# Wind power bidding in a soft penalty market

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***Abstract*— In this paper we consider the problem of offering wind power in a market featuring soft penalties, i.e. penalties are applied whenever the delivered power deviates from the nominal bid more than a given relative tolerance. The optimal bidding strategy, based on the knowledge of the prior wind power statistics, is derived analytically by maximizing the expected profit of the wind power producer. Moreover, the paper investigates the use of additional knowledge, represented by wind speed forecasts provided by a meteorological service, to make more reliable bids. The proposed approach consists in exploiting wind speed forecasts to classify the day of the bidding into one of several predetermined classes. Then, the bids are represented by the optimal contracts computed for the selected class. The performance of the optimal bidding strategy, both with and without classification, is demonstrated on experimental data from a real Italian wind farm, and compared with that of the naive bidding strategy based on offering wind power forecasts computed by plugging the wind speed forecasts into the wind plant power curve.**

## I. Introduction

In recent years, the interest in generating power from renewable energy sources (RES) has grown rapidly, pushed by the expected benefits both in environmental and economic terms (mainly reduction of $CO_2$ emissions and energy market prices [1]). On the other hand, RES integration in the grid is causing serious problems to transmission and distribution system operators [2]. Due to the intrinsic RES intermittency and variability, system operators need to procure large and costly quantities of reserve power in order to guarantee robust network operation. This may hinder the expectations of cheaper final prices applied to consumers.

One possible way to mitigate the uncertainty of RES generation is to require that producers provide day-ahead generation profiles, and to apply penalties if the delivered power differs from the nominal bid. In other words, RES producers will be soon called to take part of the risk intrinsic in the uncertainty of the intermittent production. In Italy a regulatory framework with soft penalties is active since January 1, 2013. In this framework, penalties are applied whenever the delivered power deviates from the nominal bid more than a given relative tolerance. In a first phase, 20% tolerance is allowed, while in a second phase the tolerance will be reduced to 10%. This calls for the development of suitable bidding strategies enabling the producers to offer the right amount of power without incurring penalties. In this paper, we address the above problem in the case of Wind Power Producers (WPPs).

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The problem of designing optimal WPP bidding strategies has been addressed in [3], [4], [5], [6], [7], and very recently in [8], where the authors derive explicit formulae for optimal contracts in a market where penalties are applied whenever the delivered power deviates from the schedule. Motivated by the aforementioned Italian regulatory framework, as in [8] we consider in this paper the problem of maximizing the expected profit of a WPP, but in a market where a tolerance interval around the nominal value of the bid is allowed, and penalties are applied if the delivered power falls outside this interval. We derive analytically the optimal bidding strategy based on the knowledge of the prior wind power probability distribution, and show that the optimal contract in [8] can be recovered from ours when the tolerance tends to zero.

The second contribution of this paper is to investigate the use of additional information, represented by wind speed forecasts provided by a meteorological service, to derive better suited day-ahead bids. Although wind speed forecasts may be inaccurate for wind power prediction (see the survey paper [9] for a review of techniques for wind speed and wind power forecasting), still they can give a rough indication about the wind conditions of the next day (e.g. windy or still day, implying high or low wind power generation). We show how to use such information to classify the next day into one of several energy classes and then size the optimal contract by using the *conditional* wind power probability distribution of the selected class.

The paper is organized as follows. In Section II we formulate the bidding problem and derive the optimal bidding strategy. The use of wind speed forecasts to make more reliable bids is investigated in Section III, where the approach combining classification and the optimal bidding strategy of Section II is described. Section IV reports experimental results obtained under different pricing scenarios with data from a real Italian wind farm. Finally, conclusions are drawn in Section V.

## II. Optimal bidding strategy

In this section, the problem of optimizing the bids of wind power for a market featuring soft penalties is formulated. The optimal solution is then derived, in terms of the wind power statistics and the imbalance penalties.

Let $w_m$, $m = 1, \ldots, M$, be a discrete-time random process denoting the average active power generated by the wind power plant during the $m$-th sampling interval of the day. Let $C_m$ denote the corresponding bid of active power for the same interval and let $h$ be the sampling time (typically $h = 1$ hour). Defining $\bar{P}$ as the nominal power of the wind plant (i.e. the maximum power the plant may generate), it turns out that $w_m \in [0, \bar{P}]$ and $C_m \in [0, \bar{P}]$, for all $m = 1, \ldots, M$. Denote by $w = [w_1, \ldots, w_M]^T$ and $C = [C_1, \ldots, C_M]^T$ the vectors containing the generated power and the offered contracts for a whole day.

It is assumed that the WPP is remunerated according to the actual generated power $w$ and its deviation from the bid $C$ in the following way. Let $p$ denote the unitary price at which the WPP sells its energy, $\bar{q}$ be the unitary penalty for energy shortfall and $\bar{\lambda}$ be the unitary penalty for energy surplus. The WPP receives $p$ units of money for each unit of delivered energy $hw_m$. Let $t \in [0, 1]$ represent a given relative tolerance on the deviation of the delivered power from the nominal bid. For instance, according to the recently introduced regulations, in Italy 20% tolerance $(t = 0.2)$ is currently allowed, while in a second phase such a tolerance will be reduced to 10% $(t = 0.1)$. In case the delivered power $w_m$ is smaller than a fraction $1-t$ of the nominal bid $C_m$, i.e. $w_m < (1-t)C_m$, the WPP is penalized by $\bar{q}$ units of money for each unit of energy shortage $h((1-t)C_m - w_m)$. Similarly, if the delivered power $w_m$ is greater than a multiple $(1+t)$ of the nominal bid $C_m$, i.e. $w_m > (1+t)C_m$, a penalty of $\bar{\lambda}$ units of money for each unit of energy surplus $h(w_m - (1+t)C_m)$ is applied. Hence, the net daily profit amounts to

$$\Pi(C, w) = h \sum_{m=1}^{M} \Big( pw_m - \bar{q} \max\{(1-t)C_m - w_m, 0\} - \bar{\lambda} \max\{w_m - (1+t)C_m, 0\} \Big). \tag{1}$$

Throughout the paper the price $p$ and the penalties $\bar{q}$ and $\bar{\lambda}$ are supposed to be constant and known beforehand.

*Assumption 1:* The price $p$ and the penalties $\bar{q}$ and $\bar{\lambda}$ are such that $p > 0$, $\bar{q} \geq 0$ and $p \geq \bar{\lambda} \geq 0$.

Assumption 1 serves to rule out meaningless scenarios. Notice that $\bar{\lambda} > p$ means that the net profit for an energy surplus exceeding the threshold is actually negative. While such a scenario could be of interest in general, we can still assume $\bar{\lambda} \leq p$ without loss of generality if the WPP has curtailment capabilities.

*Remark 1:* The market scenario considered in this work can be seen as a generalization of that addressed in [8], [10]. As a matter of fact, when $t = 0$, i.e. there is no tolerance interval around the nominal bid, with a proper selection of the penalty prices, the profit (1) coincides with that considered in those papers.

Since the profit $\Pi(C, w)$ is a stochastic quantity due to the uncertainty on the generated wind power $w$, the optimal bidding problem consists in determining the bid $C^*$ maximizing the expected profit $J(C) = \mathbf{E}[\Pi(C, w)]$, i.e. $C^* = \arg\max_C J(C)$. Notice that the bid $C_m$ offered by the WPP for the $m$-th time interval depends only on the expected energy generated during the same period. Hence, two bids $C_i$ and $C_j$, $i \neq j$, related to different intervals are independent from each other [8]. As a result, the previous optimization problem, involving an $M$-dimensional optimization variable $C$, boils down to $M$ scalar optimization problems

$$C_m^* = \arg \max_{C_m \in [0, \bar{P}]} J_m(C_m), \qquad m = 1, \ldots, M, \tag{2}$$

where

$$J_m(C_m) = h\mathbf{E}\Big[\Big(pw_m - \bar{q} \max\{(1-t)C_m - w_m, 0\} - \bar{\lambda} \max\{w_m - (1+t)C_m, 0\}\Big)\Big]. \tag{3}$$

Since the optimal solution to (2) depends on the wind power statistics over the considered interval, let $F_m(\omega)$ denote the cumulative distribution function (*cdf*) of the random variable $w_m$, i.e. $F_m(\omega) = \Pr(w_m \leq \omega)$, and $f_m(\omega)$ be the probability density function (*pdf*) of $w_m$ wherever the derivative of $F_m(\omega)$ exists. Moreover, for $\beta \in [0, 1]$, let $F_m^{-1}(\beta) = \inf\{\omega \in [0, \bar{P}] : F_m(\omega) \geq \beta\}$ be the quantile function.

*Assumption 2:* The *cdf* $F_m(\omega)$ is continuous and differentiable for all $\omega \in (0, \bar{P})$. The *pdf* $f_m(\omega)$ is integrable over $(0, \bar{P})$.

The optimal solution to (2) is given by the following result.

*Proposition 1:* Under the Assumptions 1 and 2, a contract $C_m^*$ is a solution to the optimization problem (2)-(3) if and only if it satisfies the equation:

$$\bar{q}(1-t)F_m\left((1-t)C_m^*\right) = \bar{\lambda}(1+t)\left[1 - F_m\left((1+t)C_m^*\right)\right]. \tag{4}$$

The optimal expected profit is given by

$$J_m(C_m^*) = h\Big(p\mu_m + \bar{q}\int_0^{F_m((1-t)C_m^*)} F_m^{-1}(\alpha)d\alpha - \bar{\lambda}\int_{F_m((1+t)C_m^*)}^{\bar{P}} F_m^{-1}(\alpha)d\alpha\Big), \tag{5}$$

where $\mu_m = \mathbf{E}[w_m]$.

*Proof:* From the definition (3) one gets

$$J_m(C_m) = h\Big(p\mu_m - \bar{q}\int_0^{(1-t)C_m} [(1-t)C_m - \omega]f_m(\omega)d\omega - \bar{\lambda}\int_{(1+t)C_m}^{\bar{P}} [\omega - (1+t)C_m]f_m(\omega)d\omega\Big).$$

Under regularity assumptions on the *pdf* $f_m(\omega)$, the application of the Leibniz integral rule yields

$$J_m'(C_m) = h\Big( - \bar{q}(1-t)F_m\left((1-t)C_m\right) + \bar{\lambda}(1+t)\left[1 - F_m\left((1+t)C_m\right)\right]\Big). \tag{6}$$

Besides, the second derivative of $J_m$ takes on the form

$$J_m''(C_m) = h\Big( - \bar{q}(1-t)^2 f_m\left((1-t)C_m\right) - \bar{\lambda}(1+t)^2 f_m\left((1+t)C_m\right)\Big). \tag{7}$$

From Assumption 1, it follows that

$$J_m'(0) \geq 0 \quad \text{and} \quad J_m'(\bar{P}) \leq 0. \tag{8}$$

Since $J_m'(x)$ is a continuous function under Assumption 2, there exists a stationary point $\bar{x} \in [0, \bar{P}]$, i.e. $J_m'(\bar{x}) = 0$. Moreover, from (7), $J_m(x)$ is concave. Therefore, $J_m(x)$

attains its maximum over $[0, \bar{P}]$ in $\bar{x}$, i.e. $C_m^* = \bar{x}$. Condition (4) follows from (6), while, as in [8], equation (5) follows by evaluating $J_m$ at a point $C_m^*$ satisfying (4). ∎

Notice that Assumption 2 is a technical condition to permit differentiation under the integral sign when deriving equation (6) (see, e.g., [11]). The previous results can be generalized to the scenario where the penalties are stochastic variables independent of the generated power $w$, by replacing $\bar{q}$ and $\bar{\lambda}$ in (4)-(5) with their respective mean value. Given the monotonicity of (6), numerical computation of the optimal contract $C_m^*$ can be performed very efficiently through bisection.

The following corollary of Proposition 1 establishes the connection between our result for $t = 0$ and the analogous result found in [8].

*Corollary 1:* When $t = 0$, the optimality condition (4) boils down to $F_m(C_m^*) = \frac{\bar{\lambda}}{\bar{q}+\bar{\lambda}}$.

## III. Exploiting wind forecasts

In the previous section, contracts were determined assuming to know the prior wind power statistics. In this section, we assume that additional information is available, namely the wind speed forecasts $\hat{v}_m$, $m = 1, \ldots, M$, provided by a meteorological service for the day the bid refers to, and investigate how to exploit these forecasts in the bidding strategy. The most intuitive approach would be to offer the forecasted wind power profile computed using wind speed forecasts. We will discuss this approach, and show on the basis of both theoretical arguments and experimental results that offering wind power forecasts may lead to unsatisfactory performance for the WPP. With this motivation, we propose an alternative approach which combines classification methods and the optimal bidding strategy of Section II. Wind forecasts are exploited to classify the day of the bidding into one of several predetermined classes. Then, the bid is represented by the optimal contract computed as in Proposition 1 for the selected class.

### A. Offering wind power forecasts

Wind power forecasting is a challenging problem which has recently attracted increasing attention from researchers. The main difficulty is represented by the inherent intermittency of wind, which makes the prediction task very hard. The interested reader is referred to the survey paper [9] for a review and categorization of different approaches. In many cases, the focus is on wind speed forecasts, which are then converted to power through the power curve of a wind turbine. Since we assume that wind speed forecasts are provided by a meteorological service, in this section we only focus on describing mathematically the power curve of a wind turbine.

By plotting the power $w$ generated by a wind turbine versus the wind speed $v$, it can be observed that the plotted points can be very well approximated by a sigmoid function (see Fig. 1). The range of validity of the sigmoid approximation is limited below by the *cut-in* speed (i.e. the minimum wind speed at which the wind turbine generates usable power) and above by the *rated* wind speed (i.e. the minimum wind speed at which the wind turbine generates its nominal power). Between the rated and the *cut-off* wind speed (i.e. the wind speed at which protections are activated and shut down occurs), the turbine operates at its nominal power. Generated power is zero out of the range between cut-in and cut-off speed.

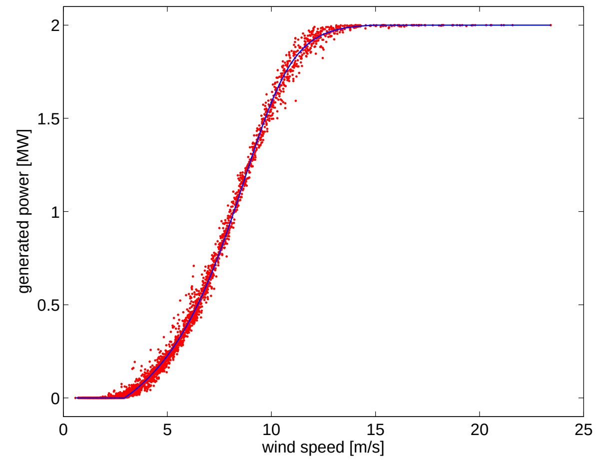


Fig. 1. Generated power vs wind speed for a 2 MW wind turbine (red points), and power curve (10) fitted to the data (solid blue curve).

There exist several expressions for sigmoid functions. The one considered in this paper has the following form:

$$P_\sigma(v) = b + (a - b)\left(1 + e^{\frac{v - v_0}{c}}\right)^d, \qquad (9)$$

where $a > 0$, $b < 0$, $c < 0$, $d < 0$ and $v_0 > 0$ represent the parameters of the sigmoid function. By using (9), the power curve of a wind turbine can be expressed as:

$$P(v) = \begin{cases} \min(\max(0, P_\sigma(v)), \bar{P}) & \text{if } v \le v_{off} \\ 0 & \text{otherwise,} \end{cases} \qquad (10)$$

where $\bar{P}$ and $v_{off}$ are the nominal power and the cut-off wind speed of the wind turbine, respectively. Note that the cut-in and rated wind speed do not appear explicitly in (10), being implicitly determined by the $\min$ and $\max$ functions. The nominal power and the cut-off wind speed can be found in wind turbine data sheets, while the parameters $a$, $b$, $c$, $d$ and $v_0$ are generally estimated from recorded measurements.

If wind speed forecasts $\hat{v}_m$, $m = 1, \ldots, M$, are available, the bid can be formed by offering the wind power forecasts computed using $\hat{v}_m$ and (10):

$$C_m = P(\hat{v}_m), \quad m = 1, \ldots, M. \qquad (11)$$

Note that, in this case, the bid is different every day, depending on the wind forecasts for that day.

### B. Day classification based on wind forecasts

The bidding strategy based on offering wind power forecasts has a number of drawbacks. First, inaccurate wind speed forecasts may induce unacceptable errors when predicting wind power through the power curve (10). This will be shown in the experimental results of Section IV. Second, and most importantly, the bids do not take into account the penalties $\bar{q}$ and $\bar{\lambda}$. This implies that offering these forecasts may not be the best one can do. For instance, consider the limit case $\bar{q} = 0$, i.e. power shortfalls are not penalized. Clearly, under this assumption the optimal strategy is to offer $C_m = \bar{P}$, $m = 1, \ldots, M$, thus having all the generated power remunerated at price $p$. Similarly, if $\bar{\lambda} = 0$, i.e. power surplus is remunerated at the same price as the bids, then the optimal strategy is to offer $C_m = 0$, $m = 1, \ldots, M$.

Motivated by the above discussion, in this paper we propose a different approach to mitigate the effects of inaccurate wind speed forecasts and, simultaneously, take explicitly into account the imbalance penalties. The idea is to combine the optimal bidding strategy described in Section II with a suitable classification strategy based on wind speed forecasts. Roughly speaking, the proposed approach consists in training a classifier which maps a day (represented by the corresponding wind speed forecasts) to one of several classes associated to different levels of daily generated energy. Then, the bid made for that day is the optimal contract computed as in Proposition 1, but using the *conditional* wind power probability distribution of the corresponding class.

Denote by $\bar{E} = 24\bar{P}$ the maximum amount of energy that the wind power plant may generate daily, and partition the interval $[0, \bar{E})$ into $s$ contiguous, non overlapping intervals $\mathcal{E}_i = [E_{i-1}, E_i)$, $i = 1, \ldots, s$, such that

$$0 = E_0 < E_1 < \cdots < E_{s-1} < E_s = \bar{E}. \tag{12}$$

The wind energy generated during day $d$ is computed as

$$E^{(d)} = h \sum_{m=1}^{M} w_m^{(d)}, \tag{13}$$

where $w_m^{(d)}$ denotes the average wind power generated during the $m$-th sampling interval of day $d$. Then, the classification rule is defined as:

$$d \in \mathcal{C}_i \quad \Leftrightarrow \quad E^{(d)} \in \mathcal{E}_i, \quad i = 1, \ldots, s, \tag{14}$$

where $\mathcal{C}_i$ represents the $i$-th day class. Clearly the delivered daily energy $E^{(d)}$ can be computed only a posteriori. Hence, since the bids must be made in advance, day $d$ is classified a priori on the basis of the corresponding wind speed forecasts $\hat{v}_m^{(d)}$, $m = 1, \ldots, M$. To this aim, we train an automatic classifier, which takes as inputs the wind speed forecasts and returns the class the day will likely belong to. Training is performed by creating a training set from past data of generated power and wind speed forecasts. First, each day $d$ of the training set is assigned to the corresponding true class $\mathcal{C}^{(d)} \in \mathcal{C} = \{\mathcal{C}_1, \ldots, \mathcal{C}_s\}$ according to (14). Then, a scalar feature $f^{(d)} \in \mathcal{F} \subseteq \mathbb{R}$ for day $d$ is computed as

$$f^{(d)} = \sum_{m=1}^{M} \left(\hat{v}_m^{(d)}\right)^3. \tag{15}$$

This choice is motivated by the fact that total wind energy flowing through a given section is proportional to the cube of the wind speed. The pairs $\left(\mathcal{C}^{(d)}, f^{(d)}\right)$, $d = 1, \ldots, D_T$, where $D_T$ is the cardinality of the training set, are used to train a classifier $H : \mathcal{F} \to \mathcal{C}$ which, given a feature $f \in \mathcal{F}$, returns a class $H(f) \in \mathcal{C}$. Several approaches can be adopted to estimate the function $H$ [12]. In this paper, since the features are scalar, we adopt the approach based on pairwise separation and Robust Linear Programming (RLP) [13].

Having the classifier $H$ available, the last step is to determine the optimal bidding strategy for each of the classes $\mathcal{C}_i \in \mathcal{C}$. This boils down to substituting the *cdf* $F_m(\cdot)$ in (4) with the conditional *cdf* $F_m(\omega \,|\, \mathcal{C}_i) = \Pr(w_m \leq \omega \,|\, \mathcal{C}_i)$ for each class $\mathcal{C}_i$, where $\Pr(\cdot \,|\, \mathcal{C}_i)$ means that the statistics is restricted only to those days belonging to the class $\mathcal{C}_i$.

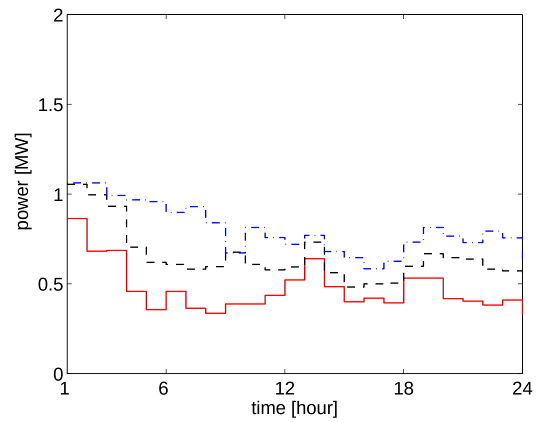

Fig. 2. Bids made with the bidding strategy OB in Scenario I: $t = 0$ (solid), $t = 0.1$ (dashed) and $t = 0.2$ (dot-dashed).

## IV. Experimental results

In this section, the bidding strategies previously introduced are validated on experimental data taken from a real Italian wind farm. We denote by OB the optimal bidding strategy of Proposition 1. The bidding strategy described in Section III-A, which uses wind speed forecasts and plant power curve to compute (and offer) wind power forecasts is denoted by WF+PC. The bidding strategy proposed in Section III-B, which combines the use of wind speed forecasts for classification and Proposition 1 is denoted by WF+OB.

The considered wind farm is composed of 35 wind turbines with nominal power $\bar{P} = 2$ MW. For each turbine, the following data are available:

- generated power $w_m^{(d)}$;
- wind speed $v_m^{(d)}$;
- wind speed forecasts $\hat{v}_m^{(d)}$,

where $m = 1, \ldots, M$, $d = 1, \ldots, D$, $M = 24$ and $D = 150$ is the number of days spanned by the data set (about 5 months of recordings). The data set is split into a training set composed of the data of the first 100 days ($D_T = 100$) and a validation set containing the data of the remaining 50 days.

For the bidding strategy OB, the training set is used to estimate the *cdf*s $F_m(\cdot)$ by building the empirical *cdf*s with the generated power $w_m^{(d)}$. Then, for fixed penalties $\bar{q}$ and $\bar{\lambda}$ and tolerance $t$, the bids $C_m$ are computed using (4). These

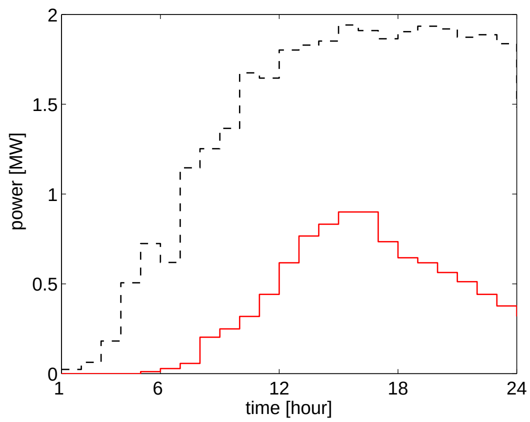


Fig. 3. Example of forecasted (solid) and generated (dashed) wind power profiles.

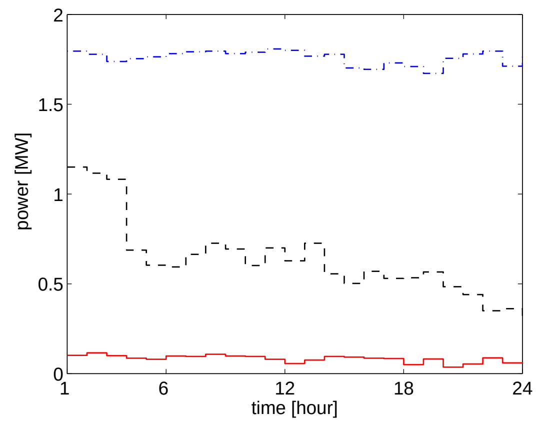


Fig. 4. Bids made with the bidding strategy WF+OB for $t = 0.1$ in Scenario I: classes $\mathcal{C}_1$ (solid), $\mathcal{C}_2$ (dashed) and $\mathcal{C}_3$ (dot-dashed).

bids are repeated every day in the validation phase. Figure 2 shows three bid profiles for different values of the tolerance $t$. Note that, as the tolerance $t$ is increased, the bids become higher and higher. Indeed, from (4), if $t$ tends to 1, $C_m$ tends to $\bar{P}$ for all indexes $m$.

Concerning the bidding strategy WF+PC, data points $(v_m^{(d)}, w_m^{(d)})$ in the training set are used to estimate the power curve (10) of each wind turbine by solving a nonlinear least squares problem. Figure 1 shows the power curve fitted to the data for one of the considered wind turbines. In the validation phase, the estimated power curve and the wind speed forecasts $\hat{v}_m^{(d)}$ are used to compute the bids $C_m$ through (11). Recall that, in this case, the bids are different from one day to another since they depend on the wind speed forecasts. Figure 3 shows an example of bid profile compared with the corresponding actual wind power profile. It can be seen that the wind power forecasts (used as bids) underestimate the actual wind power. This is quite common in the considered data set, and is due to the fact that the available wind speed forecasts are inaccurate for the site of interest (they are averaged at regional level). As a matter of fact, statistically reliable wind forecasts for lead times from 24 to 36 hours are very difficult to obtain.

In the case of the bidding strategy WF+OB, the energy range $[0, \bar{E})$, with $\bar{E} = 24\bar{P} = 48$ MWh, is partitioned into three intervals by choosing $s = 3$, $E_1 = 4$ MWh and $E_2 = 12$ MWh in (12). Training data are then used to train a classifier $H$ and to determine the bids $C_m$ for each of the three classes $\mathcal{C}_1$, $\mathcal{C}_2$ and $\mathcal{C}_3$, as described in Section III-B. The resulting classifier has the form:

$$H(f) = \begin{cases} \mathcal{C}_1 & \text{if } f < f_1 \\ \mathcal{C}_2 & \text{if } f_1 \leq f < f_2 \\ \mathcal{C}_3 & \text{if } f \geq f_2, \end{cases} \tag{16}$$

where $f_1$ and $f_2$ are thresholds depending on the wind turbine (average values are $f_1 = 534$ and $f_2 = 2802$), and the feature $f$ is computed from wind speed forecasts according to (15). Figure 4 shows the bid profiles for the three classes. Differences are apparent and consistent with the fact that the classes $\mathcal{C}_1$, $\mathcal{C}_2$ and $\mathcal{C}_3$ contain days characterized by low, medium and high energy generation, respectively. Note that bids are always the same for days associated to the same class, while they change from one day to another according to the different classes the days belong to.

The performance of the bidding strategies OB, WF+PC and WF+OB has been evaluated using the validation data set under two possible scenarios described in the following.

## A. Scenario I

In the first scenario, we set $p = 72$ €/MWh, and assume that the power exceeding the upper bound is not remunerated at all, i.e. $\bar{\lambda} = p$. Note that latter choice corresponds to the same price scenario considered in [10]. Moreover, we set $\bar{q} = 0.2p$. The average daily profits for the three bidding strategies and for different values of the tolerance $t$ are reported in Fig 5. By comparing the solid with the dashed line of the figure, it is apparent that WF+OB performs significantly better than OB for all considered values of $t$. This is not surprising and due to the fact that additional information provided by wind speed forecasts makes it possible to offer bids that are closer to the actual realization of the wind power process. Moreover, looking at the dot-dashed line of the figure, the unsatisfactory performance of WF+PC is unquestionable. As described above and shown in Figure 3, this is due to the fact that available wind speed forecasts typically underestimate the actual wind speed acting on the blades of the wind turbines, thus resulting into an underestimate of the generated power. Since the power exceeding $(1 + t)C_m$ is not remunerated at all in the considered scenario, this explains why the average daily profit guaranteed by WF+PC is so low. Notice that WF+PC yields average profits which are smaller than WF+OB (which uses the same wind speed forecasts for day classification) and even than OB, which does not use wind forecasts at all. These results also suggest that the classification strategy proposed in Section III-B is able to extract sufficient information from inaccurate wind speed forecasts to improve consistently the bidding strategy.

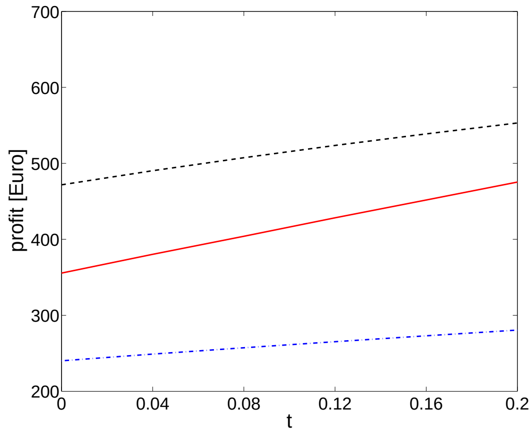


Fig. 5. Average daily profit in Scenario I: OB (solid), WF+OB (dashed) and WF+PC (dot-dashed).

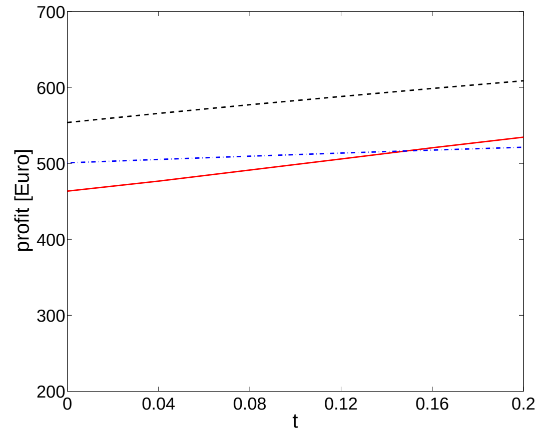


Fig. 6. Average daily profit in Scenario II: OB (solid), WF+OB (dashed) and WF+PC (dot-dashed).

### B. Scenario II

In the second scenario, we set $p = 72$ €/MWh, and assume that the power exceeding the upper bound is penalized, but nevertheless remunerated at $0.5p$, i.e. $\bar{\lambda} = 0.5p$. Moreover, we set $\bar{q} = 0.2p$ as in Scenario I. The average daily profits for the three bidding strategies and for different values of the tolerance $t$ are reported in Fig. 6. By comparing again the solid with the dashed line of the figure, it is apparent that WF+OB performs significantly better than OB for all considered values of $t$, thus confirming the benefits of the classification strategy based on wind speed forecasts on performance. Different from Scenario I, also WF+PC performs better than OB for most of the considered values of $t$. In fact, the delivered power exceeding $(1+t)C_m$ is remunerated at half the price $p$ in Scenario II, and this occurs very often since typically $w_m \gg C_m$. However, WF+OB always performs better than WF+PC, which confirms the better use of the additional information made by the former bidding strategy.

## V. Conclusions

In this paper, the problem of wind power optimal bidding has been addressed in a scenario where a fixed relative deviation from the nominal bids is explicitly tolerated by the energy market regulations. A stochastic optimization approach has been adopted, and the optimal bidding strategy has been derived. This strategy depends on the maximum admissible tolerance and boils down to the optimal strategy known in the literature when the tolerance is zero. The strategy has been embedded in a day classification approach which exploits wind speed forecasts provided by a meteorological service to classify the plant working days. A numerical comparison of different bidding strategies has been performed on real data from an Italian wind farm, showing that the approach with classification enhances consistently the performance of the bidding strategy, both with respect to the case without classification and to the case in which bids are computed simply by offering the wind power forecasts.

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