SIMULTANEOUS LOCALIZATION AND MAP BUILDING USING LINEAR FEATURES

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ABSTRACT

In this paper the simultaneous localization and map building (SLAM) problem, for a robot navigating in indoor environments, is addressed. A line-based representation of the environment is adopted. Line parameters are extracted from range scans, and the corresponding covariance matrices are computed from the statistical characteristics of the raw data. An Extended Kalman Filter is then used to simultaneously estimate the robot pose and update the line-based map. Feature matching is enhanced by separately keeping track of the segment associated to each line. The proposed technique is validated through numerical simulations and experimental tests, featuring the mobile robot Pioneer 3AT within a real-world indoor environment.

1. INTRODUCTION

Since long time, self-localization of mobile robots has been recognized as a fundamental issue in autonomous navigation. Typical solutions rely upon the integration of the information coming from robot sensors with the a priory knowledge of the surrounding environment *(map)*. Unfortunately, in many applications of practical relevance (like exploration tasks or operations in hostile environments), a map is not available or it is highly uncertain. In these cases, the autonomous agent must build a map of the surroundings while at the same time localizing itself within it (Simultaneous Localization And Map building, SLAM). Several solutions have been proposed, mainly differing for the environment representation and uncertainty description adopted (see [1] for a comprehensive review of map building techniques). A broad class of localization and mapping algorithms represent the environment in terms of characteristic elements detectable by the robot sensory system (*feature-based* maps). Pointwise landmarks are commonly used features, whose relative range and/or bearing w.r.t. the vehicle are measured. In this scenario, localization algorithms with known landmarks and SLAM techniques have been devised both for a statistical description of the sensor uncertainty (e.g. [2, 3]) and in a bounded-error framework (e.g. [4, 5]). A different class of widely used features, especially suited for indoor applications, are lines and segments [6, 7]. Such features can be effectively extracted from range scans and consequently exploited for localization and/or mapping purposes. In [8] an algorithm for computing the relative displacement between two different robot poses by aligning the corresponding range scans is presented. In this context, fitting lines are instrumental to the solution of the point-to-point correspondence problem. A method for building a line-based map, accounting for both robot pose and measurement uncertainty, can be found in [9]. An alternative mapping technique is proposed in [10]. It is based on an improved line extraction scheme, which explicitly takes into account each point uncertainty in the computation of the line parameters. However, both [9] and [10] do not explicitly address the SLAM problem. Recently, a segment-based SLAM algorithm exploiting 3D laser scans has been presented in [11]. It builds a 2D map of the environment, which is at same time used for localizing the robot, by projecting on the horizontal plane readings of a 3D laser range finder.

In this paper we present a SLAM algorithm, adopting a line-based description of the environment. The problem is cast as a state estimation problem for a unique uncertain dynamic system, including both the robot pose and line parameters in a global reference frame. From range scans delivered by a 2D laser range finder, linear features are extracted and the corresponding covariance matrices are explicitly computed. This information is then processed by an Extended Kalman Filter (EKF) to simultaneously estimate the current robot pose and update the map. In order to facilitate data association, the EKF is enhanced by keeping track separately of segments associated to line features.

The paper is structured as follows. The formulation of the SLAM problem is presented in Section 2. Section 3 describes the line extraction algorithm and the computation of the corresponding covariance matrix. Section 4 illustrates the matching strategy. Results of numerical simulations and experimental tests are reported in Section 5. Finally, in Section 6 possible lines of future research are outlined.

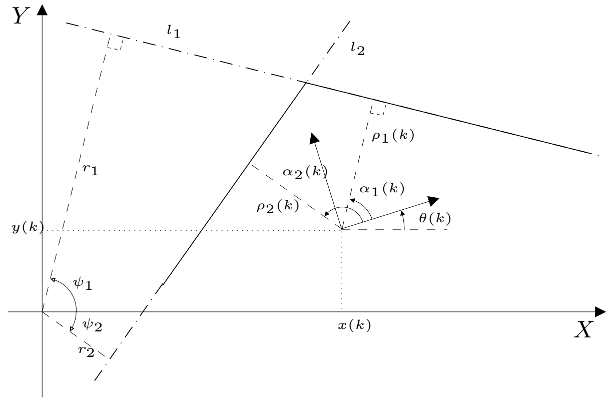

Fig. 1. Line parameters $[r,\ \psi]'$ are expressed w.r.t. the global reference frame. Line parameters $[\rho,\ \alpha]'$ are expressed w.r.t a robot-centered reference frames. $[x,\ y,\ \theta]'$ denote the robot pose w.r.t. the global reference frame.

2. PROBLEM FORMULATION

Let us consider an autonomous vehicle navigating in a 2D environment and let $p(k) = [x(k),\ y(k),\ \theta(k)]'$ denote its *pose* (i.e., the position $x(k), y(k)$ and orientation $\theta(k)$) at time k, in a global reference frame. The environment is supposed to be populated of static rectilinear features (like segments of walls, doors, shelves) detectable by the robot sensory system. In this scenario, a suitable environment representation can be given in terms of the lines underlying each feature. A line $l_i = [r_i,\ \psi_i]'$ is parametrized by its distance $r_i \geq 0$ from the origin and the direction $\psi_i \in (-\pi,\ \pi]$ of the normal passing through the origin (see Figure 1). Given a robot kinematic model and a measurement equation, the simultaneous localization and map building problem can be cast as a state estimation problem for an uncertain dynamic system. Let us suppose that the agent pose evolves according to the following LTI model[1]:

$$p(k+1) = p(k) + u(k) + w(k), \qquad k = 0, 1, \ldots \quad (1)$$

where $w(k) \in \mathbb{R}^3$ models the noise affecting the odometric measurements $u(k) \in \mathbb{R}^3$.

At each time instant k, the robot is able to measure the distance $\rho_i(k)$ and orientation $\alpha_i(k)$ of the i-th feature from its current pose $p(k)$ (see Figure 1). Let $z_i(k) = [\rho_i(k),\ \alpha_i(k)]'$, $i = 1, \ldots, n$ denote the parameter measurements of the i-th feature in a robot centered reference frame. Then, $z_i(k)$ can be expressed as a function $\mu_i(p(k), l_i)$ of the current vehicle pose $p(k)$ and the parameters l_i of the sensed feature in the global reference frame:

$$z_i(k) = \mu_i(p(k), l_i) + v_i(k), \quad (2)$$

[1]Nonetheless, the proposed framework encompasses more general kinematic models of the form $p(k+1) = f(p(k), u(k), w(k))$.

where $v_i(k) \in \mathbb{R}^2$ models the noise affecting the i-th measurement. Let us rearrange all the quantities to be estimated into a state vector $\xi(k) = [p(k)', l_i', \ldots, l_n']'$. Since static features are considered, from (1) the time evolution of the state vector is given by:

$$\xi(k+1) = \xi(k) + E_3 u(k) + E_3 w(k) \quad (3)$$

where $E_3 = [I_3\ 0_{3\times 2n}]' \in \mathbb{R}^{(3+2n)\times 3}$. Finally, if we stack the measurements taken at time k into a vector $z(k) \in \mathbb{R}^{2n}$, the measurement equation (2) can be rewritten as:

$$z(k) = \mu(\xi(k)) + v(k) \quad (4)$$

where $\mu(k) = [\mu_1(p(k), l_1)', \ldots, \mu_n(p(k), l_n)']'$ and $v(k) = [v_1(k)', \ldots, v_n(k)']'$. Consequently, the simultaneous localization and map building problem can be stated as follows.

SLAM problem. *Let $\hat{\xi}(0)$ be an estimate of the initial robot pose and feature parameters. Given the dynamic model* (3) *and the measurement equation* (4)*, find an estimate $\hat{\xi}(k)$ of the robot pose and feature parameters $\xi(k)$, for each $k = 1, 2, \ldots$.*

The main advantage of the above formulation is that any state estimation technique can be used to address the SLAM problem. In this work a statistical description of the uncertainty is adopted, so that a natural solution to the estimation problem is represented by the Extended Kalman Filter (EKF).

3. FEATURE EXTRACTION

The extraction of lines from range scans is a widely studied problem and several solutions are available (see e.g., [9], [10] and reference therein). In this section we briefly review the approach adopted in order to obtain the measurements $z(k)$, as well as the corresponding covariance matrix, from a range scan. The robot is supposed to be equipped with some kind of proximity sensor (e.g., sonar rings or laser rangefinder) providing N range and bearing measurements $[d_j,\ \phi_j]'$. The sensor readings are processed in order to extract the parameters $[\rho_h(k),\ \alpha_h(k)]'$ of the linear features present in the surroundings, by iteratively alternating *segmentation* and *line fitting* steps. Let us denote by

$$s_j = [x_j,\ y_j]' = [d_j \cos(\phi_j),\ d_j \sin(\phi_j)]', \quad (5)$$

the Cartesian coordinates of the j-th point in the robot-centered reference frame. The segmentation phase consists in partitioning the sensor readings into subsets S_h (called *segments*) of "almost collinear" points: $S_h = \left\{s_1^{(h)}, \ldots, s_{n_h}^{(h)}\right\}$, $h = 1, \ldots, q$, where n_h denotes the cardinality of the h-th segment. Each set is built iteratively. All the points s_j are processed sequentially, with the first two initializing the segment S_1. Then, a point s_j, is added to the current segment

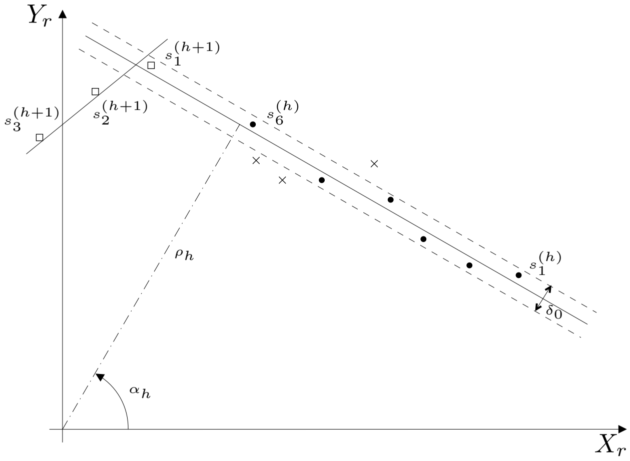

Fig. 2. Extraction of segments $S_h = \{s_1^{(h)}, \ldots, s_6^{(h)}\}$ (dots •) and $S_{h+1} = \{s_1^{(h+1)}, s_2^{(h+1)}, s_3^{(h+1)}\}$ (squares □). The points marked with a cross × are not added to S_h since their normal distance to the fitting line is greater than δ_0, whereas $s_1^{(h+1)}$ is not added to S_h because its distance from $s_6^{(h)}$ is greater than δ_1. Assuming $N_p = 3$, when three consecutive points cannot be inserted into S_h, the new segment S_{h+1} is instantiated. Assuming $N_0 = 5$, the segment S_{h+1} is discarded since it does not contain enough points.

if: i) its normal distance from the current fitting line is below a threshold δ_0 and ii) its Euclidean distance from the last point in the current segment is below a threshold δ_1.

When a new point is inserted in the current segment, the parameters of the fitting line are recomputed on the basis of the new set of points. If N_p consecutive points do not meet the above criterion, the current segment S_h is assumed to be completed, and a new one S_{h+1} is instantiated, starting from the next point with respect to the last one inserted into S_h. Notice that the condition which determines the end of a segment allows to filter out spurious readings (outliers) due to small projections, indentations or occlusions of a flat surface. Moreover, in order to increase the robustness of the extraction phase and reject false features, segments shorter than a minimum length L_0 or made up of less than N_0 points are deemed unreliable and discarded. An example of the segmentation procedure is depicted in Figure 2.

Once a segment S_h has been identified, the parameters of the corresponding linear feature $[\rho_h,\ \alpha_h]'$ are computed by fitting a line through all the points belonging to S_h. Specifically, denoting by $s_i^{(h)} = [x_i^{(h)},\ y_i^{(h)}]'$, the coordinates of the i-th point in S_h, the parameters of the fitting line are computed by minimizing the cost function:

$$E(\rho, \alpha) = \sum_{i=1}^{n_h} \left(\rho - x_i^{(h)} \cos(\alpha) - y_i^{(h)} \sin(\alpha)\right)^2 . \qquad (6)$$

The solution of the above optimization problem can be analytically computed as a function of the points $s_i^{(h)}$ concurring to define the current line. Recalling equation (5), the parameters of the h-th feature can be written as a function $g : \mathbb{R}^{2n_h} \to \mathbb{R}^2$ of the sensor readings (see [12]):

$$[\rho_h,\ \alpha_h]' = g\left(d_1^{(h)}, \phi_1^{(h)}, \ldots, d_{n_h}^{(h)}, \phi_{n_h}^{(h)}\right). \qquad (7)$$

The line parameters extracted according to the above procedure represent the measurements (4) used to update the state estimate. However, the EKF requires the knowledge of the covariance matrix of the observation noise $v(k)$. An approximation of such a matrix can be obtained, through linearization of equation (7), from the statistical properties of the errors affecting the raw data delivered by the sensor (see [12]).

4. DATA ASSOCIATION

In order to be processed by the EKF, each measurement extracted from the sensor data must be associated to the corresponding line present in the map (matching problem). Let

$$\hat{\xi}(k|k-1) = [\hat{p}(k|k-1)', \hat{l}_1(k|k-1)', \ldots, \hat{l}_n(k|k-1)']'$$

be the state estimate at time k, before the measurements are processed. Given the current state estimate $\hat{\xi}(k|k-1)$ and the observations $z_h(k)$, the matching problem consists in determining the feature l_i (if it exists) in the map, originating the h-th measurement, $h = 1, \ldots, q$. Several heuristics can be devised to this purpose, involving different comparison criteria. In this respect, the following observations have to be considered:

i) the comparison requires to express the extracted line parameters w.r.t. the global reference frame;

ii) the uncertainty affecting the parameters involved in the comparison must be taken into account;

iii) different features in the environment may lie on the same line (e.g., two aligned walls separated by a hallway).

The first issue is addressed by solving $z_h(k) = \mu_h(\hat{p}(k|k-1), l_h)$ with respect to l_h. At the same time, the covariance of the extracted parameters is propagated, through linearization, according to the covariance matrix of the current robot pose estimate. To face the last problem, for each line $\hat{l}_i(k|k-1)$ in the map the endpoints of an associated segment are computed and updated beside the EKF recursion, in order to trace the length and the position of the physical feature along the corresponding line. We stress that the segment endpoints are not included in the state vector, but are used as an instrumental tool to enhance the matching stage.

The matching algorithm proceeds through two stages. First, for each measurement $z_h(k)$ all possible associations are determined by using three validation gates. The parameters involved in this test are:

a) the squared difference of orientation, weighted by the inverse of its variance, between the extracted line and the feature estimate $\hat{l}_i(k|k-1)$;
b) the squared normal distance, weighted by the inverse of its variance, of the midpoint of the extracted segment to the line estimate $\hat{l}_i(k|k-1)$;
c) the overlapping percentage between the extracted segment and the one associated to the line estimate $\hat{l}_i(k|k-1)$.

Then, among all feasible correspondences, the one minimizing a cost function involving the quantities previously computed in a)-c) (typically a weighted sum) is selected. When a measurement $z_h(k)$ is associated to a feature $\hat{l}_i(k|k-1)$, the endpoints of the i-th segment are updated, by projecting the endpoints of the extracted segment onto the corresponding line $\hat{l}_i(k|k-1)$. If a measurement $z_h(k)$ does not match any of the lines currently present in the state vector, it has to be considered as a new feature and the state vector is properly augmented (see [12] for details). In order to avoid the introduction of spurious features in the state vector, an unmatched measurement can be first inserted into a list of tentative features and then added to the state only when it is deemed sufficiently reliable (e.g., if it is detected at least a prescribed number of times over a given length of time as suggested in [3]).

Despite the cautions taken in the extraction and matching phases, it may still happen that two initially distinct features turn out to be related to the same environment item (e.g., a long wall with temporary occlusions, or duplication of the same feature due to a poor estimation of the current robot pose). A possible way to deal with this problem is to periodically inspect the state vector and check whether any pair of features pass the aforementioned validation gates. If that is the case, the two state components can be actually considered as two estimates of the same feature, and they can be consequently merged according to their current uncertainties.

5. SIMULATION AND EXPERIMENTAL RESULTS

In order to evaluate the performance of the line-based SLAM algorithm, synthetic indoor environments have been considered first. The robot motion has been simulated according to the model (1), where the odometric errors are modelled as a white, Gaussian process with standard deviation equal to 5% of the nominal displacement. The robot is supposed to be equipped with a proximity sensor (like a laser range finder), featuring limited field of view (maximum distance $10\ m$ and scanning angle $180°$). The raw data $[d_i, \phi_i]'$ are corrupted by white, Gaussian noise with standard deviations $\sigma_{d_i} = 0.015\ m$ and $\sigma_{\phi_i} = 10^{-4}\ rad$. Figure 3 shows four snapshots of the exploration of a simple four rooms environment. The robot starts from the upper left room and covers

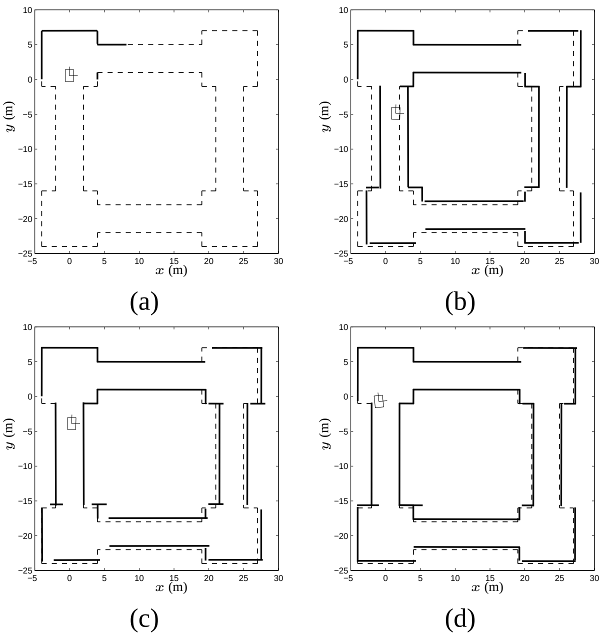

Fig. 3. Synthetic environment (dashed line) and estimated map (solid line) at different time instants: a) initial map; b) before closing the first loop; c) after closing the first loop; c) after closing the second loop.

two clockwise loops at an average speed of $0.5\ m/s$, while taking scans of the surroundings at a frequency of $2\ Hz$. Figure 3-(a) depicts the estimated map after the first laser scan; while the vehicle goes away from its initial position the estimation accuracy of the newly discovered feature degrades (Figure 3-(b)); the completion of the first loop allows to reduce the map uncertainty (Figure 3-(c)); the resulting map after closing the second loop is very close to the actual environment structure (Figure 3-(d)). Figures 4 and 5 show the evolution of the estimation errors, as well as the estimated 3σ bounds, of the robot pose and of a line present in the map, respectively. It can be noticed how the estimation uncertainty remarkably shrinks whenever a loop closure is recognized.

The proposed SLAM algorithm has been validated also on real data, gathered during several experiments performed by a mobile robot Pioneer 3AT. This vehicle has a differential drive guide and is equipped with a SICK LMS laser range finder, providing $180°$ planar scans of the environment, with a $0.5°$ resolution. The results of a typical run are shown in Figure 6. The robot explores an environment constituted by four rooms of our laboratory (labelled from (A) to (D) in Figure 6) according to the sequence: (A)-(D)-(A)-(B)-(C)-(B)-(A). The total distance travelled by the vehicle is about $170\ m$, at an average speed of $0.4\ m/s$. Along the path, the robot collects range scans at a frequency of $4\ Hz$ and accordingly updates its pose estimate and the line-based

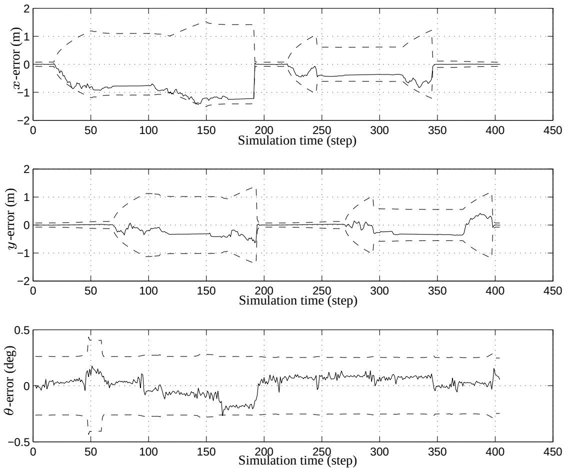

Fig. 4. Robot pose estimation error (solid line) and 3σ bounds (dashed line).

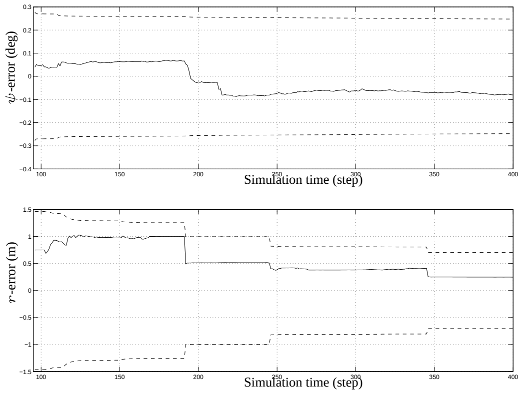

Fig. 5. Estimation error (solid line) and 3σ bounds (dashed line) of feature #15.

map. In Figure 6, the trajectory estimated by the SLAM algorithm (solid line) and the one reconstructed from encoder readings (dashed line) are depicted, together with the segments associated to the final line estimates in the map (thick solid lines). It can be noticed the poor quality of odometric estimates which after the first turns rapidly begin to drift away from the actual vehicle position; the error accumulated at the end of the run is about $4.5\ m$. The effectiveness of the SLAM algorithm to compensate for odometric errors is clear from the final position and orientation errors, smaller than $0.07\ m$ and $0.5°$, respectively. Although the ground truth is available only for the initial and final positions, nonetheless satisfactory estimation accuracy along the path can be observed, by looking at Figures 7-8 where 70 range scans taken during the experiment are plotted according to the current odometric estimate (Figure 7) or to the current SLAM algorithm estimate (Figure 8).

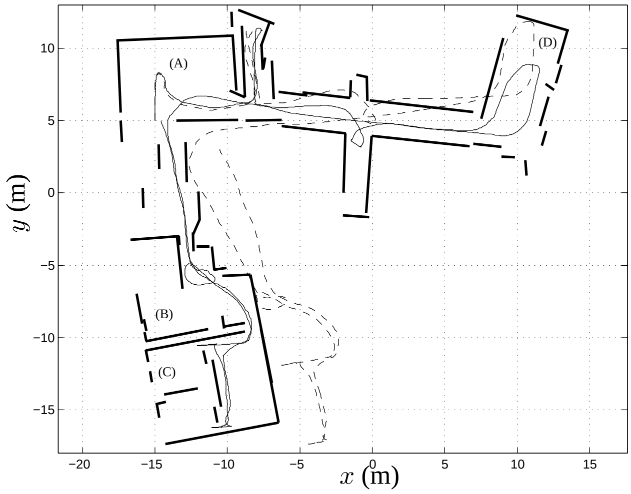

Fig. 6. Estimated trajectory by the SLAM algorithm (solid line) and by odometry (dashed); segments associated to final map (thick solid lines).

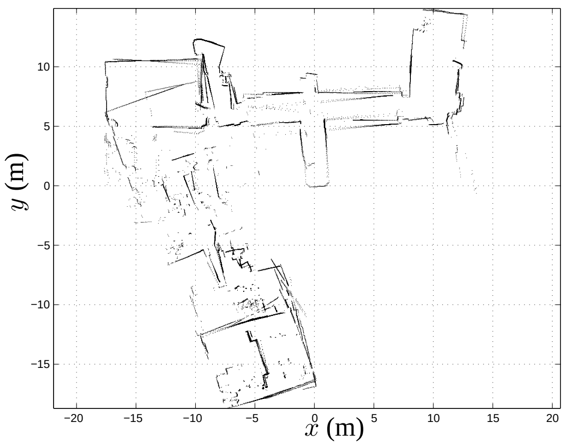

Fig. 7. Raw data relative to odometric pose estimates.

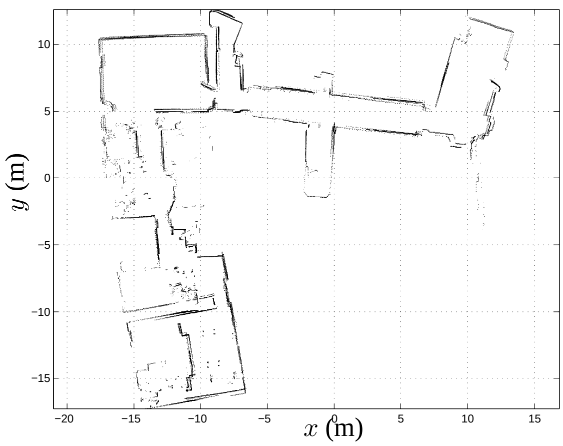

Fig. 8. Raw data relative to SLAM algorithm pose estimates.

The final map built by the SLAM technique is composed of 66 features. The parameters of the segmentation procedure have been tuned trading-off the accuracy of the extracted lines and the need to account for several unevennesses characterizing real-world features. The overall map management based on a tentative list and periodical line merging, resulted in the rejection of 48 spurious lines and the fusion of 19 map elements. It is worth noticing that the misalignment among the rooms actually resembles the shape of the building (dating back to the 15th century) and is not due to mapping faults. The presence of slightly curved walls or occlusions caused an over-segmentation of the map which in some cases, as a result of a wrong matching, generated overlapping segments corresponding to nominally different features. However, despite these drawbacks, the map accuracy proved to be suited for navigation purposes, allowing the robot to traverse back and forth different rooms without getting lost.

6. CONCLUSION AND FUTURE WORK

In this paper, a simultaneous localization and mapping technique for mobile robots navigating in indoor environments has been presented. By adopting a line-based representation of the environment, the problem is cast as a state estimation problem and it is tackled via the Extended Kalman Filter. The covariances of the extracted lines are computed directly from the uncertainty affecting the robot sensors. The results of experimental validation, carried out using the mobile platform Pioneer 3AT, confirm the viability of the proposed approach in unstructured indoor environment.

Current directions of research include the integration of additional features (e.g, corners), the adoption of different segmentation algorithms (e.g., [10]), more sophisticated matching policies and techniques to address scalability issues (e.g., [13]). The consistency of the line-based map in the long run is currently under investigation. Finally, extensions of the proposed technique to cooperative SLAM for a multi-robot system will be pursued.

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