1)

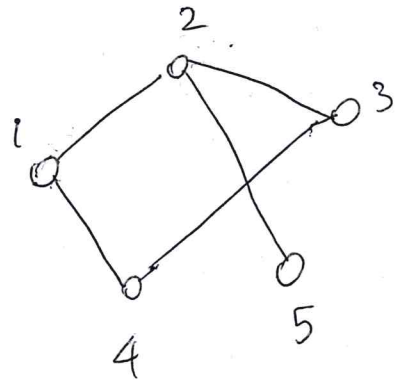
2/10/19

Grafi

$G = (N, A)$

N nodi

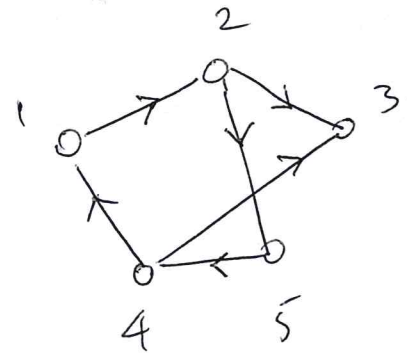
A archi $\quad A \subseteq N \times N$



$A = \{(1,2), (1,4), (2,3), (2,5), (3,4)\}$

$(i,j) \quad i \neq j$

grafo non orientato



$A = \{(1,2), (2,3), (2,5), (4,1), (5,4), (4,3)\}$

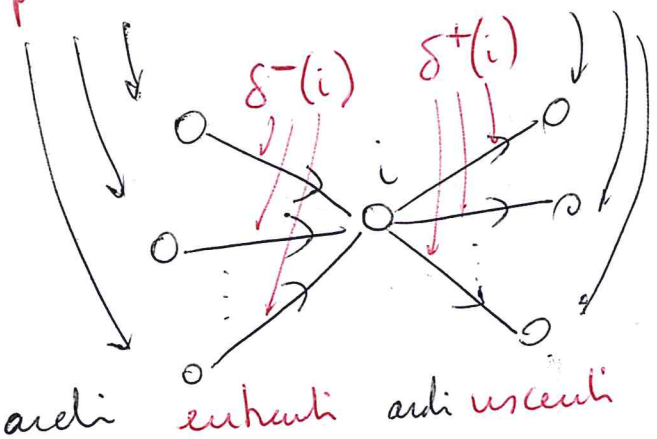
grafo orientato

Grafi non orientati: $\delta(i)$

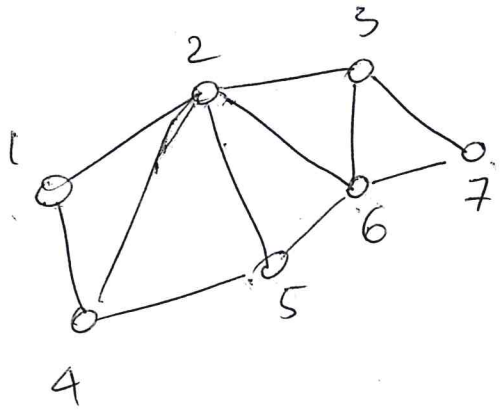
nodi adiacenti: collegati da un arco

Grafi orientati:

nodi predecessori e successori di un nodo



archi entranti archi uscenti



grado di un nodo: numero di archi incidenti il nodo

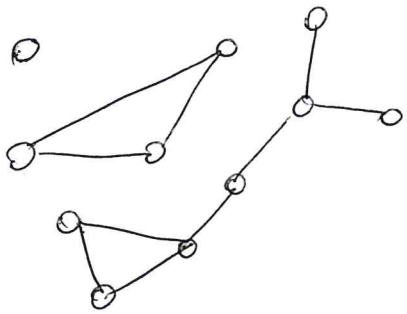
$|\delta(2)| = 4$

$|\delta(3)| = 3$

Grafi orientati:

grado in ingresso $= |\delta^-(i)|$

grado in uscita $= |\delta^+(i)|$



Componente connessa, nodi isolati

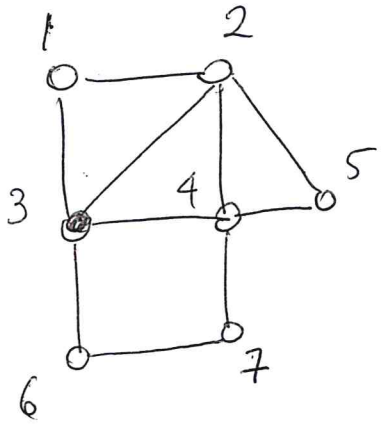
La somma dei gradi di tutti i nodi è un numero pari ("Handshaking lemma")

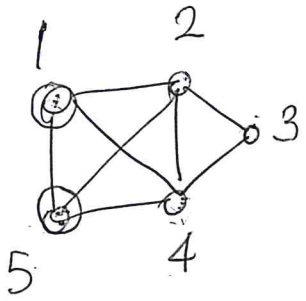
Cammino: in un grafo non orientato, è una successione di archi in cui due archi consecutivi hanno un nodo in comune

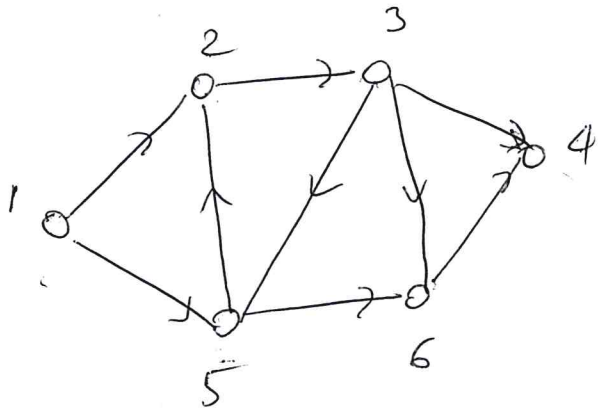
Cammino semplice: cammino che non passa più volte per uno stesso nodo

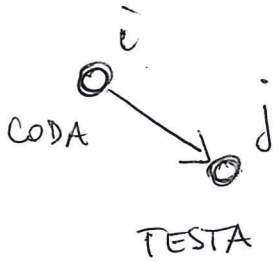
Ciclo : cammino che inizia e termina nello stesso nodo

Ciclo semplice : ciclo che non passa due volte per lo stesso nodo





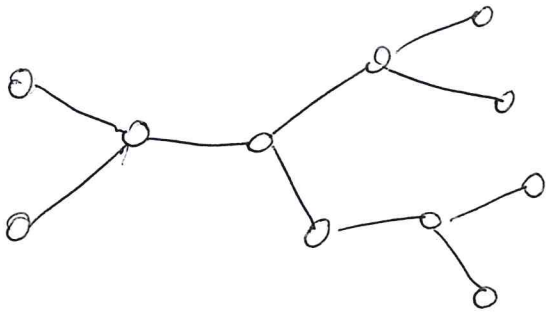




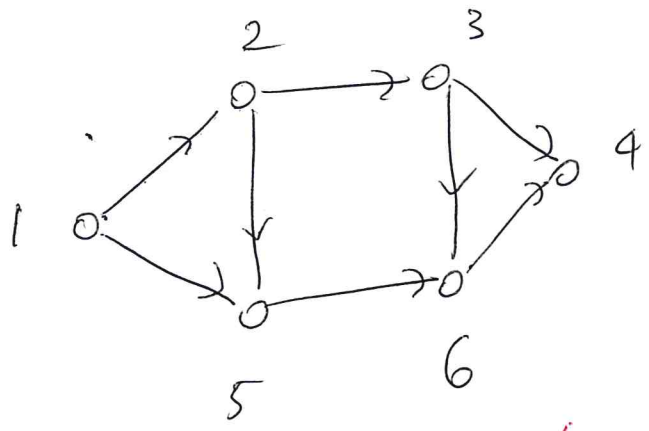
Cammino : successione di archi in cui la testa di un nodo è la coda del successivo

Cammino semplice : cammino che non passa due volte per lo stesso nodo

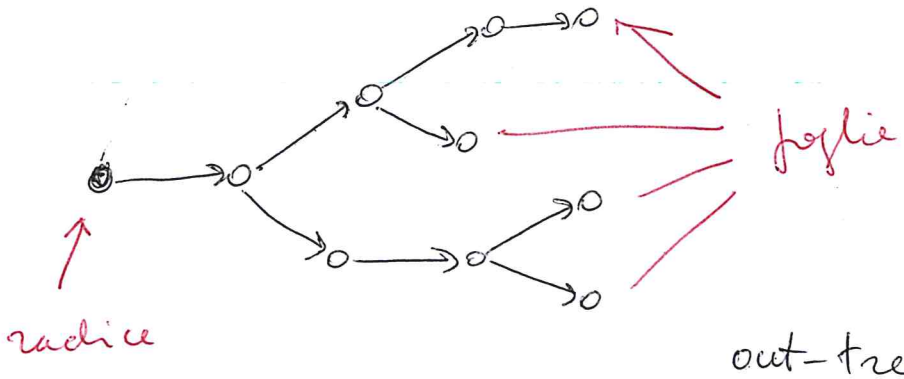
④



Albero : grafo non orientato aciclico

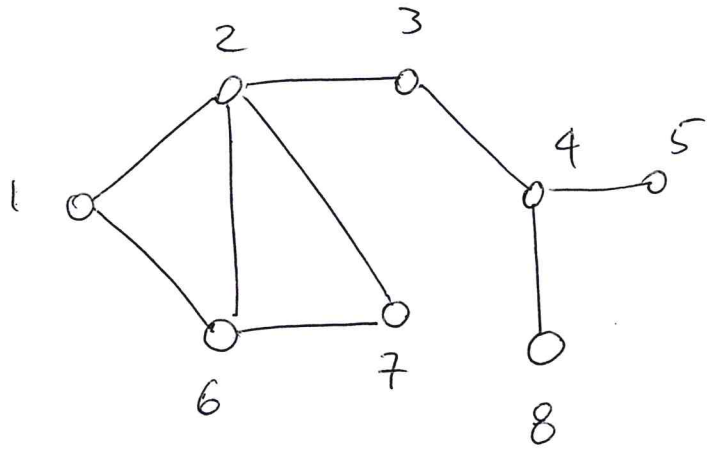


Grafo orientato aciclico (DAG)



out-tree
in-tree

(5)



$$\begin{pmatrix} 0&1&0&0&0&1&0&0 \\ 1&0&1&0&0&1&1&0 \\ 0&1&0&1&0&0&0&0 \\ 0&0&1&0&1&0&0&1 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&1&0&0&0&1&0&0 \\ 0&0&0&1&0&0&0&0 \end{pmatrix}$$

Matrice di adiacenza

$n \times n$

$a_{ij} = 1$ se $(i,j) \in A$

ARCHI

NODI

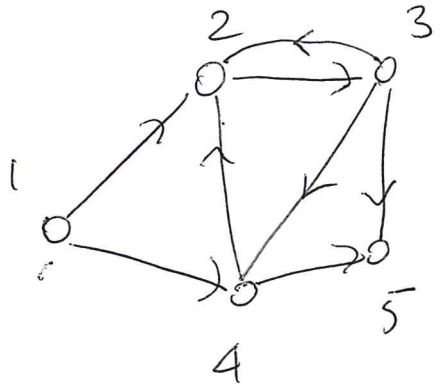
$$\begin{pmatrix} 1&1&0&0&0&0&0&0&0 \\ 1&0&1&1&0&1&0&0&0 \\ 0&0&0&0&0&1&1&0&0 \\ 0&0&0&0&0&0&1&1&1 \\ 0&0&0&0&0&0&0&1&0 \\ 0&1&1&0&1&0&0&0&0 \\ 0&0&0&1&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1 \end{pmatrix}$$

Matrice di incidenza

$n \times m$

$a_{ik} = 1$ se l'arco k è incidente i

(6)



Matrice di adiacenza
$a_{ij}=1$ se $(i,j)\in A$

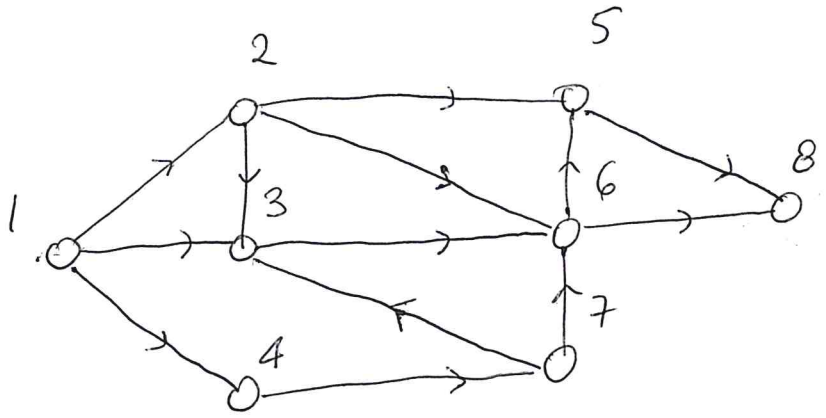
$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrice di incidenza

$a_{ik}=1$ se
i è la coda di k
$a_{ik}=-1$ se
i è la testa di k

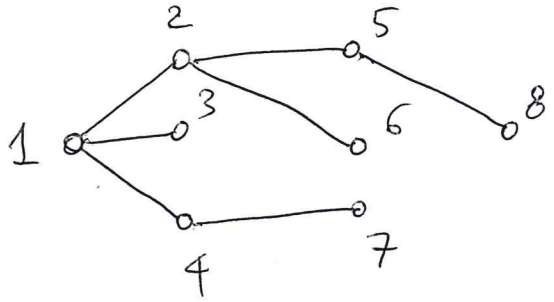
	ARCHI							
NODI	1	1	0	0	0	0	0	0
	-1	0	-1	1	-1	0	0	0
	0	0	0	-1	1	1	1	0
	0	-1	1	0	0	-1	0	1
	0	0	0	0	0	0	-1	-1

(7)



1: 2, 3, 4
2: 3, 5, 6
3: 6
4: ~~8~~, 7
5: 8
6: 5, 8
7: 3, 6
8: ∅

~~1~~ ~~2~~ ~~3~~ ~~4~~ ~~5~~ ~~6~~ ~~7~~ ~~8~~



~~1~~ ~~2~~ ~~3~~ ~~4~~ 5 6 7

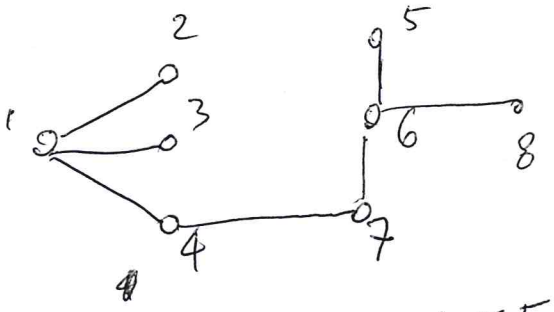
FIFO

Visita in ampiezza

(8)

1 : 2, 3, 4
2 : 3, 5, 6
3 : 6
4 : 7
5 : 8
6 : 5, 8
7 : 3, 6
8 : ∅

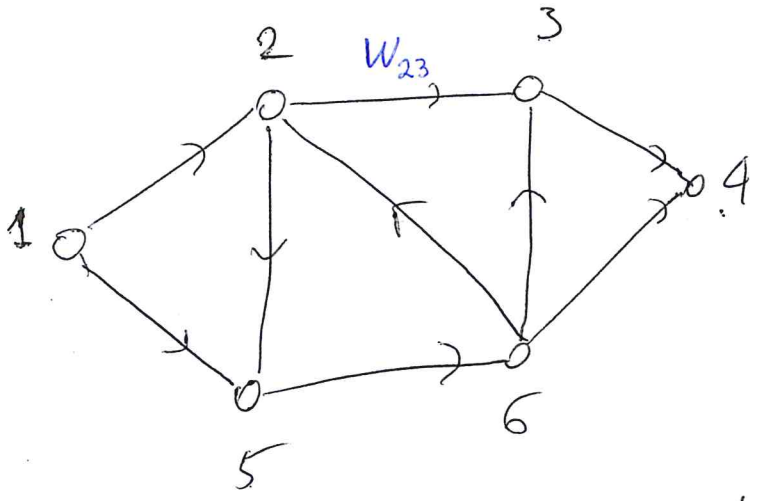
~~1~~ ~~2~~ ~~3~~ ~~4~~ ~~7~~ ~~6~~ ~~5~~ ~~8~~



LIFO

Visita in profondità

(9)



W_{ij} : peso dell'arco (i,j)

Peso di un cammino: somma dei pesi degli archi che lo costituiscono.

Problema del cammino minimo: dato un grafo orientato $G = (N, A)$, dati due nodi s e t, trovare il cammino da s a t di peso minimo.

Come si può formulare in termini di PL?

$$X_{ij} = \begin{cases} 1 & \text{se } (i,j) \in P \\ 0 & \text{se } (i,j) \notin P \end{cases}$$

Variabili binarie