, i.e. $F_m(\omega) \triangleq \Pr(w_m \leq \omega)$. Moreover, we let $F_m^{-1}(\nu) = \inf\{\omega : F_m(\omega) \geq \nu\}$, $\nu \in [0, 1]$, be the corresponding quantile function. It turns out (see [Bitar et al., 2012]) that the optimal solution to (2) is given by:

$$
C_m^* = F_m^{-1} \left( \frac{\bar{\lambda}}{\bar{\lambda} + \bar{q}} \right). \tag{3}
$$

Note that the optimal solution (3) depends only on the penalties $\bar{q}$ and $\bar{\lambda}$, and the *cdf* of $w_m$. If the penalties are stochastic variables independent of the generated energy $w_m$, the optimal bid (3) still holds by replacing $\bar{q}$ and $\bar{\lambda}$ with their mean values. Concerning $F_m(\cdot)$, in real applications it must be typically estimated from historical energy generation data. As will be discussed in the next section for the specific case of PV power producers, the way $F_m(\cdot)$ is estimated may have an important impact on the practical performance of the optimal bidding strategy (3).

In some cases, deviations from the bid are tolerated within a specified threshold. This applies, for instance, to the regulatory framework recently introduced in Italy. An extension of problem (1)-(2) to the framework with soft penalties is presented in [Giannitrapani et al., 2013b].

*Remark 1.* In some markets (e.g., the Italian one), it may happen that, depending on the network contingency, $\bar{q} < 0$ and/or $\bar{\lambda} < 0$. This means that deviations from the schedule are actually rewarded, rather than penalized, since they contribute to mitigate the overall network imbalance. In this case, the optimal solution to problem (2) boils down to offering either zero or the maximum producible power (see [Bitar et al., 2012] for details). In this paper, we will restrict our attention to a scenario in which $\bar{q} \geq 0$ and $\bar{\lambda} \geq 0$, so that the existence of a nontrivial solution (3) is guaranteed.

3. NON STATIONARY POWER GENERATION

As recalled in Section 2, the solution of the optimal bidding problem requires the knowledge of the generated
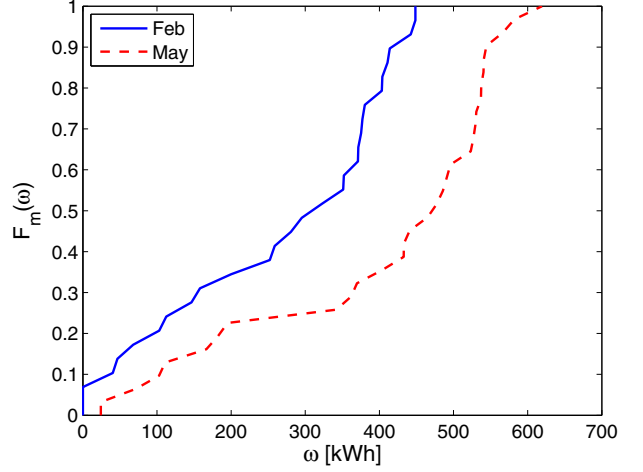


Fig. 1. Example of empirical *cdf* of the random variable $w_{11}$ in two different months of year (solid: February, dashed: May).

power *cdf* at each hour of the day. In real applications, such a distribution is to be estimated on the basis of historical data of generated power. A distinctive feature of renewable sources is that the generated power statistics is strictly dependent on weather variables, e.g., wind velocity and direction for wind plants, or irradiance and air temperature for PV plants. It is well known that meteorological variables exhibit strongly non stationary behavior, which implies that special care must be taken in the estimation of power statistics from historical data.

With regard to PV generation, which is of concern in this paper, non stationarity is due to the time-varying patterns of solar irradiance in days of different periods of the year. To realize the importance of this issue, consider Fig. 1, which shows the estimated *cdf*s of the energy generated by a 825 kWp PV plant at a certain hour of the day, in two different months of the year, i.e., February and May. For example, it is clear even from a visual inspection of the two curves, that both the maximum and the average generated energy are different in the two cases. To deal with the time-varying nature of the statistics of PV power, we propose two alternative approaches, whose effectiveness will be successively tested on real data in Section 4.

The first approach consists in transforming the past power data according to a multiplicative deseasonalization model exploiting the concept of "clear-sky" power generation profile. This profile can be reliably computed for a PV plant at any fixed day of the year, by assuming that the plant is subject to the maximum solar irradiance achievable at the plant site, i.e., under cloudless sky. Clear-sky solar irradiance can be computed by means of well-known analytical models, e.g., [Wong and Chow, 2001].

The second approach consists in devising an adaptive mechanism for updating daily or weekly the estimates of power *cdf*s. The simple technique adopted consists in estimating the power *cdf* at a given hour based on most recent historical data, by selecting a moving window whose width in the past is optimized according to the best profit obtainable by the bidding strategy.
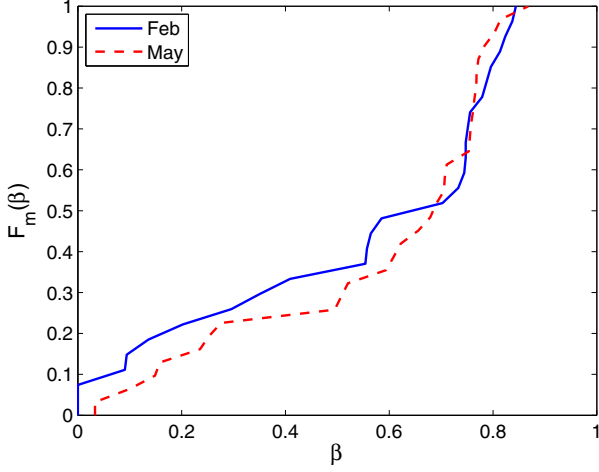
Fig. 2. Example of averaged *cdf*s of the normalized variable $\beta_{11}$ (solid: February, dashed: May).

### 3.1 Exploiting clear-sky generation profiles

In the first approach, we choose to normalize both the generated power and the bid at a given hour with respect to the maximum power obtainable from the plant at the same hour, i.e. under clear-sky conditions.

The solar irradiance at ground level takes maximum values in a cloudless day and is defined as clear-sky solar irradiance ($I_{cs}$). The generation profile of a PV plant hit by clear-sky solar radiation is called clear-sky generation profile ($w_{cs}$). It can be estimated by using clear-sky solar irradiance and the power curve of PV modules. An analytical form of the power curve of PV modules is provided by the PVUSA model (see [Dows and Gough, 1995]), which expresses the generated power $w_m$ as a function of solar irradiance $I_m$ and air temperature $T_m$ according to the equation:

$$w_m = aI_m + bI_m^2 + cI_mT_m, \qquad (4)$$

where $a$, $b$ and $c$ are the model parameters (typically $a > 0$, $b < 0$, $c < 0$). Although model (4) is linear-in-the-parameters, parameter estimation is complicated by the fact that measurements of solar radiation and air temperature may not be available at the plant site. A heuristic approach to estimate such parameters in the partial information case is presented in [Bianchini et al., 2013], which relies on historical data of generated power, air temperature forecasts and clear-sky solar irradiance. The clear-sky generation profile can be computed from (4) by replacing $I_m$ with the clear-sky solar irradiance $I_{cs,m}$ and $T_m$ with commonly available temperature forecasts.

Let us denote by $w_{cs,m}$ the clear-sky PV energy over the $m$-th hour of the day (to simplify notation, we omit the dependence of $w_{cs,m}$ on the day of the year), and let $w_m = \beta_m w_{cs,m}$, $\beta_m \in [0,1]$. Moreover, the bid is parameterized as $C_m = \alpha_m w_{cs,m}$, $\alpha_m \in [0,1]$. By substituting the expressions of $w_m$ and $C_m$ into (1), we obtain that $J(C_m, w_m) = w_{cs,m} J(\alpha_m, \beta_m)$, where

$$\begin{aligned} J(\alpha_m, \beta_m) =& p\beta_m - \bar{q}\max\{\alpha_m - \beta_m, 0\} \\ & - \bar{\lambda}\max\{\beta_m - \alpha_m, 0\}. \end{aligned} \qquad (5)$$
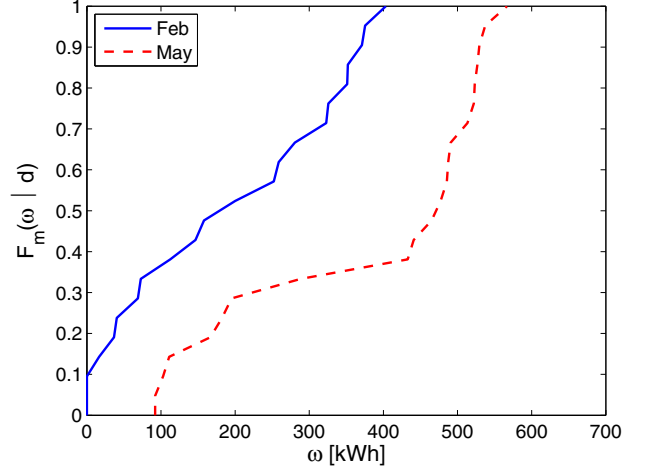


Fig. 3. Example of two moving window *cdf*s: $F_{11}^{(20)}(\omega \mid 46)$ and $F_{11}^{(20)}(\omega \mid 135)$ (solid: February, dashed: May).

The considered bidding problem can thus be reformulated as finding

$$\alpha_m^* = \arg\max_{\alpha_m} \mathbf{E}[J(\alpha_m, \beta_m)], \qquad (6)$$

where $\mathbf{E}[\cdot]$ here denotes expectation with respect to the statistics of $\beta_m$. Let $F_{cs,m}(\beta)$ denote the *cdf* of the random variable $\beta_m$. Similarly to the previous case, the optimal solution to (6) is given by:

$$\alpha_m^* = F_{cs,m}^{-1}\left(\frac{\bar{\lambda}}{\bar{\lambda} + \bar{q}}\right). \qquad (7)$$

The optimal bid is finally computed as

$$C_m^* = \alpha_m^* w_{cs,m}. \qquad (8)$$

In this way, the seasonal variations of PV power generation are captured by the clear-sky PV energy profile $w_{cs,m}$. As a consequence, to a first approximation, the resulting normalized energy $\beta_m$ can be regarded as a stationary process, thus mitigating the adverse effect of seasonality on the bidding strategy. Figure 2 shows the empirical *cdf*s of the normalized generated energy relative to the same hours of the day and months of the year as those of Fig. 1. Note the reduction of the discrepancies between the two curves, if compared with Fig. 1. The main advantage of the proposed bidding strategy is that the power *cdf* can be estimated on the basis of the entire historical data set of the generated power.

### 3.2 Moving Window

An alternative approach to tackle the non stationary behavior of PV power generation is to estimate the *cdf*s of the random variables $w_m$ by using only the most recent portion of the data set.

Let $F_m^{(L)}(\omega \mid d)$ be the time-varying *cdf* describing the statistics of the random variable $w_m$ estimated from the realizations of the random variables

$$w_{m,d-1}, w_{m,d-2}, \ldots, w_{m,d-L},$$

where $d = 1, \ldots, 365$ is the day the random variable $w_m$ refers to, and $L$ is the width of the window. In this case, the optimal bid for the $m$-th hour of day $d$ is computed as:

$$C_m^* = F_m^{(L)^{-1}}\left(\frac{\bar{\lambda}}{\bar{\lambda} + \bar{q}} \mid d\right). \qquad (9)$$

This idea leads to an adaptive mechanism which aims at tracking the seasonal variations by selecting only most recent power data to estimate the *cdf*s. Differently from the approach in Section 2, in which for fixed hour of the day one has always the same *cdf*, independently of the day of the year, here the *cdf* changes every day. Figure 3 shows this adaptation process using a moving window of length $L = 20$ days.

Note that the length $L$ of the moving window must be selected as a suitable trade-off between estimation accuracy and adaptation capability. If $L$ is chosen too large, the effect of removing non stationarity by tracking the seasonal variations is not reached (the conditional *cdf* $F_m^{(L)}(\cdot \mid d)$ tends to resemble the unconditional one $F_m(\cdot)$, i.e. the *cdf* estimated using the whole dataset). On the other hand, if $L$ is chosen too small, the conditional *cdf* turns out to be statistically inaccurate, since it is estimated using few data. In the experimental results of the next section, it will be shown how to tune the width of the moving window by evaluating the performance of the bidding strategy (9) for different values of $L$.

## 4. EXPERIMENTAL RESULTS

The performance of the bidding strategies described so far is evaluated in this section using experimental data from an Italian PV plant.

The basic bidding strategy introduced in Section 2 will be denoted by OB. The bidding strategies developed in Section 3, which use different techniques to mitigate the effects of seasonality of PV power generation, will be denoted by OB+N and OB+WI for the approaches exploiting normalization and moving window, respectively. Furthermore, the results obtained with the aforementioned bidding strategies are compared with those of two additional bidding strategies fully exploiting weather forecasts.

The first one, which uses weather forecasts along with the PV power curve (4) to compute the energy bids, will be denoted by WF+PC. This intuitive approach consists in offering the forecast energy derived by substituting the forecasts of solar irradiance and air temperature into the equation of the power curve (4).

The second one, which combines the normalization technique of Section 3.1 and the use of weather forecasts for the classification of the next day, will be denoted by WF+OB+N. In other words, this alternative approach consists in training a classifier which, given energy forecasts for the next day, in the simplest implementation labels the next day as "sunny" or "cloudy", depending on the level of total daily generated energy. Then, the bid made for that day is the optimal contract computed as in (7)-(8), but using the conditional normalized PV power *cdf* of the corresponding class. The interested reader is referred to [Giannitrapani et al., 2013a] for further details.

The following data from a 825 kWp PV power plant are available:
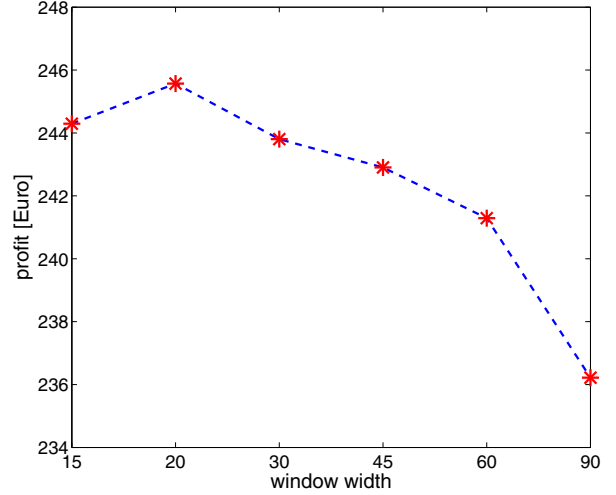
- generated power $w_m$,



Fig. 4. Average daily profit for the bidding strategy OB+WI in the market scenario where $p = 0.1027$ €/kWh and $\bar{q} = \bar{\lambda} = 0.4p$.

- solar irradiance forecasts $\hat{I}_m$,
- air temperature forecasts $\hat{T}_m$.

The number of days spanned by the data set corresponds to one year of recordings in 2012. The data set is split into a training set (about two third of the data) and a validation set, containing the remaining data.

### 4.1 Selection of the window width

The performance of the bidding strategy OB+WI depends on the selection of the window width. Figure 4 shows how the results could change significantly for different values of the parameter $L$. Here, the average daily profits have been computed over the entire year.

The optimal value of the window width is chosen by simply selecting the one providing the highest average daily profit. According to the results shown in Fig. 4, in the next simulations we set $L = 20$ days.

### 4.2 Optimal bidding strategies

For the bidding strategies OB and OB+N, the training set is used to compute the empirical *cdf*s $F_m(\cdot)$ and $F_{cs,m}(\cdot)$. Then, the bids $C_m$ are computed using (3) or (7)-(8), according to the strategy adopted. Concerning the strategy OB+WI, the bids are computed as in (9), where $F_m^{(L)}(\cdot \mid d)$ is estimated from the data gathered over the most recent $L$ days. The proposed strategies have been evaluated using the data contained in the validation data set under four market scenarios. The values of the

Table 1. Simulation setup.

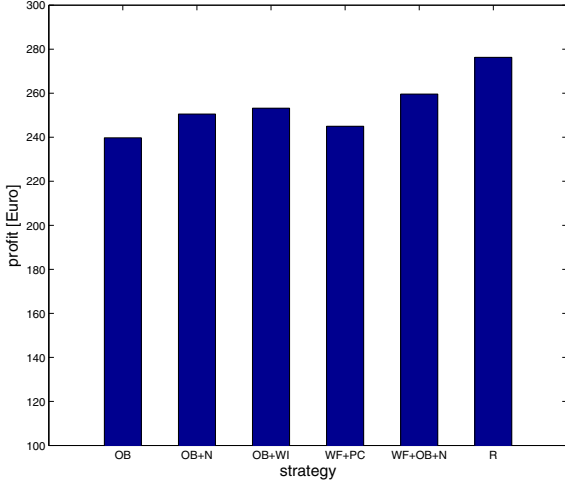|  | $\bar{q} = \bar{\lambda}$ |
| --- | --- |
| Scenario I | $0.25p$ |
| Scenario II | $0.5p$ |
| Scenario III | $0.75p$ |
| Scenario IV | $p$ |

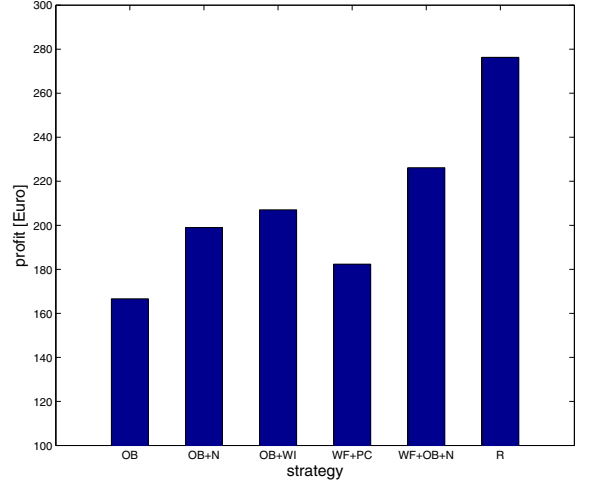Fig. 5. Average daily profits in Scenario I.
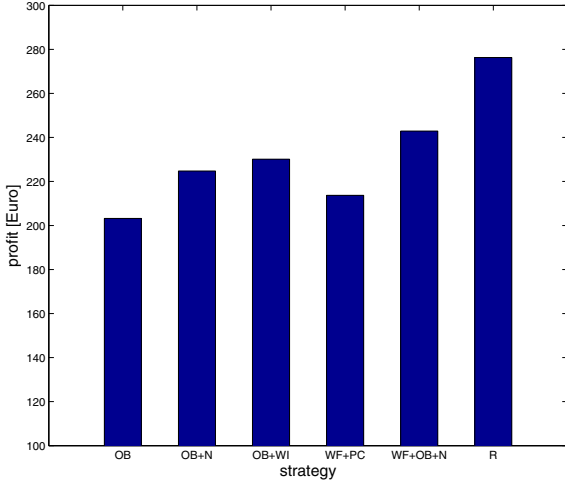


Fig. 7. Average daily profits in Scenario III.



Fig. 6. Average daily profits in Scenario II.

surplus and shortfall penalties are summarized in Table 1 (it is always assumed $\bar{q} = \bar{\lambda}$), whereas the price $p = 0.1027$ €/kWh is taken to be the same in all scenarios.

The performance achieved by the proposed bidding strategies in the four test scenarios is reported in Figs. 5, 6, 7 and 8. The bars represent the average daily profit computed over 1000 simulations. In each simulation, different training and validation data sets have been obtained by selecting the days belonging to each set randomly but without overlapping. This method avoids that presented results could be biased by a specific choice of the two data sets (e.g. the first eight months as training set and the last four months as validation set).

### 4.3 Discussion

In all scenarios, OB performs significantly worse than the bidding strategies OB+N and OB+WI. In this respect, the approaches adopted to manage the non stationary behaviour of PV power generation seem to work by enhancing consistently the results of the base line strategy OB. In
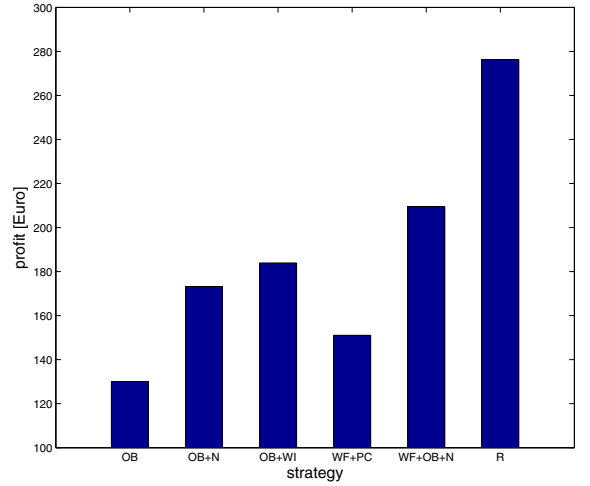


Fig. 8. Average daily profits in Scenario VI.

Scenario I the profit of the PV power producer increases up to 5.3%, in Scenario II up to 11.6%, in Scenario III up to 19.2% and in Scenario IV up to 28.7%. Note that the improvement increases with the entity of the penalty. Moreover, the strategy OB+WI performs slightly better than OB+N.

The strategy WF+PC is ranked poorly with respect to OB+N e OB+WI in all scenarios, despite using weather forecasts. Typically, such a naive approach may lead to unsatisfactory performance because it is strictly dependent on the accuracy of the weather forecasts and does not take into account the price $p$ and the penalties $\bar{\lambda}$ and $\bar{q}$. On the other hand, the strategy WF+OB+N, which overcomes the above mentioned drawbacks through a different use of weather forecasts, turns out to be the most profitable one among all the strategies presented in the paper. We stress that both WF+PC and WF+OB+N exploit the same information, i.e. the weather forecasts provided by a commercial meteorological service. It is apparent that a classification-based approach to the use of weather

forecasts turns out to be more robust to forecast errors, whereas the performance of a power curve-based approach degrades quickly as the forecast inaccuracy increases. This makes strategy WF+OB+N particularly favorable when having access to only moderately accurate weather forecasts.

The bar plots in Figs. 5, 6, 7 and 8 show the average results for each scenario. However, it is stressed that we observed profit(WF+OB+N) ≥ profit(OB+WI) ≥ profit(OB+N) ≥ profit(WF+PC) ≥ profit(OB) in 98% of the trials.

For comparison purposes, the ideal strategy R is also considered, where it is assumed that the exact generation profile of the next day is known in advance. This makes it possible to evaluate the performance of the proposed bidding strategies with respect to the maximum achievable. Although the profits go down when the penalties raise, for each scenario the bidding strategy OB+WI fills approximately 37% of the gap between OB and R, while WF+OB+N fills 54% of the same gap.

## 5. CONCLUSIONS

The optimal bidding strategy for a power producer from non-dispatchable renewable energy sources participating in a competitive market with financial penalties for generation imbalance, requires the knowledge of the cumulative distribution function of the power generation. However, when dealing with PV plants, the statistics of the power generation differ significantly over the year according to the seasonality of the solar irradiance. This work has focused on the development of suitable methodologies able to cope with the non stationary nature of PV power generation. Two approaches have been proposed. The first one aims at removing the non stationarity by normalizing the energy generated hourly with the energy obtainable under clear-sky conditions. The second one consists in tracking the actual time-varying cumulative distribution function through the use of a moving window containing the most recent generation data.

Experimental results have shown that both solutions reach comparable performance and provide an effective means to adapt the bidding strategy to the case of a PV power producer. Indeed, a significant increase of the average daily profit has been observed, with respect to the bare application of a bidding strategy which simply neglects the power generation non stationarity. Remarkably, the proposed solutions perform even better than offering the predicted power generation profile computed by substituting the day-ahead forecasts of solar irradiance and air

temperature into the equation of the PV plant power curve.

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