A Local Market Model for Community Microgrids

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***Abstract*— This work presents a model of community microgrids, whose members can exchange energy and services among themselves. Pricing of energy exchanges within the community is obtained by designing an internal local market based on the marginal pricing scheme. The market aims at maximizing the social welfare of the community, thanks to the more efficient allocation of resources and the reduction of the peak power to be paid, achieved at an aggregate level. Revenues and costs are redistributed among the members, in such a way that no one is penalized within the community as compared to acting individually. The overall framework is formulated in the form of a bilevel optimization model, where the lower level problem clears the market, while the upper level problem implements the community sharing policy.**

I. Introduction

Microgrids are becoming increasingly popular nowadays, thanks to their enormous potential to make a more efficient use of resources at a local level [1], and to promote self-consumption of distributed generation [2]. The interest in microgrids has grown also in the control community, where a number of relevant problems have been addressed, such as voltage stabilization [3], reactive power compensation [4], and optimal operation planning [5], just to mention a few. Recently, participation of microgrids in frequency regulation markets has been studied in [6], where all resources are assumed to be price takers in a regulation market ruled by a regional transmission operator. Beyond to participation to external markets, aggregation of heterogenous resources into a microgrid might further be fostered by the definition of suitable pricing schemes for internal energy exchanges.

Community microgrids, where members of a community exchange energy and services among themselves, represent promising models to support socioeconomic development and community well-being [7]. In order to trade locally generated energy, suitable energy market models need to be designed. In most cases, the internal community market is managed by a third party. In [8] a virtual entity coordinates the sharing activities of a set of photovoltaic prosumers, relying on a heuristic pricing scheme. A fair benefit distribution among members of a microgrid is achieved in [9] by a central operator, who makes the best decision through a Nash bargaining model, assuming discrete price levels for market prices. In [10], a distributed market structure is proposed, where all prosumers optimize their assets individually. Optimality is achieved as prosumers are coordinated by a non-profit virtual node.

In this paper, the problem of market-based pricing of energy trades within a community microgrid is addressed. A community microgrid architecture featuring an internal market is presented. Joining the community brings several advantages to its members (in the following referred to as entities), such as the possibility to trade energy at more favorable prices and the reduction of peak power costs. The *market clearing problem*, i.e. determining the energy exchanges and the market prices within the community, is addressed by applying a social welfare maximization approach, within the marginal pricing framework, which ensures the efficient allocation of resources [11],[12]. A Pareto superior-type criterion is introduced to ensure that none of the entities is penalized with respect to acting individually. The market clearing problem, as well as the redistribution of the benefits among the community members, are formulated as a bilevel optimization problem, that can be efficiently solved thanks to its specific structure. The proposed model is illustrated on a toy example and tested on a real data set.

The paper is organized as follows. Section II describes the considered community microgrid architecture. Section III formalizes the proposed framework as a bilevel model, and discusses different strategies for its solution. Section IV reports numerical examples to illustrate the main features of the model. Finally, Section V summarizes the obtained results and outlines possible directions of future research.

Notation: The subscript t is used to denote a discrete time instant within the considered time horizon $\mathcal{T}$. The time between two consecutive time instants is denoted by Δ_T. The subscript u is used to denote an entity within the entity set $\mathcal{U}$.

II. Problem setting

A *community microgrid* is a collection of entities that share their resources and provide services to each other. An *entity* is characterized by its own generation, load and/or storage devices. When several entities are connected to the same local bus (see Fig. 1a), the community microgrid provides a virtual layer to allow local energy flows between entities, i.e. energy flows that do not cross the boundary of the local bus (see Fig. 1b).

Let $e_u^{\mathrm{gri}} \geq 0$ and $i_u^{\mathrm{gri}} \geq 0$ be the energy exported to and imported from the grid, and $e_u^{\mathrm{com}} \geq 0$ and $i_u^{\mathrm{com}} \geq 0$ be the energy exported to and imported from the community by entity u, respectively. Then, the net energy flowing from entity u to the local bus amounts to $(e_u^{\mathrm{gri}} + e_u^{\mathrm{com}}) - (i_u^{\mathrm{gri}} + i_u^{\mathrm{com}})$. Since the energy balance at the community level implies

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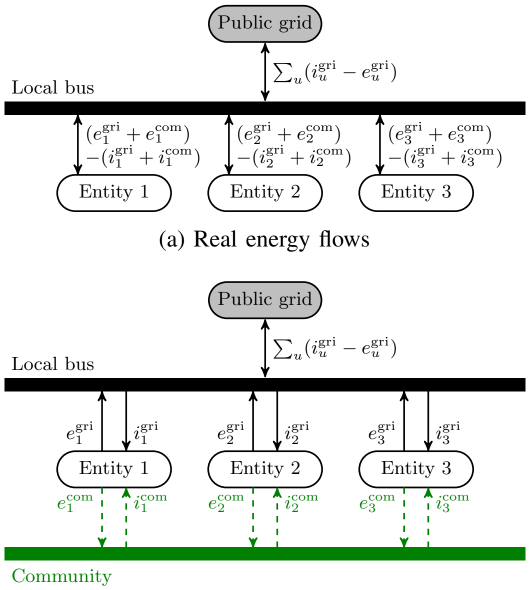


(a) Real energy flows

(b) Virtual energy flows

Fig. 1: Representations of the entities and of the community.

$\sum_{u\in\mathcal{U}}(i_u^{\text{com}} - e_u^{\text{com}}) = 0$, the net energy flowing from the grid to the local bus amounts to $\sum_{u\in\mathcal{U}}(i_u^{\text{gri}} - e_u^{\text{gri}})$. Figure 1a shows the actual energy flows in the considered distribution system, whereas the energy flows over the virtual community layer are shown in Fig. 1b.

For the energy exchanged with the external grid, namely e_u^{gri} and i_u^{gri}, entities are subject to the same mechanism as if they would not be part of a community (electricity tariffs, fees, taxes, etc.). The peak power penalty, which remunerates the distribution system operator for the capacity of the public grid, is assumed to be applied to the aggregate net energy flow $\sum_{u\in\mathcal{U}}(i_u^{\text{gri}} - e_u^{\text{gri}})$.

In a day-ahead planning stage, a *community operator* is in charge of optimizing the behavior of the entities and their interactions with the public grid, with the purpose of maximizing the social welfare of the community. In turn, the entities pay a fee to the operator for the remuneration of its activity. In the next section, an optimization model is designed in order to solve the market clearing problem and to share the corresponding benefits among the entities of the community.

For the sake of exposition, a simplified setup is considered in the following. In particular, each entity u is assumed to be equipped with a *non-flexible load* whose power consumption at time t is $C_{u,t}^{\text{nfl}}$, a *non-steerable generator* whose power generation at time t is $P_{u,t}^{\text{nst}}$ and an *energy storage device* whose charging and discharging power at time t are

$$\overline{P}_u a_{u,t}^{\text{cha}} \quad \text{and} \quad \underline{P}_u a_{u,t}^{\text{dis}}, \tag{1}$$

respectively. In (1), $\overline{P}_u$ and $\underline{P}_u$ denote the maximum charging and discharging rate of the storage, whereas $a_{u,t}^{\text{cha}}$, $a_{u,t}^{\text{dis}} \in [0,\ 1]$ are the actual charging and discharging control inputs, respectively. However, we stress that a more general setup including flexible loads, steerable generators, and demand-side models can be easily embedded in the proposed framework. Similarly, extended tariff schemes accounting for additional services, such as symmetric reserve provision, can be considered. The interested reader is referred to [13] for extensions of the model that is presented next.

III. Optimization model

The community operator is responsible for two main tasks. The first one is to clear the local energy market of the community, i.e. to determine the energy flows between the entities and the corresponding energy prices. The second task is to redistribute the profit of the community among all the entities. In Section III-A, both tasks are simultaneously formulated as a nonlinear bilevel model. Practical aspects on how to tackle the solution of the proposed bilevel model are discussed in Section III-B.

A. The bilevel model

A bilevel model is a mathematical program composed of two nested optimization problems, termed upper and lower level [14]. Formally,

$$\max_{x\in\mathcal{X}} F(x, y^*) \tag{2}$$

$$\text{s.t.} \quad y^* \in \arg\max_{y\in\mathcal{Y}} f(y; x)\,, \tag{3}$$

where F and f are the objective functions of the upper and lower level problems (2) and (3), respectively. In general, the optimizer y^* and the feasible set $\mathcal{Y}$ of the lower level depend on the unknown x of the upper level. In turn, the feasible set $\mathcal{X}$ of the upper level may depend on y^*.

In power system economics, bilevel programming is typically used to access dual variables (see, e.g., [15], [16]), which in turn represent market prices within the marginal pricing framework [11]. In the proposed bilevel model, the lower level solves the community microgrid market clearing problem by determining the local energy flows between the entities and the prices at which energy is traded within the community. The objective is to maximize the social welfare of the community. Then, the role of the upper level is to share the profit of the community among the entities, by ensuring that no entity is penalized with respect to acting individually. The latter objective is achieved by enforcing the condition

$$J_u \geq J_u^{\text{SU}} \qquad \forall u \in \mathcal{U}, \tag{4}$$

where J_u^{SU} is the optimal profit of entity u when acting individually, while J_u is the profit of entity u within the community. If at least one inequality in (4) is satisfied strictly, the state of the community is termed *Pareto superior* to the case when entities act individually.

1) Lower level problem: This section describes the lower level problem (3). The vector y of decision variables is composed of the executed quantities $e_{u,t}^{\text{gri}}$, $i_{u,t}^{\text{gri}}$, $e_{u,t}^{\text{com}}$, $i_{u,t}^{\text{com}}$, the community prices $\pi_{u,t}^{\text{com}}$ at which the local energy flows $e_{u,t}^{\text{com}}$ and $i_{u,t}^{\text{com}}$ are valued, the community peak power

$$\overline{p} = \max_{t\in\mathcal{T}} \sum_{u\in\mathcal{U}} \left(i_{u,t}^{\text{gri}} - e_{u,t}^{\text{gri}} \right) / \Delta_T, \tag{5}$$

the storage charging and discharging control inputs $a^{\text{cha}}_{u,t}$, $a^{\text{dis}}_{u,t}$, and the storage states of charge $s_{u,t}$.

The objective function f is defined as follows:

$$-\sum_{u\in\mathcal{U}}\sum_{t\in\mathcal{T}}\Big(\pi^{\text{igr}}_t i^{\text{gri}}_{u,t}-\pi^{\text{egr}}_t e^{\text{gri}}_{u,t}+\gamma^{\text{com}}\left(e^{\text{com}}_{u,t}+i^{\text{com}}_{u,t}\right)$$
$$+\gamma^{\text{sto}}_u\Delta_T\Big(\overline{P}_u\eta^{\text{cha}}_u a^{\text{cha}}_{u,t}+\frac{\underline{P}_u}{\eta^{\text{dis}}_u}a^{\text{dis}}_{u,t}\Big)\Big)-\pi^{\text{peak}}\overline{p},\quad (6)$$

where π^{igr}_t and π^{egr}_t denote the price of energy when purchased from or sold to the grid, respectively, and π^{peak} is the peak power penalty. In (6), η^{cha}_u and η^{dis}_u are the charging and discharging efficiencies of the storage devices. The fees γ^{com} and γ^{sto}_u account for the remuneration of the community operator and the use of storage. The objective function represents the social welfare of the community, composed of the costs of purchasing energy from the grid, the revenues from selling energy to the grid, the remuneration of the community microgrid operator, the costs for using storage, and the penalty paid for the community peak power.

The feasible set $\mathcal{Y}$ of the lower level problem (3) is defined by different constraints. The energy balance for each entity can be written as

$$e^{\text{gri}}_{u,t}-i^{\text{gri}}_{u,t}+e^{\text{com}}_{u,t}-i^{\text{com}}_{u,t}=\Delta_T\big(P^{\text{nst}}_{u,t}-C^{\text{nfl}}_{u,t}$$
$$+\underline{P}_u a^{\text{dis}}_{u,t}-\overline{P}_u a^{\text{cha}}_{u,t}\big)\quad\forall u\in\mathcal{U},\forall t\in\mathcal{T}.\ [\pi^{\text{com}}_{u,t}\in\mathbb{R}]\quad (7)$$

The variable $\pi^{\text{com}}_{u,t}$ between square brackets in (7) represents the dual variable of the corresponding constraint. It has an important economic interpretation within the marginal pricing framework [11]. Indeed, $\pi^{\text{com}}_{u,t}$ represents the market price at which entity u exchanges energy with the community at time t. The balance of the energy flows within the community at each time period is given by

$$\sum_{u\in\mathcal{U}}\left(i^{\text{com}}_{u,t}-e^{\text{com}}_{u,t}\right)=0\qquad\forall t\in\mathcal{T}.\quad (8)$$

Bounds on the maximum energy exchanged with the grid at each time instant can be included as

$$(e^{\text{gri}}_{u,t}-i^{\text{gri}}_{u,t})/\Delta_T\le E^{\text{cap}}_{u,t}\quad\forall u\in\mathcal{U},\forall t\in\mathcal{T}\quad (9)$$
$$(i^{\text{gri}}_{u,t}-e^{\text{gri}}_{u,t})/\Delta_T\le I^{\text{cap}}_{u,t}\quad\forall u\in\mathcal{U},\forall t\in\mathcal{T},\quad (10)$$

where $E^{\text{cap}}_{u,t}$ and $I^{\text{cap}}_{u,t}$ denote the maximum power that can be injected into or absorbed from the grid. Finally, constraint (5) is rewritten by letting the community peak power $\overline{p}$ be a free variable and by adding the constraints

$$\sum_{u\in\mathcal{U}}\left(i^{\text{gri}}_{u,t}-e^{\text{gri}}_{u,t}\right)/\Delta_T\le\overline{p}\quad\forall t\in\mathcal{T}.\quad (11)$$

A second set of constraints is related to the operation of the storage devices. The dynamics of the state of charge for each storage unit can be modelled as

$$s_{u,t}=s_{u,t-1}+\Delta_T\Big(\overline{P}_u\eta^{\text{cha}}_u a^{\text{cha}}_{u,t}-\frac{\underline{P}_u}{\eta^{\text{dis}}_u}a^{\text{dis}}_{u,t}\Big)$$
$$\forall u\in\mathcal{U},\quad\forall t\in\mathcal{T}\quad (12)$$
$$s_{u,0}=S^{\text{init}}_u,\quad s_{u,T}=S^{\text{end}}_u\qquad\forall u\in\mathcal{U}.\quad (13)$$

In (13), S^{init}_u and S^{end}_u are given parameters, representing the initial and final state of charge of storage unit u. Finally, bounds on the storage control inputs and storage state of charge are included as:

$$0\le a^{\text{cha}}_{u,t}\le 1\qquad\forall u\in\mathcal{U},\forall t\in\mathcal{T}\quad (14)$$
$$0\le a^{\text{dis}}_{u,t}\le 1\qquad\forall u\in\mathcal{U},\forall t\in\mathcal{T}\quad (15)$$
$$\underline{S}_u\le s_{u,t}\le\overline{S}_u\qquad\forall u\in\mathcal{U},\forall t\in\mathcal{T}.\quad (16)$$

In particular, constraints (16) impose that the state of charge of each storage unit cannot exceed its upper and lower bounds $\overline{S}_u$ and $\underline{S}_u$.

Remark 1: In [13], it is shown that at the optimum of the lower level problem, $e^{\text{gri}}_{u,t}$ and $i^{\text{gri}}_{u,t}$, as well as $e^{\text{com}}_{u,t}$ and $i^{\text{com}}_{u,t}$, cannot be simultaneously nonzero, i.e. no simultaneous export to and import from the grid can occur for entity u over a given time period. The same holds for the energy exported to and imported from the community by entity u. □

Summarizing, the lower level problem solves the community microgrid market clearing problem by maximizing the objective function (6), subject to the constraints (7)-(16). Notice that this is a linear program and solving the dual problem gives access to the community prices $\pi^{\text{com}}_{u,t}$, that are a key outcome of the market clearing process. An important result that can be obtained from duality relations, is that at the optimum of the lower level problem one has:

$$\gamma^{\text{com}}\sum_{u\in\mathcal{U}}\sum_{t\in\mathcal{T}}\left(e^{\text{com}}_{u,t}+i^{\text{com}}_{u,t}\right)=-\sum_{u\in\mathcal{U}}\sum_{t\in\mathcal{T}}\pi^{\text{com}}_{u,t}\left(e^{\text{com}}_{u,t}-i^{\text{com}}_{u,t}\right).\quad (17)$$

Equality (17) states that the monetary flows within the community (right-hand side) balance each other and cover the remuneration of the community operator (left-hand side). The proof of the above result can be found in [13]. In the following, the value of the objective function (6) at the optimum of the lower level problem will be denoted by J^*.

2) Upper level problem: The role of the upper level is to share among the entities the optimal profit J^* of the community, while ensuring that no entity is penalized with respect to acting individually. To do this, we let

$$J_u=J^{\text{energy}}_u+J^{\text{peak}}_u\quad (18)$$

be the total profit of entity u within the considered community microgrid framework. In (18), the quantity J^{energy}_u takes into account the revenues and costs for entity u related to energy flows:

$$J^{\text{energy}}_u=-\sum_{t\in\mathcal{T}}\Big(\pi^{\text{igr}}_t i^{\text{gri}}_{u,t}-\pi^{\text{egr}}_t e^{\text{gri}}_{u,t}+\pi^{\text{com}}_{u,t}\left(i^{\text{com}}_{u,t}-e^{\text{com}}_{u,t}\right)$$
$$+\gamma^{\text{sto}}_u\Delta_T\Big(\overline{P}_u\eta^{\text{cha}}_u a^{\text{cha}}_{u,t}+\frac{\underline{P}_u}{\eta^{\text{dis}}_u}a^{\text{dis}}_{u,t}\Big)\Big).\quad (19)$$

Notice that the energy exchanges with the community, $e^{\text{com}}_{u,t}$ and $i^{\text{com}}_{u,t}$, are valued at the price $\pi^{\text{com}}_{u,t}$, i.e., the market-clearing price for entity u at time t. Moreover, in (18) J^{peak}_u is the portion assigned to entity u of the cost paid by the community for the peak power, defined as

$$J^{\text{peak}}_u=-\pi^{\text{peak}}\overline{p}_u,\quad (20)$$

where $\overline{p}_u \geq 0$ is the contribution to community peak power $\overline{p}$ assigned to entity u, satisfying the constraint:

$$\overline{p} = \sum_{u \in \mathcal{U}} \overline{p}_u. \tag{21}$$

By summing (18) over all entities u, and exploiting (17) and (21), it is straightforward to obtain the identity:

$$J^* = \sum_{u \in \mathcal{U}} J_u, \tag{22}$$

which shows that the proposed framework totally shares the optimal profit J^* of the community among the entities.

In order to ensure that no entity is penalized with respect to acting individually, for each entity u the quantity J_u in (18) has to be compared with the value J_u^{SU}, representing the maximum profit that the entity would achieve over the time horizon $\mathcal{T}$ without joining the community. This value is computed for each entity by solving an optimization problem derived from the lower level problem of the community. Specifically, in (6)-(13), all summations with respect to $u \in \mathcal{U}$ are removed, the index u is fixed and refers to the entity considered, the energy exchanges $e_{u,t}^{\text{com}}$ and $i_{u,t}^{\text{com}}$ with the community are set to zero, and $\overline{p}$ is replaced with $\overline{p}_u$. Given the lower bounds J_u^{SU} for all entities, the requirement that all entities should benefit from joining the community, is translated into the following condition:

$$J_u \geq J_u^{\text{SU}} + \alpha, \quad \forall u \in \mathcal{U}, \tag{23}$$

where $\alpha \geq 0$ is a slack variable to be maximized. Notice that maximizing α corresponds to maximize $\min_u \left(J_u - J_u^{\text{SU}}\right)$, i.e., the minimum gain achieved by each single entity u of the community. Since $\alpha \geq 0$, condition (23) generalizes condition (4).

In order to satisfy (23), the upper level problem may act on the term J_u^{peak} by deciding the quantities $\overline{p}_u$, subject to (21). Conversely, the term J_u^{energy} of (18) is fixed by the considered solution of the lower level problem. Notice, however, that the lower level problem may have multiple solutions, and the upper level problem explores all of them.

Summarizing, in the proposed formulation, the upper level problem (2) is an optimization problem in the decision variables α and $\overline{p}_u$, with feasible set $\mathcal{X}$ defined by the constraints (21)-(23), and $\alpha \geq 0$. The objective function F of the upper level coincides with the slack variable α.

B. Solution strategies

The proposed bilevel formulation has a nice structure that can be exploited to solve (2)-(3) efficiently. In fact, the lower level problem is a linear program which does not depend on the decision variables of the upper level problem, i.e. $f(y;x) = f(y)$ in (3). Moreover, for a given solution of the lower level problem, the upper level problem is also a linear program, i.e. $\max_{x \in \mathcal{X}} F(x, y^*)$ is a linear program for any fixed y^*. This implies that the bilevel optimization problem can be solved very efficiently as the cascade of two linear programs, one corresponding to the lower level, and the other corresponding to the upper level. If the solution of the lower level problem is unique, this strategy exactly solves (2)-(3). The uniqueness of the solution of the lower level problem can be checked a priori via standard tools in linear programming [17].

If the lower level solution is not unique, the bilevel problem can be tackled by recasting it as a single optimization program. The (linear) lower level problem is replaced with its first-order necessary and sufficient Karush-Kuhn-Tucker conditions. Furthermore, the strong duality property is exploited in order to avoid nonlinearities coming from the complementary slackness conditions. However, the resulting model is still nonlinear, due to the bilinear terms $\pi_{u,t}^{\text{com}} e_{u,t}^{\text{com}}$ and $\pi_{u,t}^{\text{com}} i_{u,t}^{\text{com}}$ appearing in (19). Further details about the recasting procedure can be found in [13].

IV. Numerical results

This section reports numerical results obtained by applying the model proposed in Section III. In particular, Section IV-A describes an illustrative example to highlight the main features of the proposed approach, whereas Section IV-B reports a test case inspired by a real community microgrid in Belgium currently under development within a pilot project [18]. In both examples, the community operator fee is assumed to be $\gamma^{\text{com}} = 0.01$ €/kWh, whereas the unitary community peak cost is set to $\pi^{\text{peak}} = 0.18$ €/kW. The following settings are used for the storage devices. The initial and final state of charge in (13) are set to zero. The charging and discharging efficiencies are assumed to be $\eta_u^{\text{cha}} = 0.9$ and $\eta_u^{\text{dis}} = 0.95$, respectively. The unitary cost for using storage is set to $\gamma_u^{\text{sto}} = 0.04$ €/kWh. The absence of simultaneous storage charging and discharging is checked a posteriori. The proposed model is implemented in Python with Pyomo 5.5, and solved by using Ipopt 3.12.11 on a 8-core 2.40 GHz Intel Xeon CPU E5-2630 v3, with 32 GB of RAM.

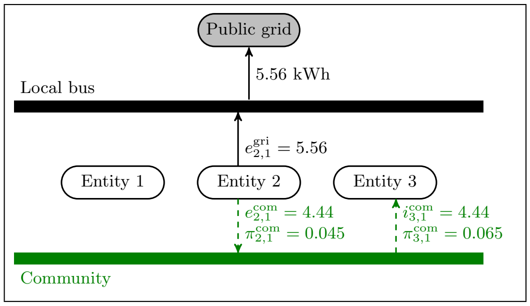
A. Illustrative example

This illustrative example encompasses two time periods, i.e. $\mathcal{T} = \{1, 2\}$. The duration of each time period is one hour, i.e. $\Delta_T = 1$ h. The community microgrid is composed of three entities with the following characteristics:

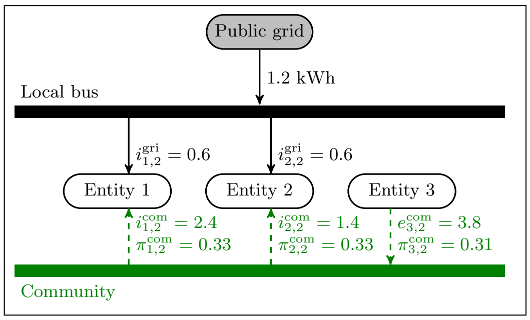
- Entity 1 is a non-flexible load with $C_{1,1}^{\text{nfl}} = 0$ kW and $C_{1,2}^{\text{nfl}} = 3$ kW;
- Entity 2 is a prosumer with non-steerable generation $P_{2,1}^{\text{nst}} = 10$ kW in period 1, and non-flexible load $C_{2,2}^{\text{nfl}} = 2$ kW in period 2;
- Entity 3 is a storage device with maximum storage capacity $\overline{S}_3 = 4$ kWh and minimum state of charge $\underline{S}_3 = 0$ kWh.

The entities can buy energy from the grid at price $\pi_t^{\text{igr}} = 0.15$ €/kWh, and can sell their energy to the grid at price $\pi_t^{\text{egr}} = 0.045$ €/kWh.

By applying the model described in Section III, clearing of the community microgrid market leads to the solution depicted in Fig. 2, where both the community market prices and the energy exchanges within the community and with the grid are reported. In particular, Fig. 2a shows the optimal solution at time period $t = 1$. Entity 2 sells $e_{2,1}^{\text{com}} = 4.44$ kWh



(a) Energy flows and prices, time period 1



(b) Energy flows and prices, time period 2

	Comm.	Entity 1	Entity 2	Entity 3
J^{MU}	-0.631	-0.882	-0.102	0.353
J^{SU}	-1.200	-0.990	-0.210	0
$J^{\text{energy,MU}}$	-0.415	-0.882	-0.102	0.569
$J^{\text{energy,SU}}$	-0.300	-0.450	0.150	0
$J^{\text{peak,MU}}$	-0.216	0	0	-0.216
$J^{\text{peak,SU}}$	-0.900	-0.540	-0.360	0

(c) Summary of costs (< 0) and revenues (> 0)

Fig. 2: Results of the example of Section IV-A.

to the community at price $\pi^{\text{com}}_{2,1} = 0.045$ €/kWh. This energy is bought by entity 3 to fully charge its storage unit, taking into account the charging efficiency. Indeed, $\overline{S}_3 = i^{\text{com}}_{3,1}\eta^{\text{cha}}_3 = 4$ kWh. The residual energy produced by entity 2 is sold to the main grid, i.e. $e^{\text{gri}}_{2,1} = 5.56$ kWh. Therefore, the grid represents the marginal unit [12], whose bid price defines the market price for entity 2 as required by the marginal pricing framework [11], i.e. $\pi^{\text{com}}_{2,1} = \pi^{\text{egr}}_1 = 0.045$ €/kWh. Notice that entity 3 buys energy from the community at price $\pi^{\text{com}}_{3,1} = 0.065$ €/kWh. The price difference between entity 2 and entity 3 is twice the fee $\gamma^{\text{com}} = 0.01$ €/kWh. This comes from the fact that, for an entity u exporting to the community and an entity u' importing from the community in the same time period t, the following relation holds (see [13]):

$$\pi^{\text{com}}_{u',t} = \pi^{\text{com}}_{u,t} + 2\gamma^{\text{com}}. \tag{24}$$

Figure 2b shows the optimal solution at time period $t = 2$. Entity 3 sells $e^{\text{com}}_{3,2} = 3.8$ kWh to the community by fully discharging its storage unit, considering the discharging efficiency. Entity 1 satisfies its demand partly from the community ($i^{\text{com}}_{1,2} = 2.4$ kWh), and partly from the grid ($i^{\text{gri}}_{1,2} = 0.6$ kWh). The price paid by entity 1 reflects all the costs incurred by the entity, including the peak power costs. Indeed, $\pi^{\text{com}}_{1,2} = \pi^{\text{igr}}_2 + \pi^{\text{peak}}/\Delta_T = 0.15 + 0.18 = 0.33$ €/kWh. The same relation holds for the price paid by entity 2, hence $\pi^{\text{com}}_{2,2} = 0.33$ €/kWh. Entity 2 satisfies its demand buying $i^{\text{gri}}_{2,2} = 0.6$ kWh from the grid and $i^{\text{com}}_{2,2} = 1.4$ kWh from the community. The market price $\pi^{\text{com}}_{3,2} = 0.31$ €/kWh for entity 3 is finally obtained from (24). In the proposed framework, it is possible to show that the prices $\pi^{\text{com}}_{3,1}$ and $\pi^{\text{com}}_{3,2}$ for entity 3 are further related by the following condition (see [13]):

$$\pi^{\text{com}}_{3,2} = \frac{\pi^{\text{com}}_{3,1}}{\eta^{\text{cha}}_3\eta^{\text{dis}}_3} + 2\frac{\gamma^{\text{sto}}_3}{\eta^{\text{dis}}_3} + \frac{\varphi^{socUP}_{3,1}}{\eta^{\text{dis}}_3}, \tag{25}$$

where the first term in the right-hand side accounts for the charging cost considering the round-trip efficiency, whereas the second term refers to usage costs. The variable $\varphi^{socUP}_{3,1}$ in the third term represents the dual variable (or shadow price) of the constraint $s_{3,1} \leq \overline{S}_3$. Complementary slackness implies that, when this constraint is active (i.e. when the storage unit is fully charged), the associated dual variable can be strictly positive. As a consequence, the selling price can rise significantly due to the storage unit becoming a scarce resource. In this example, $\varphi^{socUP}_{3,1} = 0.142$ €/kWh, and the third term in (25) accounts for almost half of the total selling price. By contrast, if the storage unit were not fully charged, then $\varphi^{socUP}_{3,1} = 0$. In this case, the selling price would represent the value allowing entity 3 to recover exactly its costs.

Table 2c reports the revenues and costs when the entities participate in the community (MU), and when they act individually (SU). It is interesting to observe that the peak power cost of the community (fifth row in Table 2c) is totally assigned to entity 3, even though entity 3 is not importing energy from the grid. This is a consequence of the allocation policy enforced by the community operator in the upper level. Among all the feasible solutions, the one that maximizes the minimum gain of the entities is selected. Different allocation policies could be imposed, depending on the preferences of the community members. Notice that, as can be observed by comparing the first and the second row in Table 2c, all the entities improve their conditions by participating in the community as compared to acting individually. In particular, entity 3 gains 0.353 €, whereas both entity 1 and entity 2 gain 0.108 €. Indeed, $\alpha = 0.108$ € at the optimum of the upper level problem.

B. Real Test Case

This section reports a test case inspired by a real pilot project currently under development in Belgium [18], where the community microgrid is composed of four entities:

- Entity 1 is a non-flexible load with an average demand of 23 kW;
- Entity 2 is a prosumer with an average demand of 17 kW and an average generation of 4 kW;
- Entity 3 is a prosumer with an average demand of 9 kW and an average generation of 75 kW;

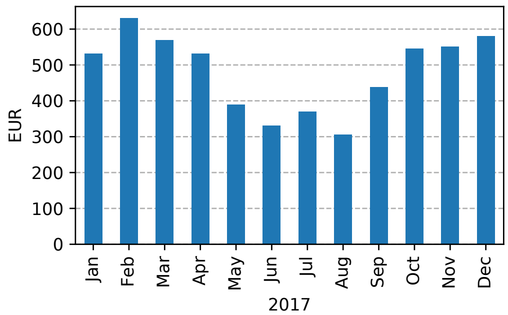


Fig. 3: Fees collected by the community operator in 2017.

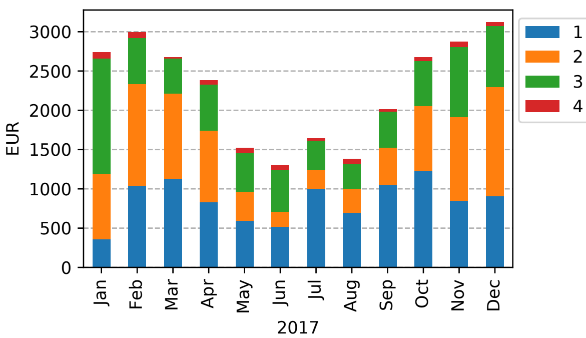


Fig. 4: Monthly gains for each entity and for the community.

- Entity 4 is a storage device with maximum capacity $\overline{S}_4 = 270$ kWh and $\eta_4^{\text{cha}} = \eta_4^{\text{dis}} = 0.95$.

Demand and generation profiles, as well as energy price profiles, refer to the whole year 2017 with time step $\Delta_T =$ 15 min. For each day of the considered year, one instance of the model proposed in Section III is solved. Every instance comprises $T = 96$ time steps. The average computation time is 18.08 s per instance.

Figure 3 shows the fee collected by the community operator as a remuneration for its activity in each month. The total amount collected over the whole year is $5,778$ €. We recall that the community operator collects $\gamma^{\text{com}} = 0.01$ €/kWh on energy imported from and exported to the community by each entity. Since γ^{com} is constant, the amount collected by the community operator is directly proportional to the total energy traded by the entities within the boundaries of the community.

Finally, Fig. 4 shows the stacked bar plot of the monthly gains of all the entities. The height of each bar represents the monthly gain of the community as a whole. As can be observed, all the entities gain from participating in the community, though with different amounts. On average, the first three entities enjoy 52% gain as compared to acting individually, whereas the total yearly gain for entity 4 is limited to 623 €. Recalling Section IV-A, this can be explained as the storage unit is seldom a scarce resource.

V. Conclusions

This paper described a bilevel programming formulation of the internal market of a community microgrid. Pricing of energy exchanges within the community is achieved by adopting a social welfare maximization approach based on the marginal pricing scheme. A Pareto superior-type condition ensures that no entity is penalized by participating in the community, as compared to acting individually. This is deemed a fundamental requirement for building a solid and long-lasting community. The proposed approach looks promising, as confirmed by the numerical results obtained on a real test case implemented in Belgium.

Future work aims at enhancing the formulation of the upper level problem, in order to achieve a better sharing of the benefits of the community among all the entities. Integration of community microgrids with their internal markets into existing electricity markets, will be also investigated.

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