Algorithms for placement and sizing of energy storage systems in low voltage networks

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***Abstract*— This paper addresses the problem of optimal placement and sizing of distributed energy storage devices in a low voltage network. The objective is to find the configuration which minimizes the total cost of storage devices, which depends both on the number of storage devices and on their size. The optimal power flow framework is adopted for formulating the overall optimization problem. Since the exact problem turns out to be intractable in realistic applications, we adopt a semidefinite programming relaxation for the power flow constraints and different heuristics for circumventing the combinatorial problem of selecting the most appropriate buses where to allocate the storage devices. The overall procedure is tested on a real application involving a portion of an Italian low voltage network.**

I. Introduction

Energy storage systems (ESS) have been recognized since several years as an instrumental tool in modernizing electricity grids in view of the numerous benefits they bring, such as stable and robust operation of the network, increased penetration of renewables, reduction of emissions (see [1], [2] for general surveys). Most of the recent papers dealing with optimal allocation and sizing of ESS in the network tackle the problem as the optimization of a cost function, subject to the non convex power flow constraints (see [3], [4] and references therein). Since the resulting problem is NP hard, three classes of approaches have been developed to reduce its complexity.

The first one adopts the linearized DC power flow (DC OPF), which is an approximation of the true power flow, especially useful when dealing with high voltage networks [4], [5], [6]. Contributions within the second class consider the full AC power flow (AC OPF) and look for appropriate convex relaxations of the non convex exact problem to circumvent the NP hardness issue [3], [7], [8], [9]. The main advantage of these approaches consists in an acceptable computational burden when dealing with scenarios of interest in real applications, in spite of the fact that the storage dynamics correlates the OPF problems at each time instant of the considered horizon. The third class of approaches formulate the optimal placement and operation of ESS by adopting the full AC OPF, either in a deterministic or stochastic setting, and propose procedures for the solution of the NP hard problem based on genetic or particle swarm optimization algorithms [10], [11]. These approaches provide good quantitative results for specific problems, while they can incur prohibitive computational burden in more general realistic applications.

In this paper, we deal with the problem of optimal placement and sizing of ESS in a Low Voltage (LV) network. The structure of LV networks makes DC OPF approaches not appropriate for tackling the problem. On the other hand, the typical high number of nodes involved in a LV network would make unfeasible the application of techniques based on the full non convex AC OPF. For this reason, our contribution falls in the second class of approaches described above. One peculiarity of the proposed approach is to look for the ESS configuration which minimizes the overall cost, which depends both on the number of devices to be installed and on their sizes. In doing this, we optimize the operation of the ESS by minimizing a cost function which also accounts for the cost of line losses. Since the exact problem translates to a mixed integer nonlinear programming problem, which is clearly intractable for realistic applications, we devise a heuristic strategy for facing the combinatorial nature of the problem, while using convex relaxations based on semidefinite programming for approximating the exact OPF problem. The devised algorithm is applied to a real example involving a portion of an Italian LV network.

The paper is organized as follows. Section II introduces the network model. In Section III we formulate the considered placement and sizing problem, and describe the proposed heuristic approach to cope with the computational complexity of the optimization problem. Section IV reports experimental results on a real LV network, while conclusions are drawn in Section V.

II. Network model

We consider LV distribution networks with radial layout, which is the most common situation in LV public distribution systems of many countries worldwide. The corresponding network model is described in this section, including equations and constraints characterizing loads, distributed generators and storage units connected to the network.

A. Network equations and constraints

Consider a radial LV network described by a tree $(\mathcal{N}, \mathcal{E})$, where $\mathcal{N} = \{1, 2, \ldots, n\}$ is the set of nodes (*buses*) and $\mathcal{E}$ is the set of edges (*lines*). The admittance-to-ground at bus i is denoted by y_{ii}, while the line admittance between nodes i and j is denoted by y_{ij}. Obviously, $y_{ij} = y_{ji}$. If $(i, j) \notin \mathcal{E}$, i.e. buses i and j are not connected by a line, $y_{ij} = 0$. The network admittance matrix $Y = [Y_{ij}] \in \mathbb{C}^{n\times n}$

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is a symmetric matrix defined as

$$Y_{ij} = \begin{cases} y_{ii} + \sum_{h \neq i} y_{ih} & \text{if } i = j \\ -y_{ij} & \text{otherwise.} \end{cases} \tag{1}$$

Consider discrete time steps denoted by $t = 1, 2, \dots$. The complex voltage and current injection at bus k and time t are denoted by $V_k(t)$ and $I_k(t)$, respectively. Moreover, $S_k(t) = P_k(t) + \jmath Q_k(t)$ denotes the complex power injection at bus k and time t, where $\jmath$ is the unit imaginary number, and $P_k(t)$ and $Q_k(t)$ are the active and reactive power injections, respectively. The power balance equation at bus k reads as

$$S_k(t) = V_k(t) I_k^*(t) = V_k(t) \sum_{j=1}^{n} V_j^*(t) Y_{kj}^*, \tag{2}$$

where $(\cdot)^*$ denotes the complex conjugate operator, and $I_k(t) = \sum_{j=1}^{n} Y_{kj} V_j(t)$.

Bus 1 is assumed to be the slack bus, representing the interconnection with the MV network. The slack bus is characterized by fixed voltage magnitude and phase. Voltage magnitude and phase at all other buses in the set $\mathcal{N}^L = \{2, \dots, n\}$ are determined by the network. For $k \in \mathcal{N}^L$, voltage quality requirements impose the voltage magnitude to remain within specified limits, i.e.

$$\underline{v}_k \leq |V_k(t)|^2 \leq \overline{v}_k, \tag{3}$$

where $\underline{v}_k < \overline{v}_k$ are given bounds.

B. Loads, distributed generators and storage units

The active and reactive power demand at bus k and time t are denoted by $P_k^D(t)$ and $Q_k^D(t)$, respectively. Similarly, the active and reactive power generation at bus k and time t are denoted by $P_k^G(t)$ and $Q_k^G(t)$. The set of buses having generation is denoted by $\mathcal{G} \subseteq \mathcal{N}^L$. A storage unit can be also connected to a bus $k \in \mathcal{N}^L$. Let $e_k(t)$ be the energy level of the storage device placed at bus k and time t. The storage level dynamics is modelled by the following first-order difference equation:

$$e_k(t+1) = e_k(t) + r_k(t)\Delta t, \tag{4}$$

where $r_k(t)$ is the average active power pumped into the storage unit at time t, and Δt is the time step. Note that $r_k(t)$ can take both positive and negative values (charging and discharging, respectively). The initial condition for the storage level is assumed to be known:

$$e_k(1) = e_k^1. \tag{5}$$

Moreover, both $r_k(t)$ and $e_k(t)$ are bounded as follows:

$$\underline{R}_k \leq \quad r_k(t) \quad \leq \overline{R}_k \tag{6}$$

$$0 \leq \quad e_k(t) \quad \leq E_k, \tag{7}$$

where $\underline{R}_k < 0$ and $\overline{R}_k > 0$ are the ramp rate limits, and E_k is the storage capacity installed at bus k. Similarly to (6), bounds $\underline{B}_k < \overline{B}_k$ can be imposed on the average reactive power $b_k(t)$ exchanged by the storage unit:

$$\underline{B}_k \leq b_k(t) \leq \overline{B}_k. \tag{8}$$

The set of buses having storage is denoted by $\mathcal{S} \subseteq \mathcal{N}^L$. For a bus k having loads, generators and storage units connected to it, the active and reactive power $P_k(t)$ and $Q_k(t)$, corresponding to the real and imaginary parts of (2), respectively, have the following general expressions:

$$P_k(t) = P_k^G(t) - P_k^D(t) - r_k(t) \tag{9}$$

$$Q_k(t) = Q_k^G(t) - Q_k^D(t) - b_k(t). \tag{10}$$

For $k \in \mathcal{N}^L \backslash \mathcal{G}$, $P_k^G(t) = Q_k^G(t) = 0$, while for $k \in \mathcal{N}^L \backslash \mathcal{S}$, $r_k(t) = b_k(t) = 0$. For the slack bus, it is assumed that

$$P_1^D(t) = Q_1^D(t) = r_1(t) = b_1(t) = 0, \tag{11}$$

while $P_1^G(t)$ and $Q_1^G(t)$ are determined by the active and reactive power balance in the network. In general, for $k \in \mathcal{N}^L$, the quantities $P_k^D(t)$, $Q_k^D(t)$, $P_k^G(t)$ and $Q_k^G(t)$ are assumed to be independent of the network state, and considered as exogenous inputs to the power flow problem. Recall that generators in LV networks are mostly of photovoltaic (PV) and micro-wind type. These small generators are typically connected to the network through grid-tie inverters. High-quality modern grid-tie inverters feature a fixed power factor close to 1, so that $Q_k^G(t) \simeq 0$. This justifies the fact that, except for the slack bus, buses with generation are treated in this paper as load buses[1].

III. Problem formulation and solution strategies

A comprehensive analysis for planning the deployment of distributed storage systems in the grid should consider *simultaneously* the optimal number, placement and size of ESS units in the distribution network. In order to reduce the complexity of the problem to be tackled, the analysis can be accomplished through two nested loops. In the inner loop, for a given ESS location, the size of each storage unit is computed by solving an OPF problem. In the outer loop, the optimal number and placement of storage devices is derived by taking into account the network topology, as well as the relative weight of their fixed and variable costs.

A. Storage sizing

Assume that the set $\mathcal{S}$ is given, i.e. the number and location of ESS units in the network have been decided. The idea is that, in the planning phase, one looks for the minimum total storage capacity allowing one to satisfy the voltage constraints (3) over a time horizon $\mathcal{T} = \{1, \dots, T\}$. Optimization variables in this problem are the storage capacity E_k and the real and reactive power $r_k(t)$ and $b_k(t)$ exchanged by each storage unit. Conversely, demand and generation profiles are assumed to be known.

In order to take into account line losses, which are of primary interest in distribution networks, we consider the cost function

$$J(C_S, C_L) = \gamma C_S + (1-\gamma) C_L, \tag{12}$$

[1] In the power flow literature, generator buses would be typically characterized by the fact that generated active power and voltage magnitude are prespecified, while generated reactive power and voltage phase are determined by the network state.

where $C_S = \sum_{k\in\mathcal{S}} E_k$ is the total installed storage capacity, $C_L = \sum_{t\in\mathcal{T}} \sum_{k\in\mathcal{N}} P_k(t)$ represents the total line losses and $\gamma \in [0,\ 1]$. Hence, the storage sizing problem can be casts a:

$$\min_{P_1(t),Q_1(t),V_k(t),r_k(t),b_k(t),E_k} J(C_S, C_L) \quad \text{s. t.} \quad (3)-(11), \quad k\in\mathcal{N}^L,\ t\in\mathcal{T}. \tag{13}$$

Problem (13) is non convex, and therefore difficult to solve. A standard way to proceed is to compute an approximated solution through convex relaxations based on semidefinite programming (see, e.g., [12]).

B. Storage placement

In this section, we propose a procedure for finding the "best" allocation of a given number of storage devices in a radial distribution network, based on clustering and sensitivity analysis (CSA). The underlying idea consists in splitting the network into n_c independent subnetworks and then, if needed, finding the most appropriate bus within each cluster in which to deploy an ESS. The proposed placement algorithm can be summarized in the following three steps.

CSA algorithm

1) *Network clustering.* The objective is to partition the original network $\Upsilon = (\mathcal{N}, \mathcal{E})$ into n_c disjoint subnetworks $\Upsilon_i = (\mathcal{N}_i, \mathcal{E}_i)$, $i = 1, \ldots, n_c$, where $\mathcal{E}_i \subseteq \mathcal{E}$, $\bigcup \mathcal{N}_i = \mathcal{N}$ and $\mathcal{N}_i \cap \mathcal{N}_j = \emptyset$, if $i \neq j$. To this aim, one of the many available graph partitioning algorithms can be used. In this work we adopt a spectral method for graph clustering, which minimizes the sum of the weights associated to each broken line [13]. The clustering algorithm is run on an auxiliary *weighted* graph $\tilde{\Upsilon} = (\tilde{\mathcal{N}}, \tilde{\mathcal{E}})$, consisting of a complete graph built over the same node set of Υ, i.e. $\tilde{\mathcal{N}} = \mathcal{N}$ and $\tilde{\mathcal{E}} = \mathcal{N} \times \mathcal{N}$. In order to associate a weight to each edge $(h, k) \in \tilde{\mathcal{E}}$, we introduce the *sensitivity matrix* $\Psi \in \mathrm{R}^{n\times n}$ whose entries are defined as

$$\Psi_{hk} = \partial |V_k| / \partial P_h, \quad h, k \in \mathcal{N}. \tag{14}$$

The value Ψ_{hk} is a measure of how much the voltage at bus k is sensitive to injection of active power at bus h. The rationale behind this choice is that the graph partitioning algorithm will tend to group together buses which are tightly coupled, while edges connecting nodes with low effect of power injection on voltage variation are likely to be removed. The outcome of the clustering algorithm is a partition $\mathcal{N}_i$, $i = 1, \ldots, n_c$, of the node set $\mathcal{N}$, from which the desired subnetworks $\Upsilon_i = (\mathcal{N}_i, \mathcal{E}_i)$ can be constructed by taking as edge set $\mathcal{E}_i \subseteq \mathcal{E}$ all the pairs $(h_i, k_i) \in \mathcal{E}$ such that both $h_i \in \mathcal{N}_i$ and $k_i \in \mathcal{N}_i$.
2) *Candidate buses*. For each subnetwork Υ_i, $i = 1, \ldots, n_c$, a set Ω_i of candidate buses is identified including all the buses where ESS can be allocated. Empirical evidence shows that when only under-voltage problems are present in a passive radial network, the buses featuring the maximum sensitivity are the leaf nodes. This is in agreement with the intuition that in a passive network, the best placement for a storage device is at the end of the distribution lines, where larger voltage drops are experienced. Similarly, when only over-voltage problems have to be compensated for, e.g., due to the presence of renewable generators in the network, the most effective ESS placement strategy is to install the storage devices directly on the same buses as the generators. In this sense, leaf nodes and generator nodes can be regarded as *critical nodes*. However, when both kinds of contingencies occur and/or only a small number of ESS can be deployed, a trade-off has to be found. From the above observations, we introduce the following definition. A node is a candidate bus if either it is a critical node or it lies on the path connecting a pair of critical nodes. Formally, let $\mathcal{L}_i \subseteq \mathcal{N}_i$ be the set of nodes in subnetwork Υ_i which are leaf nodes in the original network Υ. Let $\mathcal{G}_i \subseteq \mathcal{N}_i$ be the set of renewable generator nodes in subnetwork Υ_i. Then, the set of candidate buses for the i-th subnetwork is

$$\Omega_i = \mathcal{L}_i \cup \mathcal{G}_i \cup \{k_i : \exists l_i \in \mathcal{L}_i, \exists g_i \in \mathcal{G}_i, k_i \in \Pi(l_i, g_i)\},$$

where $\Pi(l_i, g_i)$ denotes the path connecting nodes l_i and g_i in Υ_i. Notice that, there may exist pairs of critical nodes which are not connected by a path completely contained in Υ_i, since the resulting subnetworks may have more than one connected component. Moreover, the set Ω_i can be empty, if Υ_i does not contain any critical node.
3) *Bus selection.* For each subnetwork Υ_i, $i = 1, \ldots, n_c$, such that $\Omega_i \neq \emptyset$, a criterion to select the best node among all candidate buses needs be defined. The sensitivity matrix (14) comes in handy once more, by suggesting several alternative options. For instance, adopting a worst-case approach, one may want to maximize the minimum effect that a power injection at a given node has on the voltage of all other buses belonging to the same subnetwork, in order to increase the controllability of the voltage in the system. This amounts to place the ESS for subnetwork Υ_i at the node

$$k_i^* = \arg\max_{k_i\in\Omega_i} \min_{h_i\in\mathcal{N}_i\setminus\{k_i\}} \Psi_{h_i k_i}. \tag{15}$$

The above procedure aims at covering the whole network, i.e., implicitly assumes that under- or over-voltage problems may occur at any bus. This is not typically the case, since in actual networks some buses are more prone than others to voltage fluctuations, depending on the network topology and the load profiles. Of course, if any of the subnetworks resulting from the clustering phase should not contain any node affected by voltage problems, that cluster can be safely left without ESS, thus skipping steps 2 and 3 in the above procedure. This observation, together with the fact that some Ω_i can be empty due to the lack of critical nodes in the corresponding subnetwork, leads to the conclusion that the final number of ESS n_s selected at the end of the

CSA algorithm is not larger than the number of clusters n_c, i.e. in general $n_s \leq n_c$ (usually, strictly smaller).

The clustering algorithm takes the parameter n_c as input. In view of the previous discussion, this can be seen as an upper bound on the final number of ESS that will be deployed in the network. As such, n_c must be selected by trading-off fixed and variable costs of ESS deployment. A small number of devices results in smaller installation and maintenance costs, but typically requires a larger total energy capacity to be installed in order to face all possible network contingencies with fewer storage systems. In this respect, a possible tuning strategy for the number of clusters is to repeat the CSA algorithm for increasing values of n_c, find the optimal size of each device by solving problem (13) and then evaluating the corresponding total deployment cost as

$$C_T(n_c) = c_f n_s + c_v J, \tag{16}$$

where n_s is the number of ESS resulting from running the CSA algorithm with n_c number of clusters, J is the cost function defined in (12), c_f accounts for the fixed cost related to a single device and c_v is the unitary cost associated to J.

Remark 1: The sensitivity matrix Ψ is at the heart of the proposed procedure. Typically, the expression of the bus voltages as a function of the power injected at each node cannot be computed analytically. Hence, a good estimation of Ψ is key for the success of the algorithm. One possibility is to resort to numerical simulation for approximating (14). To this aim, nominal power injections at each bus can be computed by averaging available load and generation profiles over time. Then, the voltage variation at node k after a unit power injection at node h can be computed by solving a power flow problem. While it is true that the single values of (14) do depend on the particular load profile considered, it turns out that the relative weight of the entries of Ψ is pretty much independent of the load and determined mostly by the network topology and admittances of the lines.

IV. Experimental Results

We demonstrate the strategy for ESS placement and sizing described in Section III, on a representative portion of a real LV network whose topology was provided by the largest Italian DSO. The test network is shown in Fig. 1. It consists of 17 buses and 16 lines. A total of 26 loads and 4 PV power generators are connected to the network. For all loads and generators, five typical days of active and reactive power profiles, with sampling time $\Delta t = 15$ min, are extracted from the available data. These profiles are perturbed to originate both over- and under-voltage problems. This means that, in the absence of storage units installed in the network, voltage magnitudes violate the voltage quality constraints (3) at certain buses and time steps. The objective of the ESS placement and sizing problem is to determine the minimum-cost ESS configuration which allows one to satisfy voltage quality constraints at all buses over the considered time horizon.

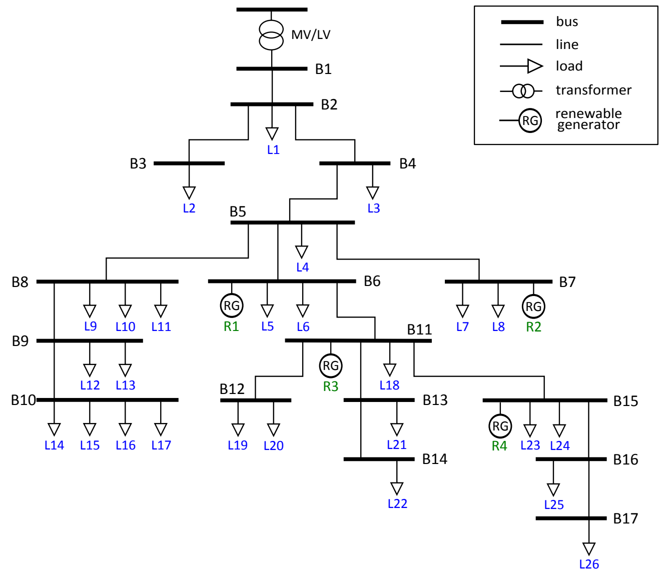

Fig. 1. Test LV network with 17 buses.

A. Simulation parameters

For all buses $k \in \mathcal{N}^L$, 10% tolerance around the nominal rated voltage is allowed in both directions, i.e. $\underline{v}_k = 0.9$ pu and $\overline{v}_k = 1.1$ pu. Storage units installed at different buses are assumed to be characterized by the same technical parameters. Ramp rate limits in (6) are chosen such that $\overline{R}_k = -\underline{R}_k = 25$ kW, whereas the bounds $\underline{B}_k$ and $\overline{B}_k$ in (8) are set to keep the angle shift between -10 and +10 deg. The initial storage level is assumed to be zero for all ESS units, i.e. $e_k^1 = 0$ kWh.

B. ESS placement and sizing

The sensitivity matrix Ψ needed by the CSA algorithm of Section III-B is computed numerically as described in Remark 1. For fixed number of clusters n_c, a suitable number of storage units n_s to be installed and the corresponding locations are determined by applying the CSA algorithm. Table I shows the results obtained for $n_c = 6$. It can be observed that clusters Υ_1, Υ_3 and Υ_4 do not contain critical buses, and therefore no ESS unit is assigned to them. The same holds for cluster Υ_2, in spite of the fact that it is composed by a single critical bus, namely bus 3. This is motivated by the fact that bus 3 is never affected by voltage problems, and therefore it can be reliably left without ESS. The procedure ends with $n_s = 2$ storage units allocated at buses 7 and 11, selected from the sets of candidate buses Ω_5 and Ω_6 through (15). The number of ESS n_s as a function of the number of clusters n_c is shown in the bottom part of Fig. 2. It can be observed that n_s is typically strictly less than n_c. When each node in $\mathcal{N}^L$ forms a cluster, i.e. $n_c = 16$, only 8 buses are deemed worth hosting an ESS.

For fixed n_c, once the CSA algorithm has provided a suitable number n_s of ESS units to be installed in the network and a corresponding set of locations $\mathcal{S}$, the capacity of the storage units has to be determined. To this aim, a convex relaxation of the OPF problem (13) based on

TABLE I

RESULTS OF THE APPLICATION OF THE CSA ALGORITHM FOR $n_c = 6$.

Subnetwork	$\mathcal{N}_i$	$\mathcal{L}_i \cup \mathcal{G}_i$	Ω_i	k_i^*
Υ_1	$\{2\}$	$\emptyset$	$\emptyset$	–
Υ_2	$\{3\}$	$\{3\}$	$\{3\}$	–
Υ_3	$\{4\}$	$\emptyset$	$\emptyset$	–
Υ_4	$\{5\}$	$\emptyset$	$\emptyset$	–
Υ_5	$\{7, 8, 9, 10\}$	$\{7, 10\}$	$\{7, 10\}$	7
Υ_6	$\{6, 11, 12, 13, 14, 15, 16, 17\}$	$\{11, 12, 14, 15, 17\}$	$\{6, 11, 12, 13, 14, 15, 16, 17\}$	11

semidefinite programming is implemented in the modelling toolbox CVX [14] and solved using SeDuMi [15]. The choice $\gamma = 1/2$ is made in the cost function (12). In order to reduce the computational burden, a practical approach is to solve the convex relaxation of (13) separately for every day in the available data set, and then take for each ESS the largest storage capacity determined in the different runs. The proposed approach is justified by the fact that load and renewable power generation profiles often exhibit daily cyclic behavior. In order to link coherently consecutive days, additional constraints are added in (13), imposing storage levels at the beginning and the end of the day to be equal:

$$\sum_{t \in \mathcal{T}_d} r_k(t) = 0, \quad \forall k \in \mathcal{S}, \tag{17}$$

where the time horizon $\mathcal{T}_d$ covers one day. Note that the original OPF problem (13) could be infeasible, if the ESS units allocated in $\mathcal{S}$ do not help solving voltage problems arising in the network for any value of the storage capacity. If this is the case, the day for which the optimization problem is solved is termed *infeasible*. The number of infeasible days provides useful information for the final decision on the ESS units to be placed in the network.

The selection of the number of clusters leading to the optimal number and locations of ESS, is carried out as described in Section III-B. For the evaluation of the total deployment cost (16), we choose $c_f = 8000$ € and $c_v = 5$ €. The total deployment cost (16) as a function of the number of clusters n_c is shown in the top part of Fig. 2 (blue dashed curve). In the same plot, the green solid curve represents the percentage of infeasible days as a function of n_c. It is apparent that a convenient choice is $n_c = 6$, to which corresponds the minimum total deployment cost, and no infeasible days occur. The corresponding number of ESS units is $n_s = 2$ (bottom part of Fig. 2). The two storage units are placed at buses 7 and 11 (see Table I).

C. Comparisons with alternative heuristics

The proposed CSA algorithm is compared to two alternative storage allocation strategies, in terms of the total deployment cost c_T. The first strategy, denoted by DS, consists in optimizing the total storage capacity under the assumption that a storage unit is available at each bus. This amounts to solve problem (13) with $\mathcal{S} = \mathcal{N}^L$. Since the optimal solution computed by DS is typically not sparse, i.e. the storage capacity $E_k \neq 0$ for most of the buses, the number

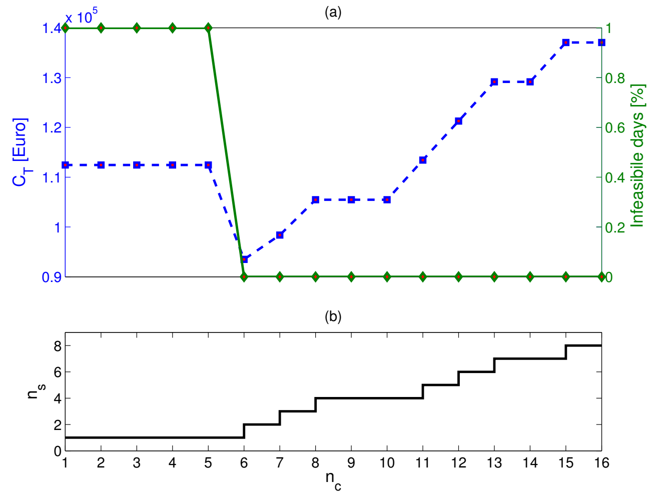

Fig. 2. (Top) Total deployment cost (blue dashed) and percentage of infeasible days (green solid) as a function of the number of clusters. (Bottom) Number of ESS units as a function of the number of clusters.

of allocated devices is high and hence the resulting total cost C_T is usually prohibitive. Nonetheless, the performance of DS provides a useful lower bound on the optimal value of the cost function (12) that can be achieved, thus allowing one to quantify the performance degradation incurred when the CSA algorithm is used. In the considered case study, it can be noticed (see Fig. 3) that $n_c = 6$ (the smallest n_c corresponding to $n_s = 2$) allows one to fill around 80% of the gap between the lower bound and the cases in which only one ESS is deployed in the network ($n_c \leq 5$). This confirms that $n_c = 6$ is a suitable choice in the considered application. The second strategy, denoted by FB, exploits the solution provided by DS in order to determine the placement and sizing of a given number of storage devices. If n_s storage units have to be allocated, FB first builds the set of storage buses $\mathcal{S}_{FB}$ by taking the n_s buses featuring the largest storage capacity E_k in the solution given by DS. Then, the optimal size of each storage unit is computed by solving problem (13) with $\mathcal{S} = \mathcal{S}_{FB}$. The total cost C_T associated to the DS strategy is equal to 201 k€ and, as expected, greatly exceeds that of the CSA solution, for $n_c = 6$ clusters and resulting in $n_s = 2$ storage units. For $n_s = 2$, strategy FB yields a total cost $C_T = 99$ k€ and an average performance index $J(C_S, C_L) = 1.66 \cdot 10^4$, which have to be compared with $C_T = 93.5$ k€ and $J(C_S, C_L) = 1.54 \cdot 10^4$ provided by

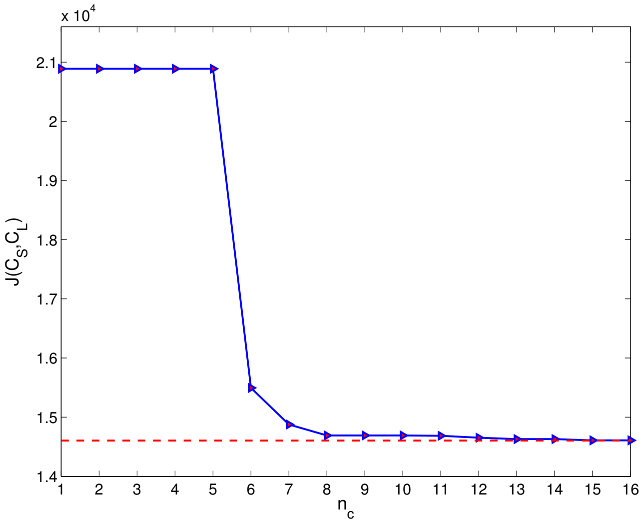

Fig. 3. Average value of the cost function $J(C_S, C_L)$ for the CSA algorithm for different values of n_c (blue solid curve) and lower bound provided by strategy DS (red dashed curve).

CSA algorithm. Strategies FB and CSA achieve comparable results in terms of J and C_T. However the storage placement of FB (buses 6 and 15), results in 20% of infeasible days (one out of the five considered), even for $n_s = 5$. The main reason is related to the fact that the FB placement process neglects the network topology, leaving a subset of the network without ESS. In the considered example, the subnetwork composed of the buses $\{8, 9, 10\}$, which suffers from under-voltage problems, can be helped only if an ESS is placed in one of its buses. Unfortunately, the FB solution selects a storage unit in the bus 10 only for $n_s > 5$.

V. Conclusions

The paper addressed the problem of optimal placement and sizing of distributed storage devices in a low voltage electricity grid. The novel contribution presented consists in a procedure allowing for the minimization of the total cost of the storage systems rather than the global storage capacity. From the optimization model viewpoint, this extension taking into account both the storage capacity and the number of storage devices to be installed, translates to a mixed integer non convex problem. A heuristic procedure has been proposed to circumvent the integer nature of the optimization problem, while a semidefinite programming based relaxation was adopted to approximate the underlying OPF. Application of the devised procedure on a portion of a LV Italian network shows very interesting and promising results. Ongoing work is devoted to investigating robustness of the selected allocation strategy to unexpected load losses or generator outages.

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