Change in the State of the Art of (Mixed) Integer Programming

1977–Vancouver–Advanced Research Institute

24 papers

16 reports

- 1 paper computational, 4 small instances
- Report on computational aspects: only instances up to 30 variables can be expected solved

2006–MIP Solution Time to 0.01% of Optimality (CPLEX–Irvin Lustig)

Total		987		
Solved within	1 sec.	521	\	
	10 sec.	209	\	
	1 min.	113	/	(94.2%)
	10 min.	87	/	
	30 min.	33		
	1 hour	9		
Unsolved after	1 hour	15		(1.5%)

Measure of success:

- Problems involving $10^4 - 10^5$ and $10^3 - 10^4$ constraints are routinely solved

  (Ex. supply chain management, portfolio optimization, combinatorial auctions, etc.)

- Some complex but relatively small problems can be solved as they arise, in time for the solution to be used ("real time IP")

  (Ex. Optimal Real-Time Traffic Control in Metro Stations)

- But: MIP remains $\mathcal{NP}$-complete

  Some small problems remain unsolved in useful time

Reasons

- More powerful computers
- Faster and more reliable LP codes
- Progress on cutting planes
    - Stronger cuts
    - Better use of cuts

- Two general approaches to cut generation

1958

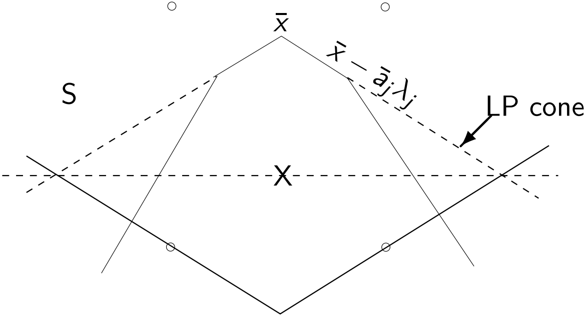
(a) “algebraic” – Gomory-Chvátal, MIG cuts

$$\begin{array}{ll} ax \leq b & \sum_J f_j s_j \geq f_0 \qquad (f_j = a_j - (a_j)) \\ \lfloor a \rfloor x \leq \lfloor b \rfloor & \sum_{J_1} f_j x_j + \sum_{J_2} \frac{f_0}{1-f_0}(1-f_j)x_j \geq f_0 \\ & \sum_{j \in J} \min\left\{ \frac{f_j}{f_0}, \frac{1-f_j}{1-f_0} \right\} x_j \geq 1 \end{array}$$

1969

(b) “geometric” – intersection cuts, disjunctive cuts

Intersection cuts (1969)



Convex set S such that

(i) $\bar{x} \in \operatorname{int} S$
(ii) $\operatorname{int} S \cap X \cap \mathbb{Z}^n = \emptyset$
The n points $\bar{x} - \bar{a}_j\bar{\lambda}_j$ where the extreme rays of the LP cone intersect $\operatorname{bd} S$ define a valid cut:

$$\sum_J \bar{\lambda}_j^{-1} x_j \geq 1$$

Possible convex sets $S_1 : \{x : 0 \leq x_k \leq 1\}$

$$x_k = a_0 + \sum_J a_j x_j$$

$\sum_J \max\{\frac{a_j}{a_0}, \frac{-a_j}{1-a_0}\}x_j \geq 1$ or, after monoidal strengthening,

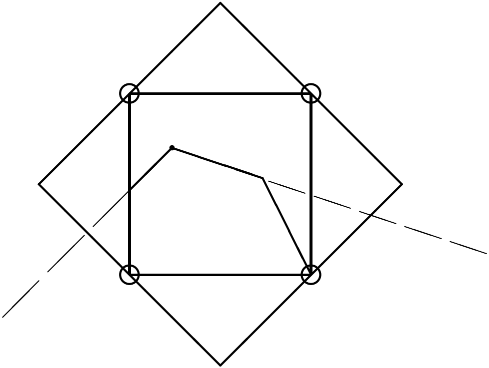
$$\sum_J \min\{\frac{f_j}{f_0}, \frac{1-f_j}{1-f_0}\}x_j \geq 1 \quad (f_j = a_j - \lfloor a_j \rfloor)$$

- Connection to algebraic approach.

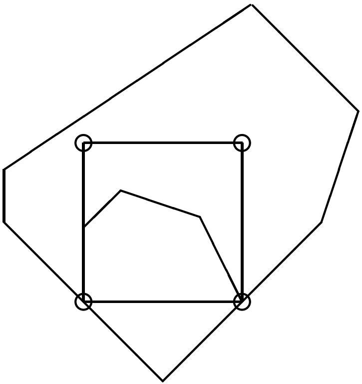
$$\left\{ \begin{array}{c} \text{MIG cut} \\ \text{from } x_k \text{ row} \end{array} \right\} \Longleftrightarrow \left\{ \begin{array}{c} \text{strengthened} \\ \text{Intersection cut from} \\ \lfloor x_k \rfloor \leq x_k \leq \lceil x_k \rceil \end{array} \right\}$$

Other suitable convex sets:

- Scaled octahedron



- Outer polar (of the feasible set)



Related recent work:
(connection with lattice-free bodies)

  - Anderson, Louveaux, Weismantel and Wolsey (2007)
  - Cornuéjols & Margot (2008)

Lift-and-Project

1974 Disjunctive Programming: Optimization over a Union of Polyhedra

1. Compact linear representation of the convex hull
2. Sequential convexifiability

1975-1990 B., Blair, Jeroslow, Pulleyblank

1990-1991 Sherali-Adams, Lovász-Schrijver

1993,1996 B., Ceria, Cornuéjols
Cut Generating Linear Program (CGLP)

1998-2002 Ceria, Pataki; Ceria, Soares; B., Bockmayr, Pisaruk, Wolsey; Bienstock, Zuckerberg

2002,2003 B., Perregaard
Solving (CGLP) in the (LP) Tableau
Commercial implementation

2007 B., Bonami
Open source implementation, 3 variants

Disjunctive Programming

A *disjunctive set*, i.e. the constraint set of a disjunctive program, can be expressed in many different forms, of which the following two extreme ones have special significance. Let

$$P_i := \{x \in \mathbb{R}^n : A^i x \geq b^i\}, \qquad i \in Q$$

be convex polyhedra, with Q a finite index set and (A^i, b^i) an $m_i \times (n+1)$ matrix, $i \in Q$, and let

$$P := \{x \in \mathbb{R}^n : Ax \geq b\}$$

be the polyhedron defined by those inequalities (if any) common to all P_i, $i \in Q$. Then the disjunctive set $\underset{i \in Q}{\cup} P_i$ over which we wish to optimize some linear function can be expressed as

$$\left\{x \in \mathbb{R}^n : \bigvee_{i \in Q} (A^i x \geq b^i)\right\}, \tag{1}$$

which is its *disjunctive normal form (DNF)* (a disjunction whose terms do not contain further disjunctions).

The same disjunctive set can also be expressed as

$$\left\{x \in \mathbb{R}^n : Ax \geq b, \bigvee_{h \in Q_j} (d^h x \geq d_0^h), j = 1, \ldots, t\right\}, \tag{2}$$

which is its *conjunctive normal form (CNF)* (a conjunction whose terms do not contain further conjunctions). Here (d^h, d_0^h) is a $(n+1)$-vector for $h \in Q_j$, all j, where each set Q_j contains exactly one inequality of each system $A^i x \geq b^i$, $i \in Q$, and t is the number of all sets Q_j with this property.

Thus the connection between (1) and (2) is that each term $A^i x \geq b^i$ of (1) contains $Ax \geq b$ and exactly one inequality $d^h x \geq d_0^h$ of each disjunction of (2) indexed by Q_j for $j = 1, \ldots, t$, and that all distinct systems $A^i x \geq b^i$ with this property are present among the terms of (1).

The DNF of a disjunctive set is a union of polyhedra.

Disjunctive Programming is optimization over unions of polyhedra

The Lift-and-Project approach relies on two basic results of disjunctive programming.

1. There is a compact representation of the convex hull of a union of polyhedra in a higher-dimensional space (lifting, extended formulation) which can be projected back onto the original space.
2. A large class of disjunctive sets, called facial, can be convexified sequentially, i.e. their convex hull can be derived by imposing the disjunctions one at a time, generating each time the convex hull of the current set.

Result 1 uses the DNF (1),
result 2 uses the CNF (2).

The convex hull of a union of polyhedra

Theorem 1. (Balas, 1974) Given polyhedra

$$P_i := \{x \in \mathbb{R}^n : A^i x \geq b^i\} \neq \emptyset, i \in Q,$$

the convex hull of $\underset{i \in Q}{\cup} P_i$ is the set of those $x \in \mathbb{R}^n$ for which there exist vectors $(y^i, y_0^i) \in \mathbb{R}^{n+1}$, $i \in Q$, satisfying

$$\begin{array}{rcll} x - \sum(y^i : i \in Q) & = & 0 & \\ A^i y^i - b^i y_0^i & \geq & 0 & \\ y_0^i & \geq & 0 & \quad i \in Q \\ \sum(y_0^i : i \in Q) & = & 1. & \end{array} \tag{3}$$

In particular, denoting by P_Q the convex hull of $\underset{i \in Q}{\cup} P_i$ and by $\mathcal{P}$ the set of vectors $(x, \{y^i, y_0^i\}_{i \in Q})$ satisfying (3),

(i) if x^* is an extreme point of P_Q, then $(x^*, \{\bar{y}^i, \bar{y}_0^i\}_{i \in Q})$ is an extreme point of $\mathcal{P}$, $(\bar{y}^k, \bar{y}_0^k) = (x^*, 1)$ for some $k \in Q$, and $(\bar{y}^i, \bar{y}_0^i) = (0, 0)$ for $i \in Q \setminus \{k\}$.

(ii) if $(\bar{x}, \{\bar{y}^i, \bar{y}_0^i\}_{i \in Q})$ is an extreme point of $\mathcal{P}$, then $\bar{y}^k = \bar{x}$ and $\bar{y}_0^k = 1$ for some $k \in Q$, $(\bar{y}^i, \bar{y}_0^i) = (0, 0)$, $i \in Q \setminus \{k\}$, and $\bar{x}$ is an extreme point of P_Q.

- The number of variables and constraints in (3) is linear in the number $|Q|$ of polyhedra in the union.
- In any basic solution of the linear system (3), $y_0^i \in \{0, 1\}$, $i \in Q$, automatically, without imposing this condition explicitly.

In the special case of a disjunction of the form $x_j \in \{0, 1\}$, when $|Q| = 2$ and

$$\begin{aligned} P_{j0} &:= \{x \in \mathbb{R}^n_+ : Ax \geq b,\ x_j = 0\}, \\ P_{j1} &:= \{x \in \mathbb{R}^n_+ : Ax \geq b,\ x_j = 1\}, \end{aligned}$$

$P_Q := \text{conv}\,(P_{j0} \cup P_{j1})$ is the set of those $x \in \mathbb{R}^n$ for which there exist vectors $(y, y_0), (z, z_0)$ such that

$$
\begin{aligned}
x - y - z &= 0 \\
Ay - by_0 &\geq 0 \\
-y_j &= 0 \\
Az - bz_0 &\geq 0 \\
z_j - z_0 &= 0 \\
y_0 + z_0 &= 1 \\
y_0 \geq 0,\ z_0 \geq 0
\end{aligned}
\tag{3'}
$$

Disjunctive cuts

Projecting out the variables (y^i, y_0^i), $i \in Q$, we obtain that $\mathrm{conv}(\cup_{i \in Q} P_i)$ is the set of those $x \in \mathbb{R}^n$ satisfying $\alpha x \geq \beta$ for all α, β such that

$$\begin{aligned} \alpha &\geq u^i A^i \\ \beta &\leq u^i b^i, \quad \forall i \in Q, \end{aligned}$$

$$\text{for some } u^i \geq 0, i \in Q$$

Thus a disjunctive cut $\alpha x \geq \beta$ is of the form

$$\begin{aligned} \alpha_j &= \max_{i \in Q} u^i A_j^i, \quad j \in N \\ \beta &= \min_{i \in Q} u^i b^i \end{aligned}$$

or, with the u^i properly scaled,

$$\beta = u^i b^i, i \in Q.$$

For a 2-term disjunction,

$$\begin{aligned} \alpha_j &= \max\{uA^i_j, vA^2_j\}, \quad j \in N \\ \beta &= ub^1 = vb^2 \end{aligned}$$

Projection and polarity

In order to generate the convex hull P_Q, and more generally, to obtain valid inequalities (cutting planes) in the space of the original variables, we project $\mathcal{P}$ onto the x-space:

Theorem

$$Proj_x(\mathcal{P}) = \{x \in \mathbb{R}^n : \alpha x \geq \beta \textit{ for all } (\alpha, \beta) \in W_0\},$$

where

$$\begin{aligned} W_0 := \{(\alpha, \beta) \in \mathbb{R}^{n+1} : \alpha &= u^i A^i, \beta \leq u^i b^i \\ &\quad \textit{for some } u^i \geq 0,\ i \in Q\}. \end{aligned}$$

Note that W_0 is not the standard projection cone of $\mathcal{P}$, which is (assuming each A^i is $m_i \times n$)

$$\begin{aligned} W := \; & \{(\alpha, \beta, \{u^i\}_{i\in Q}) \in \mathbb{R}^n \times \mathbb{R} \times \mathbb{R}^{m_1} \times \ldots \times \mathbb{R}^{m_{|Q|}} : \\ & \alpha - u^i A^i = 0, \beta - u^i b^i \leq 0, u^i \geq 0, i \in Q\}. \end{aligned}$$

Instead, $W_0 = \mathrm{Proj}_{(\alpha,\beta)}(W)$, i.e. W_0 is the projection of W onto the (α, β)-space. In fact, W_0 can be shown to be the *reverse polar cone* P_Q^* of P_Q, i.e. the cone of all valid inequalities for P_Q.

Theorem

$$\begin{aligned} P_Q^* := \; & \{(\alpha, \beta) \in \mathbb{R}^{n+1} : \alpha x \geq \beta \textit{ for all } x \in P_Q\} \\ = \; & \{(\alpha, \beta) \in \mathbb{R}^{n+1} : \alpha = u^i A^i, \, \beta \leq u^i b^i \\ & \quad \textit{for some } u^i \geq 0, \, i \in Q\}. \end{aligned}$$

Polars and Reverse Polars

Polar of $P \subseteq \mathbb{R}^n$: $P^0 := \{y \in \mathbb{R}^n : xy \leq 1, \forall x \in P\}$

Reverse polar of P: $P^\# := \{y \in \mathbb{R}^n : xy \geq 1, \forall x \in P\}$

Reverse cone polar of P:
$P^* := \{(y, y_0) \in \mathbb{R}^{n+1} : xy - y_0 \geq 0, \forall x \in P\}$

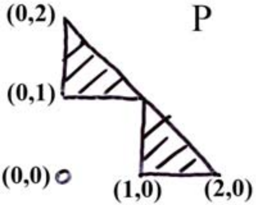
Theorem.

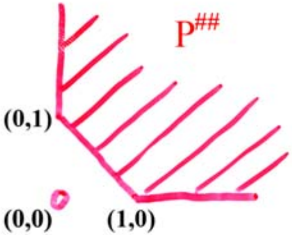
$$\begin{aligned} P^{00} &= \operatorname{conv}(P \cup \{0\}) \\ P^{\#\#} &= \operatorname{conv}(P) + \operatorname{cone}(P) \end{aligned}$$

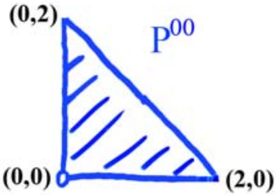
Corollary.

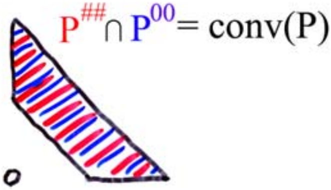
$$\operatorname{conv}(P) = P^{00} \cap P^{\#\#}$$

Example.









The reverse cone polar of

$$P_Q := \mathrm{conv}\left(P_{j^0} \cup P_{j^1}\right)$$

is

$$\begin{aligned} P_Q^* &= \{(\alpha,\beta) \in \mathbb{R}^{n+1} : \alpha x \geq \beta \text{ for all } x \in P_Q\} \\ &= \{(\alpha,\beta) \in \mathbb{R}^{n+1} \quad : \quad \alpha \geq uA - u_0 e_j \\ &\qquad\qquad\qquad\qquad\qquad\quad \alpha \geq vA + v_0 e_j \\ &\qquad\qquad\qquad\qquad\qquad\quad \beta \leq ub \\ &\qquad\qquad\qquad\qquad\qquad\quad \beta \leq vb + v_0 \\ &\qquad\qquad\qquad\qquad\qquad\quad u, v \geq 0\} \end{aligned}$$

(where e_j is the j-th unit vector).

Theorem. Assume P_Q is full dimensional. The inequality $\alpha x \geq \beta$ defines a facet of P_Q if and only if (α, β) is an extreme ray of the cone P_Q^*.

Sequential convexification

A disjunctive set is called *facial* if every one of its inequalities induces a face of the polyhedron defined by the inequalities $Ax \geq b$ common to all terms of the disjunction.

Zero-one programs (pure or mixed) are facial disjunctive programs, general integer programs are not.

Sequential convexifiability is one of the basic properties that distinguish 0-1 programs from general integer programs.

Theorem. (Balas, 1974) Let

$$D := \left\{ x \in \mathbb{R}^n : Ax \geq b, \bigvee_{h \in Q_j} (d^h x \geq d_0^h), j = 1, \ldots, t \right\},$$

where $|Q_j| \geq 1$ for $j = 1, \ldots, t$, and D is facial. Let
$P_D := \text{conv}(D)$.

Define

$$P^0 := \{x \in \mathbb{R}^n : Ax \geq b\},$$

and for $j = 1, \ldots, t$,

$$P^j := \text{conv} \left(P^{j-1} \cap \left\{ x : \bigvee_{h \in Q_j} (d^h x \geq d_0^h) \right\} \right).$$

Then

$$P^t = P_D.$$

For Mixed 0-1 Programs, the Theorem asserts that if we denote

$$P_D := \text{conv}\ \{x \in \mathbb{R}^n_+ : Ax \geq b,\ x_j \in \{0,1\},\ j = 1,\ldots,p\},$$

$$P^0 := \{x \in \mathbb{R}^n_+ : Ax \geq b, 0 \leq x \leq 1\},$$

and define recursively for $j = 1,\ldots,p$

$$P^j := \text{conv}\ (P^{j-1} \cap \{x : x_j \in \{0,1\}\}),$$

then

$$P^p = P_D.$$

Thus, a 0-1 program with p 0-1 variables can be solved in p steps. Here each step consists of imposing the 0-1 condition on one new variable and generating all the inequalities that define the convex hull of the set defined in this way.

Disjunctive rank

The Chvátal rank of a valid inequality $\alpha x \geq \beta$ derived by the rounding operation $\lfloor \lambda A \rfloor x \geq \lfloor \lambda b \rfloor$, $\lambda \geq 0$, is the minimum number of times the operation has to be iterated to obtain $\alpha x \geq \beta$.

The disjunctive rank of $\alpha x \geq \beta$ for a mixed 0-1 program is the smallest integer k such that $\alpha x \geq \beta$ is valid for P^k.

In other words, $\alpha x \geq \beta$ is of disjunctive rank k if it can be derived by k, but not fewer than k, iterations of the sequential convexification procedure.

Clearly, the disjunctive rank of a cutting plane for 0-1 programs is bounded by the number of 0-1 variables. It is known that the number of 0-1 variables is not a valid bound for the Chvatal rank of an inequality.

Another derivation of the basic result

(Balas, Ceria and Cornuejols, 1993)

Define

$$\begin{aligned} P &:= \{x \in \mathbb{R}^n : Ax \geq b, 0 \leq x \leq 1\} \\ &= \{x \in \mathbb{R}^n : \widetilde{A}x \geq \widetilde{b}\} \subseteq \mathbb{R}^n \end{aligned}$$

and

$$P_D := \text{conv}\left(\{x \in P : x_j \in \{0,1\},\ j = 1, \ldots, p\}\right)$$

1. Select an index $j \in \{1, \ldots, p\}$. Multiply $\widetilde{A}x \geq \widetilde{b}$ with $1 - x_j$ and x_j to obtain the nonlinear system

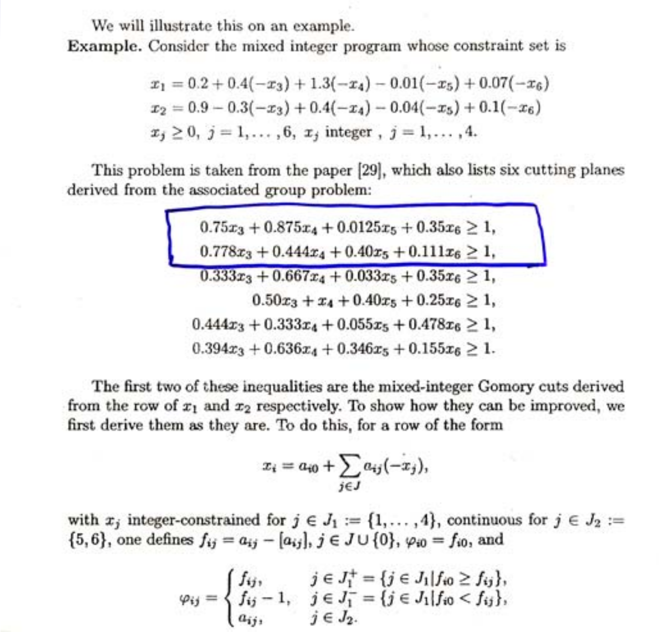
$$\begin{array}{rcl} (1 - x_j)(\widetilde{A}x - \widetilde{b}) & \geq & 0 \\ x_j(\widetilde{A}x - \widetilde{b}) & \geq & 0. \end{array} \tag{1}$$

2. Linearize (1) by substituting y_i for $x_i x_j$, $i = 1, \ldots, n$, $i \neq j$, and $y_j = x_j$ for x_j^2.
3. Project the resulting polyhedron onto the x-space.

Theorem. (Balas, Ceria, and Cornuéjols, 1993) The outcome of steps 1, 2, 3 is

$$\operatorname{conv}\left(P \cap \{x : x_j \in \{0, 1\}\}\right).$$

Corollary. (Balas, Ceria, and Cornuéjols, 1993) Repeating steps 1, 2, 3 for each $j \in \{1, \ldots, p\}$ yields P_D.



We will illustrate this on an example.

Example. Consider the mixed integer program whose constraint set is

$$\begin{aligned} x_1 &= 0.2 + 0.4(-x_3) + 1.3(-x_4) - 0.01(-x_5) + 0.07(-x_6) \\ x_2 &= 0.9 - 0.3(-x_3) + 0.4(-x_4) - 0.04(-x_5) + 0.1(-x_6) \\ x_j &\geq 0,\ j = 1, \ldots, 6,\ x_j \text{ integer },\ j = 1, \ldots, 4. \end{aligned}$$

This problem is taken from the paper [29], which also lists six cutting planes derived from the associated group problem:

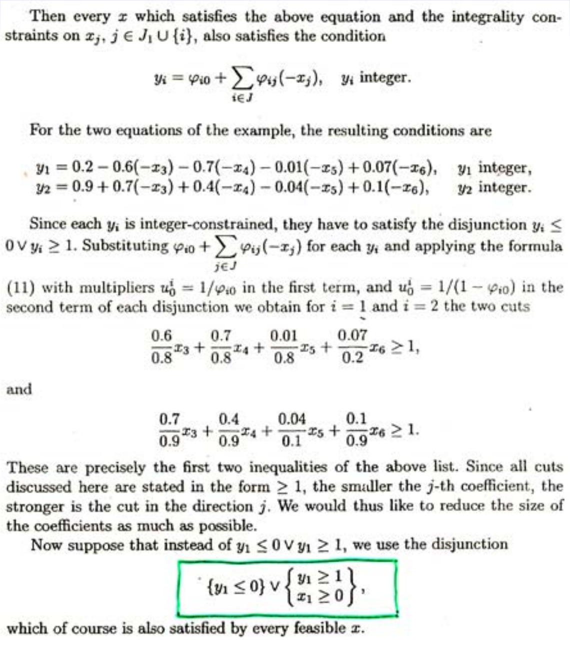
$$\begin{aligned} 0.75x_3 + 0.875x_4 + 0.0125x_5 + 0.35x_6 &\geq 1, \\ 0.778x_3 + 0.444x_4 + 0.40x_5 + 0.111x_6 &\geq 1, \\ 0.333x_3 + 0.667x_4 + 0.033x_5 + 0.35x_6 &\geq 1, \\ 0.50x_3 + x_4 + 0.40x_5 + 0.25x_6 &\geq 1, \\ 0.444x_3 + 0.333x_4 + 0.055x_5 + 0.478x_6 &\geq 1, \\ 0.394x_3 + 0.636x_4 + 0.346x_5 + 0.155x_6 &\geq 1. \end{aligned}$$

The first two of these inequalities are the mixed-integer Gomory cuts derived from the row of x_1 and x_2 respectively. To show how they can be improved, we first derive them as they are. To do this, for a row of the form

$$x_i = a_{i0} + \sum_{j \in J} a_{ij}(-x_j),$$

with x_j integer-constrained for $j \in J_1 := \{1, \ldots, 4\}$, continuous for $j \in J_2 := \{5, 6\}$, one defines $f_{ij} = a_{ij} - [a_{ij}]$, $j \in J \cup \{0\}$, $\varphi_{i0} = f_{i0}$, and

$$\varphi_{ij} = \begin{cases} f_{ij}, & j \in J_1^+ = \{j \in J_1 | f_{i0} \geq f_{ij}\}, \\ f_{ij} - 1, & j \in J_1^- = \{j \in J_1 | f_{i0} < f_{ij}\}, \\ a_{ij}, & j \in J_2. \end{cases}$$



Then every x which satisfies the above equation and the integrality constraints on x_j, $j \in J_1 \cup \{i\}$, also satisfies the condition

$$y_i = \varphi_{i0} + \sum_{i \in J} \varphi_{ij}(-x_j), \quad y_i \text{ integer.}$$

For the two equations of the example, the resulting conditions are

$$\begin{aligned} y_1 &= 0.2 - 0.6(-x_3) - 0.7(-x_4) - 0.01(-x_5) + 0.07(-x_6), \quad && y_1 \text{ integer}, \\ y_2 &= 0.9 + 0.7(-x_3) + 0.4(-x_4) - 0.04(-x_5) + 0.1(-x_6), \quad && y_2 \text{ integer}. \end{aligned}$$

Since each y_i is integer-constrained, they have to satisfy the disjunction $y_i \leq 0 \vee y_i \geq 1$. Substituting $\varphi_{i0} + \sum_{j \in J} \varphi_{ij}(-x_j)$ for each y_i and applying the formula (11) with multipliers $u_0^i = 1/\varphi_{i0}$ in the first term, and $u_0^i = 1/(1-\varphi_{i0})$ in the second term of each disjunction we obtain for $i = 1$ and $i = 2$ the two cuts

$$\frac{0.6}{0.8}x_3 + \frac{0.7}{0.8}x_4 + \frac{0.01}{0.8}x_5 + \frac{0.07}{0.2}x_6 \geq 1,$$

and

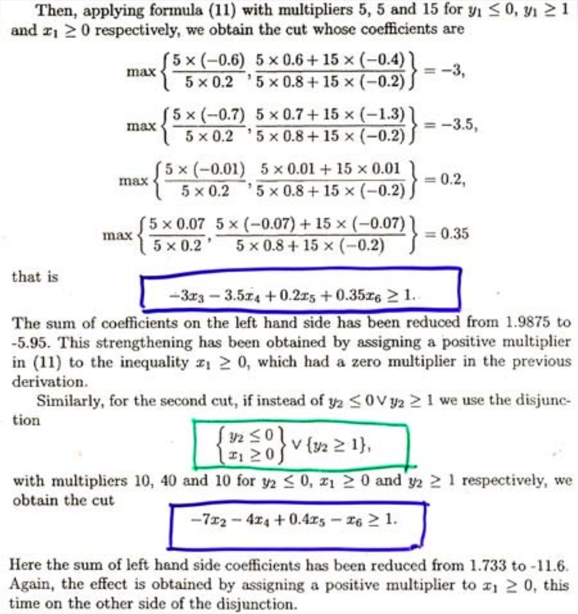
$$\frac{0.7}{0.9}x_3 + \frac{0.4}{0.9}x_4 + \frac{0.04}{0.1}x_5 + \frac{0.1}{0.9}x_6 \geq 1.$$

These are precisely the first two inequalities of the above list. Since all cuts discussed here are stated in the form ≥ 1, the smaller the j-th coefficient, the stronger is the cut in the direction j. We would thus like to reduce the size of the coefficients as much as possible.

Now suppose that instead of $y_1 \leq 0 \vee y_1 \geq 1$, we use the disjunction

$$\boxed{\{y_1 \leq 0\} \vee \left\{ \begin{matrix} y_1 \geq 1 \\ x_1 \geq 0 \end{matrix} \right\},}$$

which of course is also satisfied by every feasible x.



Then, applying formula (11) with multipliers 5, 5 and 15 for $y_1 \leq 0$, $y_1 \geq 1$ and $x_1 \geq 0$ respectively, we obtain the cut whose coefficients are

$$\max\left\{\frac{5 \times (-0.6)}{5 \times 0.2}, \frac{5 \times 0.6 + 15 \times (-0.4)}{5 \times 0.8 + 15 \times (-0.2)}\right\} = -3,$$

$$\max\left\{\frac{5 \times (-0.7)}{5 \times 0.2}, \frac{5 \times 0.7 + 15 \times (-1.3)}{5 \times 0.8 + 15 \times (-0.2)}\right\} = -3.5,$$

$$\max\left\{\frac{5 \times (-0.01)}{5 \times 0.2}, \frac{5 \times 0.01 + 15 \times 0.01}{5 \times 0.8 + 15 \times (-0.2)}\right\} = 0.2,$$

$$\max\left\{\frac{5 \times 0.07}{5 \times 0.2}, \frac{5 \times (-0.07) + 15 \times (-0.07)}{5 \times 0.8 + 15 \times (-0.2)}\right\} = 0.35$$

that is

$$\boxed{-3x_3 - 3.5x_4 + 0.2x_5 + 0.35x_6 \geq 1.}$$

The sum of coefficients on the left hand side has been reduced from 1.9875 to -5.95. This strengthening has been obtained by assigning a positive multiplier in (11) to the inequality $x_1 \geq 0$, which had a zero multiplier in the previous derivation.

Similarly, for the second cut, if instead of $y_2 \leq 0 \vee y_2 \geq 1$ we use the disjunction

$$\boxed{\left\{\begin{matrix} y_2 \leq 0 \\ x_1 \geq 0 \end{matrix}\right\} \vee \{y_2 \geq 1\},}$$

with multipliers 10, 40 and 10 for $y_2 \leq 0$, $x_1 \geq 0$ and $y_2 \geq 1$ respectively, we obtain the cut

$$\boxed{-7x_2 - 4x_4 + 0.4x_5 - x_6 \geq 1.}$$

Here the sum of left hand side coefficients has been reduced from 1.733 to -11.6. Again, the effect is obtained by assigning a positive multiplier to $x_1 \geq 0$, this time on the other side of the disjunction.