# Unicycle Steering by Brakes: a Passive Guidance Support for an Assistive Cart

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Abstract—Very often the decline of the cognitive abilities related to age determines a gradual withdrawal of older adults within the domestic walls. Part of the problem is the difficulty in navigating in large and crowded environments that are perceived as intimidating by users with declining cognitive and sensitive ability. To alleviate this problem, we propose a walking assistant endowed with autonomous sensing and active brakes able to guide the user. In this paper, we address the problem of how to guide the user with a minimal impact on her/his freedom of motion. We propose a solution based on the use of an active braking system which can be easily accommodated on commercial walkers. The paper proposes a control strategy that strikes an interesting solution to guide the user through the optimal path without restricting her/his freedom of motion.

#### I. INTRODUCTION

Very frequently people in late ages have to live with disabilities of different kind, sensory, cognitive or physical. A direct consequence of disability is a reduced mobility, which in turn exacerbates disability in a self-reinforcing loop. Several studies reveal that physical exercise ameliorates the general conditions of older adults, by increasing their physical strength and reducing the occurrence of falls [1], [2]. In this context, the goal of this research, which is part of the ongoing EU project DALi [3], is to develop a device, referred to as the c-Walker, that allows older adults to maintain confidence and mobility in such environments as would be otherwise intimidating for the emotional stress that they generate. This will increase, in our expectations, the likelihood that the user will continue to use these environments defying the anxiety that their navigation generates. The c-Walker reinvents the paradigm of the walker, extending it with sensing and cognitive abilities to acquire information on the environment in real-time, to anticipate the possible motion of other agents in the environment and to decide a safe course, where the probability of having impacts or of being trapped is minimised. The optimal path decided by the c-Walker is "suggested" to the user mainly through a mechanical guidance support (MGS), which is similar in nature to the use of force feedback in mobile haptic interfaces [4], [5].

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An important aspect of the guidance system under development is its limited intrusiveness. The assisted person remains in charge of the final decision on the direction to take and he/she can override the "suggestions" of the system. However, if he/she departs significantly from the optimal route a more authoritative action is taken by the system to avoid dangerous situations. In this paper, we show some preliminary results on the design of the MGS and on the control strategies it relies on. The system has to satisfy several requirements. First, its weight and its form factor should be chosen within the constraints of user acceptability and by avoiding any feeling of stigma associated with its use. As an example, an important feature of a walker that should be retained is the possibility of folding and transporting it in the trunk of a car. Second, the cost of the device should be limited, to allow for a mass production. Third, the type of correction made by the device should be very perceived as very soft, by limiting jerk and unnatural vibrations. In the philosophy of the DALi project, the user should retain as much as possible his/her freedom of motion and the authority of the corrective action of the MGS should be modulated with the distance from the ideal path. A possible solution that potentially allows us to cover these requirements is to use the brakes as the basis of the MGS. In this paper, we explore this paradigm and present control strategy to smoothly guide the assisted person avoiding as much as possible aggressive corrections of its trajectory. The long-term research goal is to design a shared control strategy able to trade-off the need of following a given path and a complete control of the walker by the user.

The paper is organised as follows. Section II contains a review of the work related to robotic walking assistants. Section III gives an overview of the mechanical guidance support. The path following problem maximizing the user comfort is formulated in Section IV and a possible solution is proposed in Section V. Numerical simulations are reported in Section VI to evaluate the performance of the proposed technique. In Section VII some conclusions are drawn and future directions of research are outlined.

## II. RELATED WORK

When developing robotic aids for supporting assisted people with limited motor and/or cognitive abilities the primary concern is safety. The paradigm of *passive robotics* for designing intrinsically safe devices was first introduced by Goswami et al. [6]. The authors propose a set of possible control laws that could be implemented through a mechanical wrist consisting of un-powered hydraulic cylinders and variable stiffness connections. This approach has been adopted in [7], [8] for realising the Cobot walking assistant. A Cobot is a robotic device composed of a cane with a caster wheel equipped with a servo motor for actuating the steering angle. The users supplies the motive power. Cobots usually take as reference a desired path and control the steering angle so that the force-feedback perceived by the user is negligible along the path and ideally infinite in the orthogonal direction. This way, the user is guided towards the path. Since then, a number of intelligent walking aids have been proposed in the literature, ranging from steering-only controlled walkers [9], to fully actuated assistive carts [10]. The latter features higher manoeuvrability and can actively force the assisted person to move along desired trajectories, but they can be potentially harmful if the user does not comply with the desired trajectory. The former, on the contrary, is a safer system since the walker motors can simply orient the steering wheel but the driving force is still in charge of the user. The passive walker proposed in [11] takes safety a step further, thanks to the completely lack of actuators. The device is a standard walker, with two caster wheels and a pair of electromagnetic brakes mounted on fixed rear wheels. The control of the device is based on the differential breaking principle which is at the basis of many stability control systems for cars [12]. By suitably modulating the braking torque applied to each wheel, the walker is steered toward a desired path. While this choice poses several limitations on the force and torque applicable to the cart (and hence on the achievable motion), it results in a lightweight and less expensive design thanks to the absence of motors and large batteries. The same driving principle has been recently exploited in [13] to develop novel passive walkers exhibiting more complex kinematics.

The walking assistant considered in this paper takes the model proposed in [11] as a starting point. This choice is to intake the advantage of the passive architecture in terms of safety, weight and cost. The focus of our work is however very different from that of Hirata and co-workers. Indeed, our device is not intended as a physical trainer for rehabilitation but as a cooperative moving aid. As a result, the device does not impose a strict trajectory but uses a softer strategy to gently direct the user toward the ideal path allowing him/her to move in a tunnel of configurable width. As long as the user remains inside the tunnel, he/she retains a large freedom in his/her motion.

## **III. SYSTEM OVERVIEW**

A sketch of the initial prototype of the MGS developed within the DALi project is reported in Fig. 1. The basic structure is taken from a typical commercial walker. The device is based on two front caster wheels and a pair of fixed rear wheels, equipped with mechanical brakes, which are activated using the levers mounted on the handlebars. In the current design of the c-Walker, the front wheels remain passive. The braking system will be replaced by a by–wire braking system, where the user operates on the brake through force sensors buried inside the levers; the braking signal is



Fig. 1. Initial design of the mechanical guidance support (MGS).

transduced and transmitted to the electro-actuated braking device. This way the brakes can be used independently as an MGS to guide the user across the environment. The rear wheels host incremental encoders, which can be used for the self-localization of the device, in cooperation with a exteroreceptive localization system that uses information from the environment to keep in check the error introduced by the odometry based localisation. The cart is instrumented with a vision system composed of a set of cameras and Kinect sensors, whose purpose is to collect information on the surrounding environment and on the state of the user. The collected data feed a cognitive engine which is in charge of planning an optimal course of action for the desired destination, on the basis of a number of factors including safety, environment crowd, user intentions. The interaction with the user takes place through the mechanical guidance support which can be supported by additional haptic feedback provided by ad-hoc interfaces mounted on the mobile cart.

# IV. OPTIMAL PROBLEM FORMULATION

Let  $\langle W \rangle = \{O_w, X_w, Y_w, Z_w\}$  be a fixed right-handed reference frame, whose plane  $\Pi = X_w \times Y_w$  is the plane of motion of the cart,  $Z_w$  points outwards the plane  $\Pi$  and  $O_w$ is the origin of the reference frame. Let  $\mathbf{x} = [x, y, \theta]^T \in \mathbb{R}^2 \times S$  be the kinematic configuration of the cart, where (x, y) are the coordinates of the mid-point of the wheel axle in  $\Pi$  and  $\theta$  is the orientation of the vehicle w.r.t. the  $X_w$  axis (see Figure 2). From a kinematic viewpoint, the c-Walker can be assimilated to a unicycle

$$\begin{split} \dot{x} &= v \cos(\theta), \\ \dot{y} &= v \sin(\theta), \\ \dot{\theta} &= \omega, \end{split} \tag{1}$$

where v is the forward velocity of the vehicle and  $\omega$  its angular velocity, resulting from the combined action of the



Fig. 2. Reference frames.

user thrust and the braking action. Letting d be the wheel axle length and r the radius of the wheels, we have

$$v = \frac{r}{2}(\omega_r + \omega_l), \text{ and } \omega = \frac{r}{d}(\omega_r - \omega_l),$$
 (2)

where  $\omega_r$  and  $\omega_l$  are the angular velocities of the right and left wheels, respectively. In the following, the notation  $\omega_{(.)}$  will be adopted to refer indifferently both to right and to left wheel speed. The same notation will be adopted for forces and torques acting on the wheels.

## A. Kinematic Path Following Problem

The vehicle has to converge to a desired geometric path defined in  $\Pi$  (*path following* problem). Since the desired path is the result of an on-board planning algorithm, we assume that the path is smooth (i.e., the tangent to the path is well defined in each point of the path) and that the path curvature is known. Typically, the planner yields a smooth path comprising straight segments and circular arcs.

In such a case, it is possible to design a stabilizing controller for the line segment and the circle. To this end, we assume that: a) the vehicle–to–path localization is solved (for instance, using vision apparatus) and b) a Frenet frame moving along the path, whose origin is the orthogonal projection of the vehicle position on the path, is available. Denote by s the curvilinear abscissa along the path, by l the coordinate of the vehicle position along the y-axis of the Frenet frame and by  $\theta_d$  the angle between the  $X_w$ -axis and the X-axis of the Frenet frame (see Figure 2). Define  $\tilde{\theta} = \theta - \theta_d$ , then the vehicle kinematics (1) can be rewritten as (see [14]):

$$\dot{s} = v \frac{\cos(\tilde{\theta})}{1 - c(s)l},$$
  

$$\dot{l} = v \sin(\tilde{\theta}),$$
  

$$\dot{\tilde{\theta}} = \tilde{\omega}.$$
(3)

The auxiliary input  $\tilde{\omega}$  is related to vehicle angular speed  $\omega$  through the relationship

$$\tilde{\omega} = \omega - c(s)\dot{s},\tag{4}$$

where the path curvature is defined as  $c(s) = d\theta_d(s)/ds$ . The path following problem is then solved by applying a control law that asymptotically drives l and  $\tilde{\theta}$  to zero. Assuming that the forward speed v is not always zero and that it can be measured, the control law

$$\tilde{\omega} = \tilde{\omega}_d \triangleq -lv \frac{\sin(\theta)}{\tilde{\theta}} - k_2 \tilde{\theta},\tag{5}$$

with  $k_2 > 0$ , solves the path following problem provided that the initial robot configuration is not too far from the desired path. The interested reader is referred to [14] for a thorough theoretical analysis.

## B. Dynamic Path Following Problem

In order to properly take into account the effect of the brakes acting on the cart wheels, a dynamic model is necessary. To this purpose, the linear speed v and auxiliary input  $\tilde{\omega}$  are added to the state vector. The equations governing their time evolution are

$$\dot{v} = \frac{F}{M},$$

$$\dot{\tilde{\omega}} = \frac{N}{J} - c'(s)\dot{s}^2 - c(s)\ddot{s},$$
(6)

where F is the external force acting on the vehicle along the direction of motion, N is the external torque about the  $Z_w$ -axis, M and J are the mass and moment of inertia of the cart and c'(s) = dc(s)/ds.

In this new setting, the path following problem becomes that of designing the force F and torque N such that the linear l and angular  $\tilde{\theta}$  deviations of the cart from the path asymptotically vanish. A possible solution to this problem has been proposed in [15], which additionally makes the cart track a desired linear speed profile  $v_d(t)$ . By defining  $\epsilon = \tilde{\omega} - \tilde{\omega}_d$  and  $f_1 = \dot{\tilde{\omega}}_d - \tilde{\theta} + c'(s)\dot{s}^2 + c(s)\ddot{s}$ , where  $\tilde{\omega}_d$  has been defined in (5), the control law  $F = F_d$  and  $N = N_d$ , with

$$F_d = M\dot{v}_d - (v - v_d),$$
  

$$N_d = Jf_1 - k_3\epsilon,$$
(7)

and  $k_3$  being a positive constant, makes l,  $\tilde{\theta}$  and  $v - v_d$  tend asymptotically to zero.

#### V. STEERING BY BRAKE

If the cart were driven by two independent motors exerting a torque  $\tau_l$  and  $\tau_r$  on the left and right wheel, respectively, the control law (7) could be implemented by exerting the desired torques

$$\tau_{l_d} = r \left( \frac{F_d}{2} - \frac{N_d}{d} \right),$$
  

$$\tau_{r_d} = r \left( \frac{F_d}{2} + \frac{N_d}{d} \right).$$
(8)

However, for the passive cart under investigation, the torques  $\tau_l$  and  $\tau_r$  cannot be set arbitrarily since they are the result of the human thrust  $\tau^{\star}_{(\cdot)}$  and the braking action  $\tau^{b}_{(\cdot)}$ 

$$\tau_l = \tau_l^{\star} + \tau_l^b = rF_l^h - \beta\omega_l + \tau_l^b,$$
  

$$\tau_r = \tau_r^{\star} + \tau_r^b = rF_r^h - \beta\omega_r + \tau_r^b,$$
(9)

where  $F_{(\cdot)}^h$  denotes the force exerted by the user and conveyed to the wheel hub by the mechanical structure of the cart and  $\beta$  is a damping coefficient modeling viscous friction. The braking torque  $\tau_{(\cdot)}^b$  is determined by the braking action exerted (e.g., the pressure applied to the braking surface). We assume that both the  $F_{(\cdot)}^h$  and  $\omega_{(\cdot)}$  are available. Indeed, the first one is given by a set of properly mounted accelerometers, while the latter is simply given by two incremental encoders fixed on the wheels shaft.

The idea of the passive guidance with brakes is to modulate the braking torques  $\tau_l^b$  and  $\tau_r^b$  acting on the wheels in order to make the wheel torques  $\tau_l$  and  $\tau_r$  in (9) as close as possible to the desired ones in (8). To this end, we will assume pure rolling motion without slippage of the wheels.

## A. Optimal Problem

In light of equation (9), the torques  $\tau_l$  and  $\tau_r$  are given by the solution of the following *Optimal Control by Brake* (*OCB*) problem:

$$\begin{split} \min_{\tau_l,\tau_r} c(\tau_l, \ \tau_r) \ \text{s.t.} \\ 0 &\leq \tau_{(\cdot)} \leq \tau^{\star}_{(\cdot)}, \ \text{if} \ \tau^{\star}_{(\cdot)} \geq 0 \\ 0 &\geq \tau_{(\cdot)} \geq \tau^{\star}_{(\cdot)}, \ \text{if} \ \tau^{\star}_{(\cdot)} < 0 \\ \frac{d}{2r}(\tau_r - \tau_l) &= N_d. \end{split}$$

whereas  $N_d$  is given by (7). In plain words, the first two constraints model the dissipative nature of the control by brakes approach. The cost index  $c(\tau_l, \tau_r)$  to be selected is related to the *human comfort*, which has to be intended in this paper as an inversely proportional and monotonic function of the braking action: the lower is the braking action, the lower is the control action perceived by the human. This choice has also a benefit on the autonomy of the cart since it reduces the overall power spent for the braking actuation, which is of relevance for an assistive cart. One possible choice is given by

$$c(\tau_l, \tau_r) = |\tau_l - \tau_l^{\star}| + |\tau_r - \tau_r^{\star}| = |\tau_l^b| + |\tau_r^b|,$$

i.e., among all the possible choices generating the resulting cart torque  $N_d$ , the controls that minimize the braking actions are selected.

## B. Loose Path Following

The solution to the *OCB* problem gives the vehicle torque  $N_d$  that ensures asymptotic path tracking while maximizing the human comfort. However, in order to further increase the human comfort, we may allow deviations from the planned trajectory as a function of the distance and orientation errors. An example of such a function is directly given by the Lyapunov function

$$\mathbf{V}(l,\tilde{\theta}) = \frac{1}{2} \left( l^2 + \tilde{\theta}^2 \right)$$

defined in [14] to prove stability. More in depth, the vehicle fully controls the cart, i.e., it imposes exactly the optimal solution overriding the human inputs, if  $\mathbf{V}(l, \tilde{\theta}) \geq T_d$ , where

 $T_d$  is a design parameter. In all the other cases, the braking action is a function of the error, being null when the cart is exactly on the track. For example, we may have

$$f(l, \tilde{\theta}) = \min\left(1, \frac{\mathbf{V}(l, \tilde{\theta})}{T_d}\right),$$

that leads to the *relaxed* version of the *OCB* problem by substituting the equality constraint on  $N_d$  with

$$\frac{d}{2r}(\tau_r - \tau_l) = f(l,\tilde{\theta})N_d + (1 - f(l,\tilde{\theta}))N^h, \qquad (10)$$

being  $N^h = d/(2r)(\tau_r^{\star} - \tau_l^{\star})$  the cart torque applied by the user (i.e., when the braking action is zero). Conversely, notice that letting  $f(l, \tilde{\theta}) = 1$  constantly one obtains a very firm control action, more similar in spirit to the work proposed in [11].

## C. Quadratic Formulation

A different formulation of problem OCB can be considered in order to come up with a quadratic problem, computationally easy to solve. To this end, we first notice that the parameter  $k_3 > 0$  in (7) provides an additional degree of freedom that can be used as an optimization parameter. Indeed, even if the controller changes at each iteration of the optimization algorithm, the Lyapunov function used to prove the stability does not and, hence, it can be used as a Common Lyapunov Function for this problem [16]. Moreover from (7), (8) and (10) it can be verified that the optimization variable of problem OCB are an affine function of the cart force F and the control parameter  $k_3$ 

$$\tau_l = g_f F + g_3 k_3 + g_h,$$
  

$$\tau_r = g_{\overline{f}} F - g_3 k_3 - g_h.$$
(11)

In light of the previous relationships, inequality constraints of problem *OCB* are linear with respect to F and  $k_3$ . For instance, when  $\tau_l^* \ge 0$ , we now have

$$-g_h \le g_f F + g_3 k_3 \le \tau_l^\star - g_h. \tag{12}$$

Similar linear inequalities are derived in all the other cases. In this new formulation, the optimization problem aims at minimizing the deviation of the resulting cart force F from the desired one  $F_d$  while ensuring N equal to  $N_d$ . As done in (10), such a requirement is relaxed in order to increase human comfort, thus giving the following quadratic cost function

$$U(F) = \frac{1}{2} \left( F - (f(l,\tilde{\theta})F_d + (1 - f(l,\tilde{\theta}))F^h) \right)^2, \quad (13)$$

where  $F^h$  is the resulting force exerted by the user on the cart. By setting  $\mathbf{z} = [F, k_3]^T$ , Q = diag(1, 0) and  $\mathbf{h} = [-(f(l, \tilde{\theta})F_d + (1 - f(l, \tilde{\theta}))F^h), 0]^T$  the minimization problem U(F) in (13) subject to the constraints derived from problem *OCB* can be cast in the following *Relaxed Quadratic*  Optimal Control by Brake (Relaxed QCB):

$$\begin{split} \min_{\mathbf{z}} \frac{1}{2} \mathbf{z}^T Q \mathbf{z} + \mathbf{h}^T \mathbf{z} \quad \text{s.t} \\ A \mathbf{z} \leq \mathbf{b}, \\ A_e \mathbf{z} = \mathbf{b}_e. \end{split}$$

The matrix A and the vector **b** account for the constraints written as in (12) and the bounds on the maximum and minimum values of  $k_3$ . The matrix  $A_e$  and the vector  $\mathbf{b}_e$  comes from the equality constraint (10). Notice that the *Relaxed QCB* problem is a convex optimization problem for which there exist a number of very efficient solvers.

## D. Robust Optimal Problem

The *Relaxed QCB* problem can be solved whenever the user applied forces and the wheel velocities are known. As a matter of fact, there exists several possible different choices to measure and then estimate those values. However, any estimation procedure based on noisy measurements is unavoidably affected by uncertainty. Such uncertainty reflects on the computed optimal control law and its effectiveness in a real scenario.

To take into account estimation errors, a robust optimization formulation, tractable with standard tools, can be adopted. The rationale behind the robust convex optimization is to explicitly incorporate a model of data uncertainty in the formulation of a convex optimization problem, and to optimize for the worst-case scenario under that model [17]. Based on this idea, assuming that the uncertainties in the estimates of  $\tau_r^*$  and  $\tau_l^*$  in (9) lies in the set  $[-\Delta_{\tau^*}, \Delta_{\tau^*}]$ , it is sufficient to solve the *Relaxed QCB* by subtracting from **b** a vector whose entries are all  $\Delta_{\tau}$ .

#### VI. SIMULATIONS

Simulation results are reported in this section. Based on the current prototype of cart, the wheels radius is set to r = 10 cm while the wheel axle is set to d = 50 cm. In order to evaluate the effectiveness of the proposed solution, the forces  $(F_r^h, F_l^h)$  that the user exerts on the wheels during the motion have been synthetically generated. It is assumed that the user does not try to follow the desired path, rather he/she moves freely in the surroundings. Sinusoidal forces have been simulated. In particular,  $F_r^h(t) = 1 + 1$  $\sin(\pi/10t)$  N, while  $F_l^h(t) = 0.5 + 0.5 \cos(\pi/20t)$  N. The motivation behind this choice is threefold: a) sinusoids mimic the oscillating behavior of a walking human; b) the user trajectory without braking action will be radically different from the planned path (see the inlet of Fig. 3), and hence very challenging from a control perspective; c) the amplitude of the inputs are quite small, hence reducing the set of feasible solutions for the problem. The viscous friction coefficient for the wheels is set to  $\beta = 0.01$  N m s/rad.

The effect of the braking action, computed as the solution of the *Relaxed QCB* problem formulated in Section V-C, is clearly visible in Fig. 3. The path now followed by the cart has been radically modified by the navigation assistant to



Fig. 3. Desired path (thick line) and path followed by the cart (thin line) with braking action. The inlet of this graph reports the behavior of the cart with braking action disabled.



Fig. 4. Distance errors with braking action (solid line), for fully actuated cart (dash-dotted line) and braking action with noise (dashed line).

approximate the desired one. The distance and the orientation errors of the cart from the planned trajectory can be found represented with a solid line in Fig. 4 and Fig. 5, respectively. It can be noticed how the walker is confined to a tunnel along the path, whose width is about 50 cm. As a comparison, in the same figures the tracking errors for a fully actuated cart, which is able to arbitrarily set the  $\tau_l$  and  $\tau_r$  torques as in (8), are reported with dash-dotted lines. In this case the path followed by the user is much closer to the desired one and it ends in almost half the time of the braking controller. Such results can be regarded as upper bounds on the achievable accuracy and clearly show that the tracking performance of the proposed guidance system could be improved with an active walker equipped with motors. The price to pay would be a more complicated hardware design, a sophisticated sensing apparatus and a non negligible weight increment, which plays a crucial role for an assistive cart. Most important, a fully actuated cart would be by far a more invasive and less safe system, which would force the assisted person on a predefined track, rather then gently suggesting her/him the best route to follow. The value of the Lyapunov function  $\mathbf{V}(l, \hat{\theta})$ , as well as the value of  $T_d$ , used in the formulation of the approximate path tracking problem (see Section V-B), are reported in Fig. 6. Notice how the threshold is exceeded only in the initial configuration (in which the assisted person is significantly far apart from the path), in



Fig. 5. Orientation errors with braking action (solid line), for fully actuated cart (dash-dotted line) and braking action with noise (dashed line).



Fig. 6. Lyapunov function values and threshold  $T_d$ .

which the controller exerts a firm braking action to take the user in the proximity of the path. As long as  $\mathbf{V}(l, \tilde{\theta})$ is below the threshold (which means that the assisted person is not too far from the desired configuration), the controller exerts a braking action aiming at mediating between the optimal (from a tracking viewpoint) cart torque  $N_d$  and the user applied torque  $N^h$  (see constraint (10)). In a sense, the threshold  $T_d$  plays the role of a tuning knob that allows one to trade-off between tracking accuracy and control authority.

Finally, we report a simulation for the same set-up described so far, under the assumption that the user applied forces and wheel velocities are not exactly known. It is supposed that noisy estimates of the forces are affected by an additive uniformly distributed noise  $\Delta_{F^h} \sim \mathcal{U}(-1, 1)$ , while the wheel angular velocity estimates by an independent additive uniformly distributed noise  $\Delta_{\omega} \sim \mathcal{U}(-0.5, 0.5)$ . The position and orientation errors in this case are clearly visible in Fig. 4 and Fig. 5 depicted with a dashed line. Albeit the accuracy is comparable, the braking action is quite different, and also, the time to accomplish the full path increases to 210 seconds.

## VII. CONCLUSIONS

In this paper, we have illustrated a device, the c-Walker, that operates as a navigation assistant to guide a user with moderate cognitive problems through a large and crowded environment. In particular, we have focused on the mechanical guidance support, a component of the c-Walker that guide the user along a planned trajectory. We have shown a control strategy that operates on the brakes to achieve a differential steering of the device. The controller operates very mildly when the person moves inside a tunnel that is considered safe and it becomes increasingly authoritative when the boundary of this safety region is reached.

Work is in progress to validate the guidance system on a physical prototype, to enrich the model of the c-Walker with a more accurate model of the brakes and to make the guidance policy robust with respect to the variation of parameters, e.g., apparent mass of the cart. Future work will focus on enriching the guidance support system with the additional haptic feedback.

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