A set theoretic approach to path planning for mobile robots

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***Abstract*— This paper addresses the path planning problem for mobile robots in a set theoretic framework. Under the assumption of unknown but bounded disturbances, a procedure for computing minimum average uncertainty paths is proposed. The considered scenario is that of a holonomic mobile robot moving in an environment where landmarks can be identified. Practical issues, such as limited visibility of landmarks and obstacle avoidance, are addressed. The proposed technique is validated via numerical simulations.**

I. Introduction

Mobile robot navigation implies the successfull integration of several tasks, such as sensing, environmental mapping, localization and path planning. Each of these tasks can be tackled using several techniques, depending on the robot sensory system, the environment the robot is exploring and the a priori knowledge available to the robot.

As a matter of fact, Extended Kalman Filter (EKF) turns out to be the most popular sensor fusion technique used to cope with uncertainty during the localization tasks [1], [2], [3], [4]. In recent years, several techniques based on different assumptions on the nature of the uncertainty have been investigated. In the Set Theoretic framework [5], no statistical assumptions on the nature of the errors are made, all noises being supposed to be *Unknown But Bounded* (UBB) by a known quantity. Under this hypothesis, the problem can be easily phrased in terms of *feasible sets*, defined as those sets containing all the admissible robot *poses* (positions and orientations) compatible with the whole available information and the bounds on the error. Suitable techniques have been devised to efficiently tackle the localization problem, exploiting its geometrical structure (see e.g. [6], [7], [8]).

Motion planning is recognized to be a hard problem since long time [9]. Nonetheless, as far as mobile robots (characterized by few degrees of freedom) are concerned, several solutions have been proposed, which proved to be effective in practice (see [10] for a comprehensive review). While uncertainty and disturbances are crucial issues in localization, they have been often neglected in the path planning phase, i.e., the robot is usually assumed to be perfectly known. The presence of uncertainty at execution time has been considered in motion planning techniques which aim at computing safe paths, i.e. ensuring the goal reach in spite of the uncertainty (see e.g. [11], [12]).

In this paper, we focus on a set theoretic approach to the path planning problem, in order to take into account the localization uncertainty associated to the selected paths. The main objective of the paper is to introduce a framework in which the set theoretic path planning problem can be formulated as a suitable optimization problem. In particular, the problem of finding a trajectory minimizing the overall position uncertainty along the path, is tackled. This is motivated by applications in which it is crucial to localize the robot as precisely as possible along the whole path and not only in the final target, typical examples being the exploration of unknown environments or the accomplishments of several tasks requiring a prescribed precision along the travelled path.

The paper is organized as follows. Section II briefly describes the set theoretic localization framework. Section III introduces the path planning problem in the set theoretic framework and formulates the related optimization problems. Practical issues, such as limited visibility and obstacle avoidance, are considered. Section IV presents simulation results, while some conclusions are drawn in Section V.

II. Set theoretic localization

Let us consider a robot navigating in a 2D environment, whose pose at time k is denoted by

$$p(k) = [x(k)\ y(k)\ \theta(k)]' \in \mathcal{Q},$$

with $\mathcal{Q} \stackrel{\triangle}{=} \mathbb{R}^2 \times [-\pi, \pi]$ being the set of all possible robot configurations. The coordinates $(x(k), y(k))$ represent the position of the vehicle, while $\theta(k)$ denotes its heading, w.r.t. the positive x-axis. It is assumed that a map of the environment is available, in terms of n static landmarks, having known positions:

$$l_i = [x_{l_i}\ y_{l_i}]', \quad i = 1, \ldots, n.$$

The robot is supposed to be equipped with exteroceptive sensors, such as laser range finders or stereo vision systems, providing range and bearing measurements w.r.t. the landmarks (see Fig. 1). The measurement equations take on the form

$$\begin{array}{rcl} D_i(k) & = & d(p(k), l_i) + v_{d_i}(k) \\ A_i(k) & = & \alpha(p(k), l_i) + v_{\alpha_i}(k) \end{array} \qquad i = 1, \ldots, n, \quad (1)$$

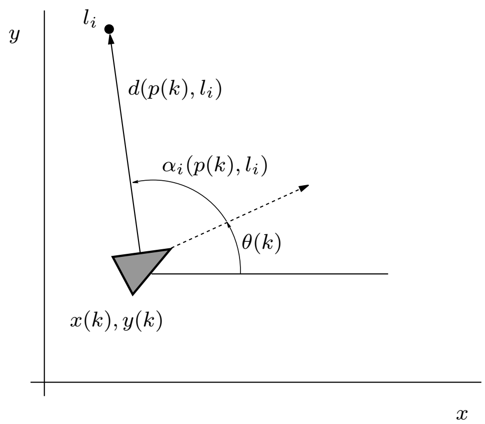


Fig. 1. Range and bearing measurement with respect to the i-th landmark.

where $D_i(k)$ and $A_i(k)$ are the actual sensor readings and $v_{d_i}(k)$, $v_{\alpha_i}(k)$ model noise affecting the distance and angular measurements, defined as

$$d(p(k), l_i) \stackrel{\triangle}{=} \sqrt{(x(k) - x_{l_i})^2 + (y(k) - y_{l_i})^2}, \tag{2}$$

$$\alpha(p(k), l_i) \stackrel{\triangle}{=} \mathrm{atan}_2(y_{l_i} - y(k), x_{l_i} - x(k)) - \theta(k). \tag{3}$$

Under the hypothesis of bounded errors

$$|v_{d_i}(k)| \leq \epsilon^{v_d}, \tag{4}$$

$$|v_{\alpha_i}(k)| \leq \epsilon^{v_\alpha}, \tag{5}$$

with ϵ^{v_d} and ϵ^{v_α} denoting known (possibly time-varying) positive scalars, one can define, for each measurement pair $D_i(k)$,$A_i(k)$, a set

$$\begin{aligned} \mathcal{M}_i(k) = \{p \in \mathcal{Q} :\ & |D_i(k) - d(p(k), l_i)| \leq \epsilon^{v_d} \\ & \text{and } |A_i(k) - \alpha(p(k), l_i)| \leq \epsilon^{v_\alpha}\}\,. \end{aligned} \tag{6}$$

This set contains all robot poses compatible with the i-th measurement readings and with the corresponding error bounds (4)-(5). In a set-theoretic framework, data fusion is obtained via set intersection. Hence, supposing that at time k the robot performs measurements w.r.t. the n landmarks, its pose is constrained to lie in the *feasible set*

$$\mathcal{M}(k) = \bigcap_{i=1}^{n} \mathcal{M}_i(k). \tag{7}$$

Notice that, if the bounds on measurement errors are correct, the set $\mathcal{M}(k)$ is not empty, whereas, an empty intersection in (7) implies that at least one of the constraints (4)-(5) has been violated. The most appealing property of the set-theoretic formulation lies in fact that, as long as the errors verify the boundedness assumption, the actual robot pose $p(k)$ is guaranteed to belong to the set $\mathcal{M}(k)$ *(feasibility property)*, regardless of the statistical nature of the noise. Such a property turns out to be especially useful whenever "certified" estimates are needed, e.g. in order to plan safe paths. As a consequence, a measure of the quality of the set-valued estimates is given by the size of the corresponding feasible sets.

The exact solution to the set-theoretic localization problem involves the computation of the set (7). Unfortunately, this set turns out to be the intersection of nonlinear, nonconvex 3D sets, whose shape can be very complex. Two major drawbacks prevent from computing its exact expression. As far as real-time applications are concerned, the computation required by (7) may be too expensive [6]. Moreover, as new measurements are processed, the shape of the feasible pose set can become arbitrarily complex, so that finding analytical expressions is a very hard problem. For these reasons, suboptimal solutions, trying to reduce the computational cost, while at the same time preserving the essential features of the set-valued estimates, have been devised. For instance, in [5] outer approximations of the feasible sets via simple structure regions have been proposed. It has been shown that, at the expense of some conservativeness, it is possible to devise efficient recursive algorithms able to compute guaranteed set estimates.

To illustrate the main idea, let us first suppose that the robot orientation $\theta(k)$ is known. It is easily verified that the projection on the xy-plane of each set $\mathcal{M}_i(k)$ corresponds to a sector of corona $\mathcal{C}_i(k)$, whose radial and angular semi-amplitude are given by ϵ^{v_α} and ϵ^{v_d}, respectively (see Fig. 2(a)). Then, all the admissible robot positions at time k, according to the error bounds and the measurements $D_i(k), A_i(k)$, are constrained into the set

$$\mathcal{C}(k) = \bigcap_{i=1}^{n} \mathcal{C}_i(k). \tag{8}$$

The goal is to bound the set $\mathcal{C}(k)$ by the minimum area set belonging to a class of simple regions. In this paper, axis-aligned boxes will be used. To this purpose, let us denote by $\overline{\mathcal{B}}\{\mathcal{Z}\}$, the minimum area box containing the set $\mathcal{Z}$. Notice that the set in (8) is still nonconvex. Hence, to further simplify the computation, rather than finding the smallest box containing $\mathcal{C}(k)$, we look for the minimum area box outbounding the set $\mathcal{T}(k)$, defined as

$$\mathcal{T}(k) = \bigcap_{i=1}^{n} \mathcal{T}_i(k), \tag{9}$$

where each $\mathcal{T}_i(k)$ denotes the minimum area trapezoid containing each sector of corona $\mathcal{C}_i(k)$, as shown in Fig. 2(a). Notice that $\mathcal{T}_i(k)$ can be analytically computed, from the landmark location l_i, the sensor readings $D_i(k), A_i(k)$ and the error bounds (4)-(5). With this choice, the problem becomes the computation of

$$\mathcal{B}(k) = \overline{\mathcal{B}}\{\mathcal{T}(k)\}, \tag{10}$$

which in turns boils down to the solution of four linear programming problems. It is worth remarking that the set $\mathcal{B}(k)$ contains, by construction, the true robot position (see Fig. 2(b)).

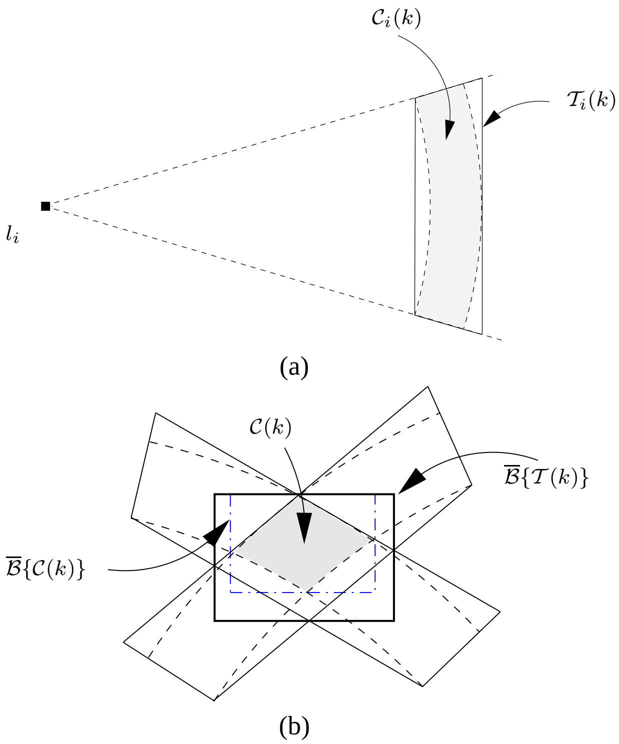


Fig. 2. (a) Trapezoidal approximation of a sector of corona. (b) Outer approximation of the exact feasible position set $\mathcal{C}(k)$ (dashed region) related to two landmarks.

To take into account also the uncertainty affecting the vehicle orientation estimates, the above procedure has to be slightly modified. At each time k an interval estimate of the actual robot heading can be derived, from a simple geometrical reasoning. Such uncertainty can be suitably taken into account by merely enlarging the angular amplitude of each sector of corona, by a quantity equal to the width of the orientation uncertainty interval.

The framework described above can easily incorporate also a dynamic model of the robot, in which uncertainties are modelled as UBB disturbances. In this case, the set estimate $\mathcal{B}(k)$ can be recursively updated by using prediction-correction schemes analogous to that adopted in EKF. The interested reader is referred to [5] for a detailed description of set membership localization algorithms.

III. Set theoretic path planning

The aim of this paper is to address the problem of computing minimum uncertainty trajectories, exploiting the set theoretic localization framework illustrated in Section II.

Consider a robot with initial pose p_0 and whose target is to reach a pose p_T, after T moves. The objective is to plan a path $\mathcal{P} = [p(0), p(1), \dots, p(T)]$, such that the average uncertainty associated to the path is minimized. This corresponds to tackle the following problem:

$$
\begin{array}{ll}
\min\limits_{\mathcal{P}} \dfrac{1}{T} \displaystyle\sum_{k=1}^{T} V\big[\mathcal{M}(k)\big] & \\
\text{s.t.} & \\
|x(k+1) - x(k)| \leq b_x & \forall k = 0 \dots T-1 \\
|y(k+1) - y(k)| \leq b_y & \forall k = 0 \dots T-1 \\
p(0) = p_0 \quad p(T) = p_T &
\end{array}
\tag{11}
$$

where $V\big[\mathcal{M}(k)\big]$ is the volume of the feasible set $\mathcal{M}(k)$, and b_x, b_y are bounds on the maximum $x-$ and $y-$displacement for each move, respectively. Bounds on the orientation displacement (rotation) are not included, because holonomic mobile robots are considered and it is assumed that the time step for a single move is such that the robot succeeds to perform the required rotation during each move.

The solution of problem (11) presents several difficulties. In particular:

a) the set $\mathcal{M}(k)$ is a function of the sensor readings $D_i(k), A_i(k)$, which in turn depend on the actual realization of measurement noises;
b) the size of $\mathcal{M}(k)$ does not depend on the robot orientation $\theta(k)$ if the visibility range is unlimited and all landmarks can be seen from any robot pose; in the more realistic situation of limited field of view, $V\big[\mathcal{M}(k)\big]$ depends on which landmarks fall in the robot field of view.

Problem a) can be faced in a worst-case approach, by maximizing $V\big[\mathcal{M}(k)\big]$ over all possible noise realizations. However, this would lead to an untractable high-dimensional min-max optimization problem. A more viable approach is that of neglecting the noise realizations in the planning phase, which amounts to consider measurements $D_i(k), A_i(k)$ produced by a zero noise realization. Notice that this is generally an unfavorable case in set theoretic localization, where the maximum uncertainty reduction is usually achieved when the noise sequence takes values on the boundary as often as possible.
In order to cope with problem b), a suitable partition of the orientation interval is introduced next. As it will become clear in the following, this will also lead to a useful simplification of the cost function in (11).

A. Limited angular visibility

In a real-world scenario, sensors usually have a limited field of view. For example, a mobile robot using a laser rangefinder to detect landmarks, has a limited angular visibility. This means that in general the robot is not able to perceive all the landmarks around it.

In this work, we consider robots which can see all the landmarks l_i such that $|\alpha(p(k), l_i)| \leq \overline{\alpha}$, where $\overline{\alpha}$ is the angular visibility (sensor range limitations can also be easily incorporated in this framework). This assumption implies that, given a position $z = [x, y]'$, the robot perceives different sets of landmarks depending on both its orientation θ and its angular visibility $\overline{\alpha}$. Therefore, the intersections in

(7), (8) and (9) do not involve all the landmarks, but only those that are actually seen from the current pose $p(k)$.

In order to establish which landmarks are seen from a given pose, we look for a partition of the orientation interval $[-\pi, \pi]$ into subsets h_i, such that each subset h_i is associated to a unique set of visible landmarks from the position z and *for all* orientations $\theta \in h_i$. For a fixed position z, let us introduce the set

$$\tau_h = \{l_j : |\alpha(p, l_j)| \leq \overline{\alpha} \quad \forall p = [x, y, \theta] \text{ with } \theta \in h\} \tag{12}$$

which is the set of visible landmarks, from position z and orientation $\theta \in h$, with $h \subseteq [-\pi, \pi]$. Then, the sought partition of $[-\pi, \pi]$ is given by

$$\begin{aligned} \mathcal{H}(z) \;=\; \{h_i \subseteq [-\pi,\pi] : & \\ \textstyle\bigcup h_i \equiv [-\pi,\pi] & \\ h_i \cap h_j = \emptyset \qquad & \forall i \neq j \\ \tau_{h_i} \neq \tau_{h_j} \qquad & \forall i \neq j \\ \tau_a \equiv \tau_{h_i} \qquad & \forall a \subseteq h_i \ \}. \end{aligned} \tag{13}$$

Notice that $\mathcal{H}(z)$ is a partition of the interval $[-\pi, \pi]$ into disjoint sets, such that the set of landmarks perceived by any couple of orientations $\theta_i \in h_i$ and $\theta_j \in h_j$ are different if $h_i \neq h_j$. Moreover, the set of perceived landmarks is the same in every subset of h_i. This makes the partition defined by (12)-(13) unique.

To clarify how the partition $\mathcal{H}(z)$ is constructed, let us consider the example depicted in Fig. 3, concerning a four landmarks scenario with a angular visibility $\overline{\alpha} = \frac{\pi}{2}$. In this case, one gets the partition $\mathcal{H}(z) = \{h_1, h_2, h_3, h_4, h_5, h_6\}$; Table I reports the sets of visible landmarks τ_{h_j}, from pose $p = [x, y, \theta]$ with $\theta \in h_j$.

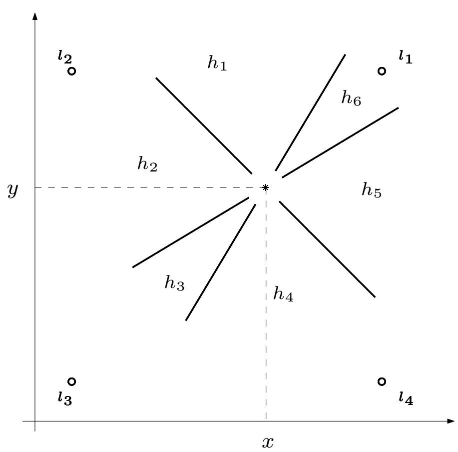


Fig. 3. Example of orientation partition: asterisk denotes the robot position $z = [x, y]'$, circles represent landmark positions, solid lines bound the angular subsets h_i.

B. A simplified optimization problem

From the above discussion, the problem of choosing the robot orientation along the path can be simplified into that of selecting a suitable orientation subset h_j. Indeed, the feasible set (7) associated to the robot pose $p = [x\ y\ \theta]'$ becomes

TABLE I

SETS OF VISIBLE LANDMARKS τ_{h_i} FOR THE EXAMPLE IN FIG. 3

τ_{h_1}	l_1, l_2
τ_{h_2}	l_2, l_3
τ_{h_3}	l_2, l_3, l_4
τ_{h_4}	l_3, l_4
τ_{h_5}	l_1, l_4
τ_{h_6}	l_1

$$\mathcal{M}^{(j)}(k) = \bigcap_{l_i \in \tau_{h_j}} \mathcal{M}_i(k) \tag{14}$$

where $h_j \in \mathcal{H}(z)$ is such that $\theta \in h_j$. Clearly, the size of $\mathcal{M}^{(j)}(k)$ in (14) depends on τ_{h_j}, and hence on the partition subset h_j in which the robot orientation lies, but it does not depend on the exact value θ of the orientation itself. For this reason, one can get rid of the dependence on θ in the cost function (11), by choosing the subset h_j for which the size of the feasible set $\mathcal{M}^{(j)}(k)$ in (14) is minimum. This amounts to consider for each position $z(k)$ the cost function

$$\mathcal{J}(z(k)) = \min_{h_j \in \mathcal{H}(z(k))} V\left[\mathcal{M}^{(j)}(k)\right]. \tag{15}$$

If in (11) $V[\mathcal{M}(k)]$ is replaced by $\mathcal{J}(z(k))$, the number of free variables in the path planning optimization problem is reduced from $3T$ to $2T$. However, the cost function (15) is still very difficult to compute, since the $\mathcal{M}^{(j)}(k)$ are nonlinear nonconvex 3D sets.

A further simplification is achieved by considering only the position uncertainty and exploiting the set approximation introduced in Section II. In particular, the feasible set approximation worked out in (9)-(10) becomes

$$\mathcal{B}^{(j)}(k) = \overline{\mathcal{B}}\left\{\bigcap_{l_i \in \tau_{h_j}} \mathcal{T}_i(k)\right\}. \tag{16}$$

Hence, one can substitute to $\mathcal{J}(z(k))$ the new cost

$$\mathcal{J}_2(z(k)) = \min_{h_j \in \mathcal{H}(z(k))} A\left[\mathcal{B}^{(j)}(k)\right]. \tag{17}$$

where $A[\cdot]$ denotes the area of the set within square brackets. Notice that, according to the discussion in Section II, in (17) the uncertainty in robot orientation is neglected: this means that in the planning phase one assumes to know exactly the orientation of the robot when computing the size of the (approximate) feasible position set.

Therefore, the set theoretic path planning problem we address can be cast as follows. Let $\mathcal{Z} = [z(0), z(1), \ldots, z(T)]$, and $\Pi = [I_2 \ 0_{2\times 1}]$. Then, solve

$$
\begin{aligned}
&\min_{\mathcal{Z}} \frac{1}{T} \sum_{k=1}^{T} \mathcal{J}_2\big(x(k), y(k)\big) \\
&\text{s.t.} \\
&|x(k+1) - x(k)| \le b_x \qquad\qquad \forall k = 1 \ldots T-1 \\
&|y(k+1) - y(k)| \le b_y \qquad\qquad \forall k = 1 \ldots T-1 \\
&\ z(0) = \Pi \cdot p_0 \quad z(T) = \Pi \cdot p_T
\end{aligned}
\tag{18}
$$

The solution $\mathcal{Z}^* = [z^*(0), z^*(1), \ldots, z^*(T)]$ of (18) provides a sequence of positions. To solve completely the path planning problem one has to specify the orientations $\theta^*(k)$ associated to the positions $z^*(k)$. In order to minimize the uncertainty along the path one should consider the set of orientations for which the minimum in (17) is achieved, i.e.

$$
h_{j^*}(k) = \arg \min_{h_j \in \mathcal{H}(z^*(k))} A\left[\mathcal{B}^{(j)}(k)\right]. \tag{19}
$$

Any value within $h_{j^*}(k)$ can be selected as the robot orientation $\theta^*(k)$ at the k-th step of the computed path. For example, $\theta^*(k)$ may be the center of the angular interval $h_{j^*}(k)$. This choice is viable because we do not constraint rotational displacement in a single robot move.

C. Obstacle avoidance

In static structured workspaces, the path planner must also deal with the presence of obstacles. One of the most popular techniques for tackling this problem is the potential field approach [10], where artificial potential functions are generated in agreement with the structure of obstacles. Once such functions are generated, they can be combined with the cost function in (17), in order to produce a new cost to be minimized in (18). A possible choice, is to replace $\mathcal{J}_2(z)$ in (18) by

$$
\mathcal{J}_2(z) + \rho \mathcal{O}(z)
$$

where $\mathcal{O}(z)$ is the artificial potential function associated to the obstacles, and ρ is a suitable weighting coefficient. Clearly, the resulting cost function accounts both for uncertainty along the path and for the presence of obstacles in the scene. On the other hand the use of potential functions usually complicates the solution of (18) increasing the occurrence of local minima. This problem is well-known and it has widely addressed in the literature (see [13]).

IV. Simulation results

In this section, simulation results concerning the proposed path planning strategy are presented. The optimization problem (18) yielding the path $\mathcal{Z}^*$ is solved via Sequential Quadratic Programming (SQP). Then, the orientations $\theta^*(k)$ are chosen as the centers of the intervals $h_{j^*}(k)$ defined in (19). The computed path has been used to simulate a mobile robot navigation, during which the localization algorithm sketched in Section II is applied. Performance in terms of localization precision are compared to those of different path choices.

Consider the four landmark scenario depicted in Fig. 3. The aim is to solve the path planning problem (18) with the constraints

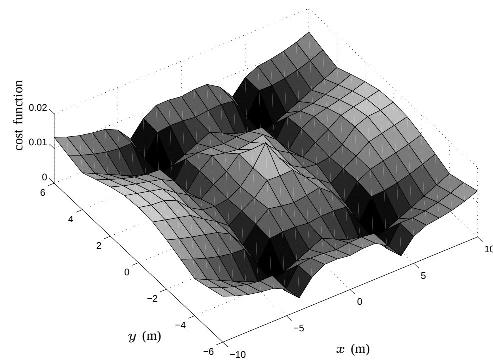
$$
p_0 = \begin{bmatrix} -3 \\ -4 \\ 0 \end{bmatrix}, \quad p_T = \begin{bmatrix} 5 \\ 4 \\ \pi \end{bmatrix}, \quad \begin{bmatrix} b_x \\ b_y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}
$$

and $T = 14$. The cost function $\mathcal{J}_2(z)$ in (17) has been computed via the set approximation techniques described in Section II, assuming the noise bounds

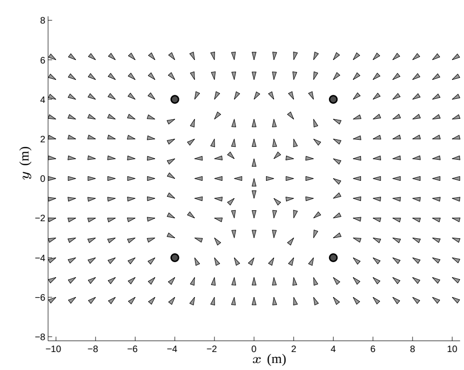
$$
\epsilon^{v_d} = 0.1 \text{ (m)}, \tag{20}
$$

$$
\epsilon^{v_\alpha} = 5 \text{ (deg)}, \tag{21}
$$

in (4)-(5). The resulting $\mathcal{J}_2(z)$ for this scenario is shown in Fig. 4(a), while the corresponding orientations are reported in Fig. 4(b). The computed path $\mathcal{Z}^*$ is shown in Fig. 5.



(a) cost function



(b) orientation

Fig. 4. The cost function (17) and the associated orientations for a four landmark scenario.

Fig. 6 shows the cost function and the computed path for the same scenario, in which two rectangular obstacles are present. The cost function has been generated according to what suggested in Section III-C.

Let us now analyze the computed paths by simulating the motion of a mobile robot, performing self-localization with respect to the landmarks at each move along the path. The localization algorithm combines the set approximation techniques outlined in Section II for processing distance and orientation measurements, with a simple dynamic model for

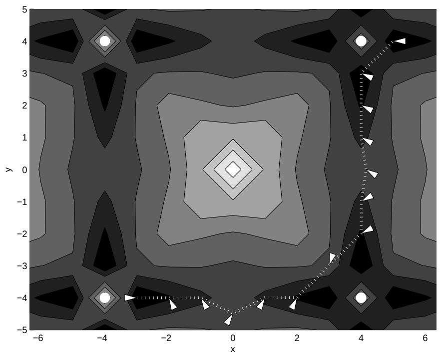
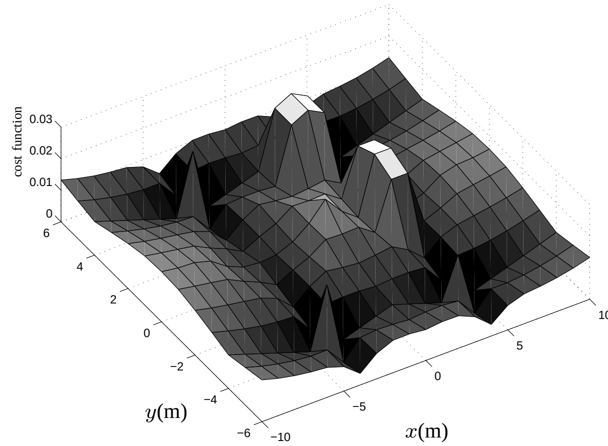
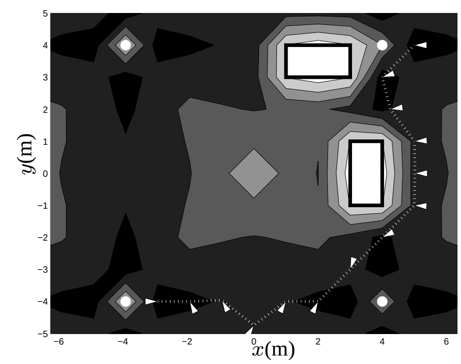


Fig. 5. Path calculated by solving (18) with the cost function in Fig. 4(a). White triangles represent the selected robot poses. White circles are landmarks; contour lines of the cost function $\mathcal{J}_2(z)$ are reported in the background.



(a) cost function



(b) computed path

Fig. 6. Cost function and computed path, for a scenario including two rectangular obstacles.

robot motion. In particular, it is assumed that the robot pose evolves as

$$p(k+1) = p(k) + u(k) + w(k) \tag{22}$$

where $u(k)$ represents translation and rotation measurements from odometric sensors, and $w(k)$ models the errors affecting $u(k)$. It is assumed that also errors $w(k)$ are UBB. For each move, bounds on translation errors are set to 10% of the $x-$, $y-$displacement, respectively, while bounds on rotation errors do not exceed 5% of the variation of θ. At each move, the set of feasible robot poses is enlarged according to the model (22); each time measurements are performed, such a set is reduced by intersecting it with the set $\mathcal{B}(k)$ in (10). A detailed description of the dynamic localization algorithm can be found in [5].

The performance of the localization algorithm has been compared for three different paths:

t_{opt}: the path obtained by solving problem (18);

t_{d_1}: the direct path joining p_0 to p_T, where the orientations are chosen as the center of the interval h_{j^*} in (19);

t_{d_2}: the direct path joining p_0 to p_T, where the orientation is kept equal to the one pointing from $z(0)$ to $z(T)$.

It is worth remarking that, while the path planning is performed assuming zero measurement noise realizations, in the simulations the disturbance signals v_{d_i}, v_{α_i}, w are generated as independent uniformly distributed random variables within the given bounds. In each simulation run, the uncertainty affecting the robot pose is computed in terms of boxes containing the robot position $(z(k))$ and intervals containing the robot orientation $\theta(k)$, for each k. Simulation results are averaged over 100 runs with different noise realizations.

Fig. 7 reports the result of typical runs for the considered paths t_{opt}, t_{d_1} and t_{d_2}, in the scenario of Fig. 6. It can be observed that the uncertainty on the robot position is smaller along the path t_{opt}, as expected. The comparison between the two direct paths t_{d_1} and t_{d_2} in Figures 7(b) and 7(c) confirms that the choice of orientations along the path plays an important role in uncertainty reduction.

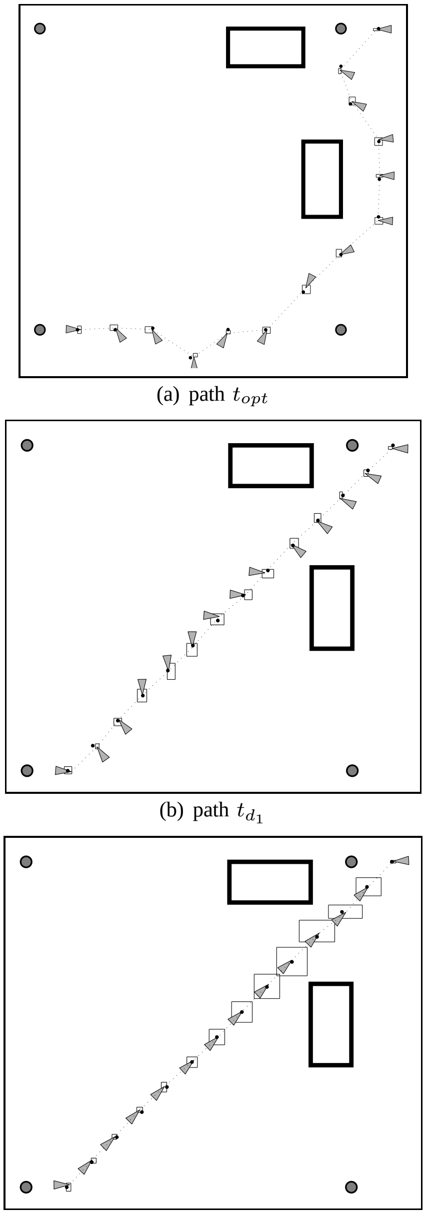
Average uncertainties on position and orientation are shown in Fig. 8. It can be noticed that, although the uncertainty values are similar at the endpoints of the path, the overall uncertainty is significantly smaller along the selected path t_{opt}.

V. Discussion and Future Work

A path planning algorithm for holonomic vehicles, which explicitly takes into account the localization uncertainty affecting the robot along its trajectory, has been proposed. A deterministic description of the uncertainty is adopted, leading to set-valued pose estimates whose size concurs to the definition of a suitable potential function to be minimized.

The presented preliminary results are obtained via general purpose optimization techniques. Current investigations include more efficient numerical methods, like the Navigation Functions [14], or computational techniques for constrained trajectory generation [15] in order to encompass more complex robot motion models (e.g., nonholonomic vehicles) .

This work has to be intended as a first step toward the development of advanced exploration strategies, through the integrated solution of different tasks, like localization,

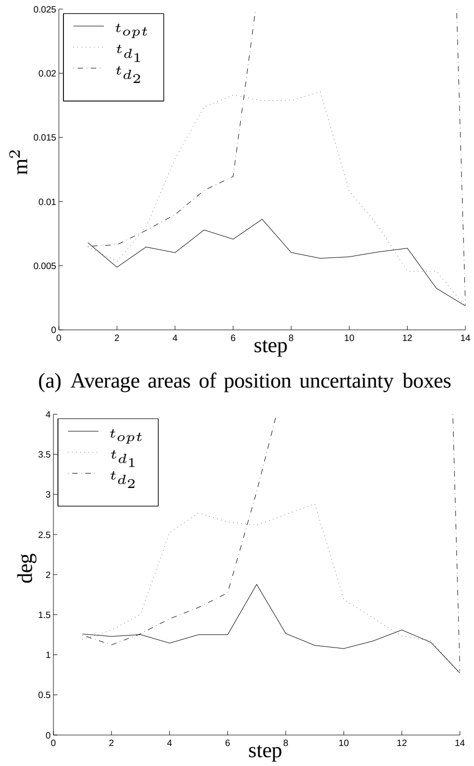


(a) path t_{opt}

(b) path t_{d_1}

(c) path t_{d_2}

Fig. 7. Simulation runs for three different paths. Black dots represent the chosen path, boxes depict the uncertainty on the robot position, triangles show the true robot pose, circles represent the landmarks.



(a) Average areas of position uncertainty boxes

(b) Average uncertainty on orientations

Fig. 8. Average uncertainties on positions and orientations over 100 runs of the localization algorithm, for the three paths of Fig. 7.

map building and path planning (see [16]). Along this line, the proposed design procedure represents an attempt to consider in the planning stage the whole information gained along the path, to localization purposes. The next goal is the extension of the proposed planner when the more challenging Simultaneous Localization and Map building (SLAM) problem has to be addressed.

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